$K^0 - \bar{K}^0$ mixing in the SM and beyond from $N_f = 2$ tmQCD

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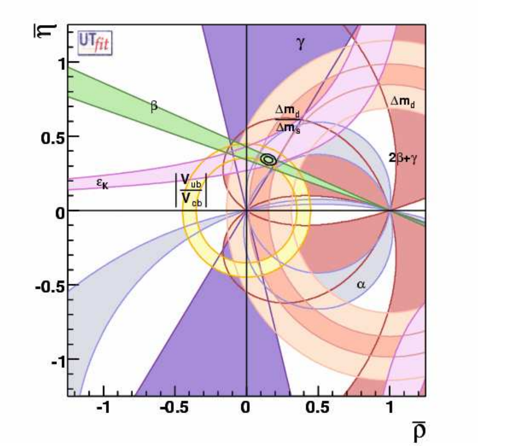
Outline

- B_K (SM)
  - ϵ_K & B_K
  - mixed (tm-OS) Wilson action:
    $\mathcal{O}(a)$-improved calculation *without*
    unwanted mixing operator under renormalisation
  - Calculation & Results
- $\bar{K}^0 - K^0$ beyond SM
  - set-up
  - Calculation & Results (Preliminary)
- Conclusions

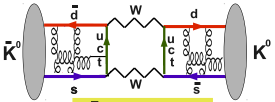
Generalities



- ϵ_K measured experimentally to sub percent precision; constrains the apex of the CKM UT
- B_K - bag parameter which parametrizes the strong interaction effects in the neutral K oscillations

Generalities

- Define $|K_{S,L}\rangle = \frac{1}{\sqrt{2(1+|\bar{\epsilon}|^2)}}[(1+\bar{\epsilon})|K^0\rangle \mp (1-\bar{\epsilon})|\bar{K}^0\rangle]$
- Experiment : $\epsilon_{\rm K} \equiv \frac{2\eta_{+-} + \eta_{00}}{3}$ with $\eta_{ij} = \frac{\mathcal{A}(K_L \to \pi^i\pi^j)}{\mathcal{A}(K_S \to \pi^i\pi^j)}$
- $Re(\epsilon_{\rm K}) = Re(\bar{\epsilon}) = cos\phi_\epsilon sin\phi_\epsilon \left[\frac{Im\ M_{12}}{\Delta m_K} + \xi\right] \Longrightarrow \epsilon_{\rm K} = e^{i\phi_\epsilon} sin\phi_\epsilon \left[\frac{Im\ M_{12}}{\Delta m_K} + \xi\right]$



  - ϕ_ϵ, $\Delta m_K \Longleftarrow$ experiment
  - $Im\ M_{12}$, ξ: calculated in SM

- $\epsilon_{\rm K} = \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{\sqrt{2}}\left[\frac{Im\ M_{12}}{\Delta m_K}\right]$ $\kappa_\epsilon = 0.94(2)$ (until 2008, $\kappa_\epsilon = 1$)

(Buras & Guadagnoli, 2008; Buras, Guadagnoli, Isidori, 2010)

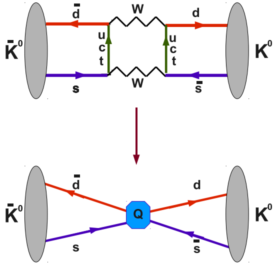
Generalities

- Calculation using the effective hamiltonian

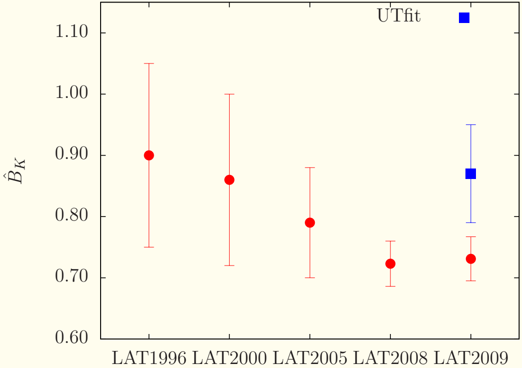
$$M_{12} \sim \langle \bar{K}^0 | H_{\rm eff}^{\Delta S=2} | K^0 \rangle = C(\mu) \langle \bar{K}^0 | \underbrace{(\bar{s}\gamma_\mu^L d)(\bar{s}\gamma_\mu^L d)}_{\mathcal{Q}(\mu)} | K^0 \rangle$$

- $\epsilon_K^{\rm SM} = \kappa_\epsilon \, C_\epsilon \, \hat{B}_{\rm K} \, |V_{cb}|^2 \lambda^2 \bar{\eta} \times (|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c)$
- $\epsilon_K^{\rm exp} = 2.237(7) \times 10^{-3} \xrightarrow{|V_{cb}|} \hat{B}_{\rm K} = 0.87(8)$

(V. Lubicz, LAT2009, 1004.3473)



evolution of B_K lattice estimate



- 1996 $\longrightarrow$ 2009: $\sigma_{B_K} \sim$ 18% $\longrightarrow$ 4%
- *tension* of $\sim 1.5\sigma$ between lattice and CKM estimate

(*source*: **V. Lubicz, LAT2009**)

B_K & Wilson fermions

- Due to Wilson term (and loss of chirality) B_K calculation is characterised by:
  1. $O(a)$ discretisation effects
  2. Complicated renormalization pattern
- Cure **(2)** using
  - WI and 4-point correlation function

    (D. Becirevic, P. Boucaud, V. Gimenez, V. Lubicz and M. Papinutto, Eur.Phys.J. 2004)
  - combinations of Wilson quarks with various combinations of twisted angle
    (ALPHA coll, P. D., J. Heitger, F. Palombi, C. Pena, S. Sint and A. Vladikas, NPB 2006)
- Cure **(1)** using
  - Symanzik program and include dim-7 counterterms $\longrightarrow$ large uncertainties
  - Mtm QCD
- Cure both **(1)** and **(2)** employing
  - Mtm QCD & Mixed action

B_K by ETMC :

an $O(a)$-improved calculation with

simplified renormalisation pattern

in collaboration with:

R. Frezzotti, V. Gimenez, V. Lubicz, F. Mescia,

GC. Rossi, S. Simula, M. Papinutto, A. Vladikas

Mixed action (Frezzotti-Rossi, JHEP 2004)

$$S = S^{\rm tlSym}_{YM} + S^{\rm Mtm}_{\psi,\rm sea} + S^{\rm OS}_{q_f,\rm valence} + S_{\rm gh,valence}$$

with Mtm $N_f = 2$ sea action:

$$S^{\rm Mtm}_{\psi,\rm sea} = a^4 \sum_x \bar{\psi}(x) \left(\gamma\tilde{\nabla} - i\gamma_5\,\tau_3\,W_{cr} + \mu_{sea}\right)\psi(x)$$

and $W_{cr} = -\frac{a}{2} r\nabla^*\nabla + M_{cr}(r)$

Using the non-anomalous chiral transformation for the light sea doublet

$$\psi = exp(i\pi\gamma_5\tau_3/4)\chi \quad \bar{\psi} = \bar{\chi}exp(i\pi\gamma_5\tau_3/4)$$

pass from the *physical* to the *twisted* basis

$$S^{\rm tm}_{\chi,\rm sea} = a^4 \sum_x \bar{\chi}(x)\left(\gamma\tilde{\nabla} - W_{cr} + i\gamma_5\tau_3\mu_{sea}\right)\chi(x)$$

Mtm version of Wilson fermions enjoys desired chiral properties (for the charged pion) due to the chirally rotated W_{cr} wrt quark mass.

Mixed action (Frezzotti-Rossi, JHEP 2004)

Valence action *à* la Osterwalder-Seiler

$$S^{\rm OS}_{q_f,\rm valence} = a^4 \sum_x \sum_f \bar{q}_f(x) \left(\gamma\tilde{\nabla} - i\gamma_5\, r_f\, W_{cr} + \mu_f\right) q_f(x)$$

and $\quad W_{cr} = -\frac{a}{2}\nabla^*\nabla + M_{cr}(r_f; r_{\rm sea})$

q_f are single quarks with chiral transformation: $q_f = exp(i\pi\gamma_5 r_f/4)q_f^{'}$.

- isospin restoration
- $O(a^2)$-unitarity violations

Partial Quenched set-up: $(q_1 = d,\ q_2 = s,\ q_3 = d^{'},\ q_4 = s^{'})$

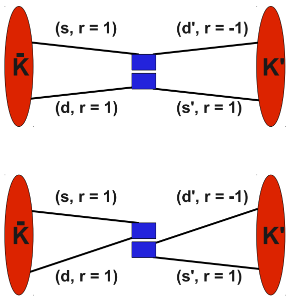
- $M_0^{\rm sea} = M_0^f = M_{cr}(r_f = \pm 1; 1)$
- $\mu^u_{\rm sea} = \mu^d_{\rm sea} = \mu_d = \mu_{d'}$
- $\mu_s = \mu_{s'}$
- $r^u_{\rm sea} = -r^d_{\rm sea} = -r_{d'} = r_d = \pm 1$
- $r_s = r_{s'} = \pm 1$

Calculate the 3-point correlator

$$C_{K'QK}(z_0 - x_0, z_0 - y_0) = \sum_{\bar{x},\bar{z},\bar{y}} \langle (\bar{d}' \gamma_5 s')|_x \, Q^{\Delta S=2}_{VV+AA}|_z \, (\bar{d}\gamma_5 s)|_y \rangle$$

with the 4-fermion operator

$$Q^{\Delta S=2}_{VV+AA} = 2\{(\bar{s}\gamma_\mu d)(\bar{s}'\gamma_\mu d')(\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}'\gamma_\mu\gamma_5 d') + (\bar{s}\gamma_\mu d')(\bar{s}'\gamma_\mu d)(\bar{s}'\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d')\}$$



- $\Phi_{K'} = \bar{d}'\gamma_5 s' \quad (-r_{d'} = r_{s'} = 1)$ **(tm-like)**
  $\Phi_K = \bar{d}\gamma_5 s \quad (r_d = r_s = 1)$ **(OS-like)**

  - **At maximal twist**
    - $O(a)$-improvement of ME
    - Continuum-like renormalization *i.e* no operator mixing

R_{B_K}

$$R_{B_K} = \frac{3}{8}\frac{C_{\mathrm{K}'\mathrm{QK}}(z_0-x_0, z_0-y_0)}{C^{(2)}_{K'}(z_0-x_0)C^{(2)}_{K}(z_0-y_0)} \qquad [\text{with } \mathrm{x}_0-\mathrm{y}_0 = \frac{\mathrm{T}}{2}]$$

$$= \frac{3}{8}\frac{C_{\mathrm{K}'\mathrm{QK}}(z_0-x_0, z_0-y_0)}{(\Phi_{K'}A_0)(z_0-x_0)|^{\mathrm{tm}}\ (A_0\Phi_K)(z_0-y_0)|^{\mathrm{OS}}}$$

$$\xrightarrow{x_0 \ll z_0 \ll y_0} \frac{3}{8}\frac{\langle \bar{K}|Q^{\Delta S=2}_{VV+AA}|K'\rangle}{(f_{K'}m_{K'})^{\mathrm{tm}}\ (f_K m_K)^{\mathrm{OS}}}\ [\frac{Z_Q}{Z_V\ Z_A}]$$

$$= B_K$$

- $m_K - m_{K'} = \mathcal{O}(a^2)$
- Factors $e^{-m_{K'}(z_0-x_0)}$ and $e^{-m_K(T/2-z_0+y_0)}$ cancel between numerator and denominator.
- Same for factors $m_{K'}$ and m_K

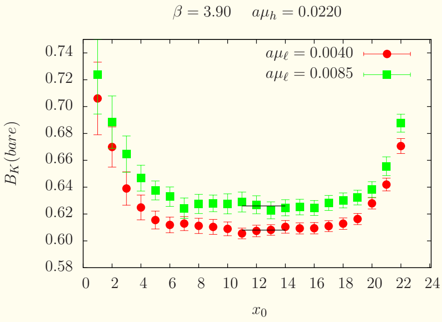
Simulation Data

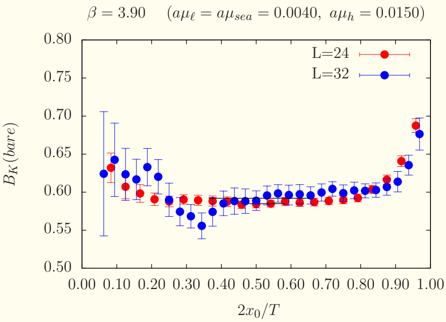
β	$a^{-4}(L^3 \times T)$	$a\mu_{\rm sea} = a\mu_\ell$	$a\mu_h$	$N_{\rm meas}$
3.80	$24^3 \times 48$	0.0080	0.0165, 0.0200, 0.0250	170
($a \sim 0.10$ fm)	"	0.0110	"	180
3.90	$24^3 \times 48$	0.0040	0.0150, 0.0220, 0.0270	400
($a \sim 0.085$ fm)	"	0.0064	"	200
	"	0.0085	"	200
	"	0.0100	"	150
	$32^3 \times 64$	0.0040	"	160
	"	0.0030	"	300
4.05	$32^3 \times 64$	0.0030	0.0120, 0.0150, 0.0180	200
($a \sim 0.07$ fm)	"	0.0060	"	150
	"	0.0080	"	220

280 MeV $\lesssim \; M_{\rm PS}^{\ell\ell} \; \lesssim$ 520 MeV $\qquad M_\pi L \gtrsim 3.3$

450 MeV $\lesssim \; M_{\rm PS}^{\ell h} \; \lesssim$ 670 MeV

Signal quality





- Typical error of bare

  $B_K \sim 0.5 - 1.2\%$

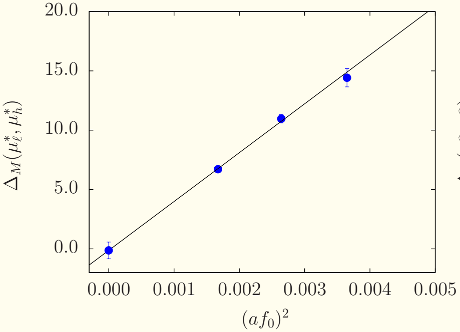
- $B_K^{\rm bare}(L = 24) = 0.585(5)$

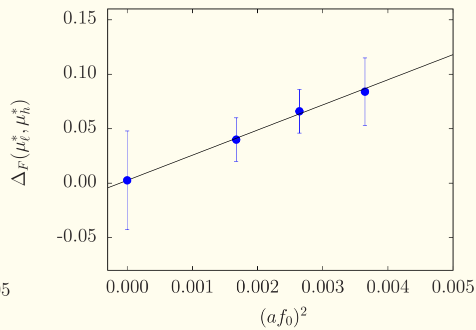
  vs.

  $B_K^{\rm bare}(L = 32) = 0.592(5)$

tm vs. OS: O(a²) discr. effects

- $\Delta_{\rm M} = [M_{\rm ps}^{OS}]^2 - [M_{\rm ps}^{tm}]^2$
- $\Delta_{\rm F} = |F_{\rm ps}^{OS} - F_{\rm ps}^{tm}|$





- calculation at (μ_ℓ^*, μ_h^*) (fixed for all β) renormalised q-masses & in f_0 units.

Renormalization Constants

- RI-MOM method

β	Z_4^{RGI} (M1)	Z_4^{RGI} (M2)	Z_A (M1)	Z_A (M2)	Z_V (WI)
3.80	0.591(18)	0.616(11)	0.746(11)	0.727(07)	0.5816(02)
3.90	0.617(10)	0.633(06)	0.746(06)	0.730(03)	0.6103(03)
4.05	0.693(10)	0.694(06)	0.772(06)	0.758(04)	0.6451(03)

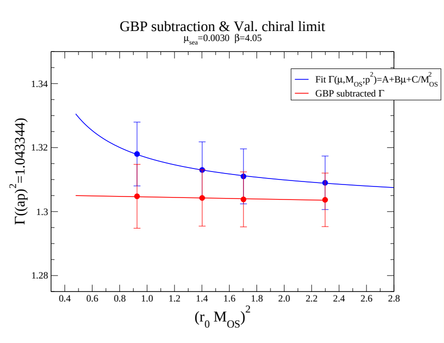
- (M1): $\mathcal{O}(a^2p^2)$ fitted.
- (M2): $\mathcal{O}(a^2p^2)$ not fitted.

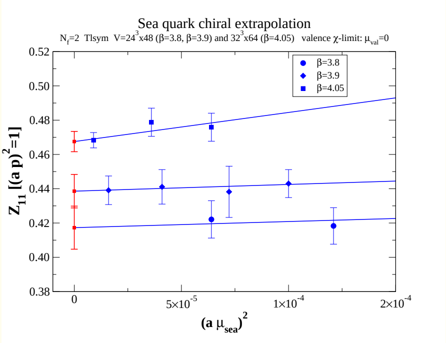
4-f RCs: valence & sea chiral limit

- GBP subtraction

$$\Gamma_{\mathcal{O}}(\mu, M_{\rm ps}; p^2) = A + B\mu + C/[M_{\rm ps}^{\rm OS}]^2 \quad \textit{for } \mathcal{O}_{n=1}$$

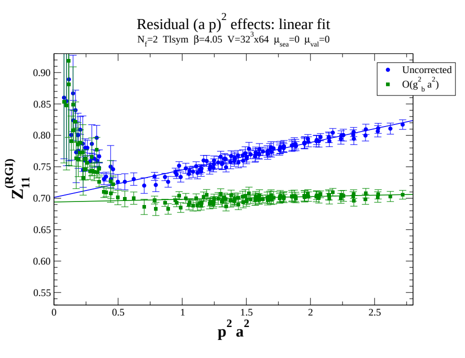
$$\Gamma_{\mathcal{O}}(\mu, M_{\rm ps}; p^2) = A' + B'\mu + C'/([M_{\rm ps}^{\rm tm}]^2 + [M_{\rm ps}^{\rm OS}]^2) \quad \textit{for } \mathcal{O}_{n>1}$$

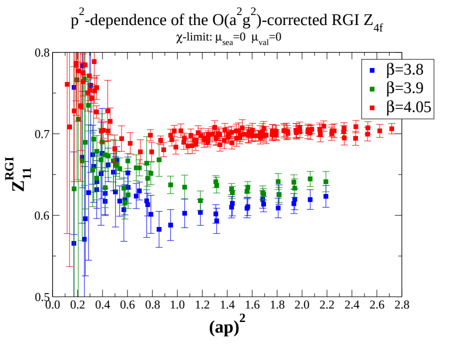




4-f RCs:

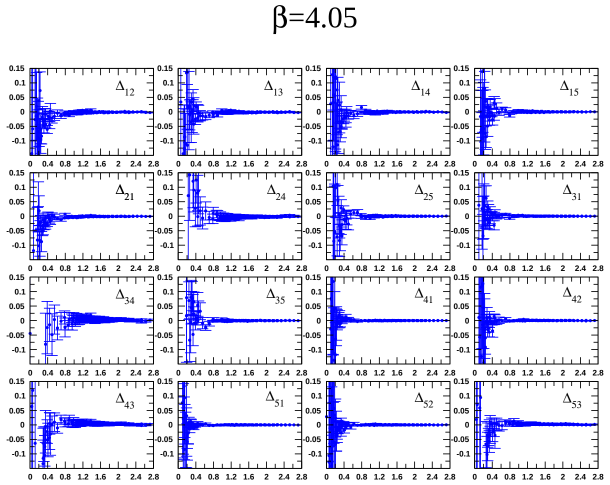
- Z_{11} vs. $(ap)^2$





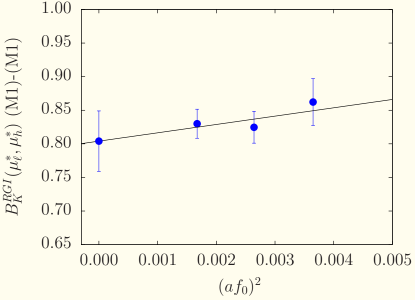
4-f Renormalization Constants

- Operator Mixing

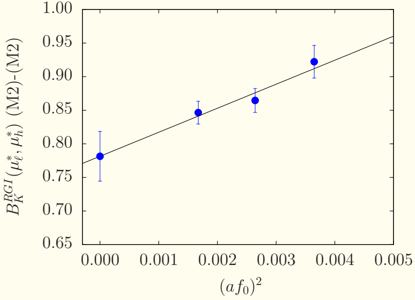


"B_K^{RGI}" at fixed (ℓ, h) q-masses

'M1':RI-MOM RCs with $\mathcal{O}(a^2p^2)$ fitted



'M2': RI-MOM RCs with $\mathcal{O}(a^2p^2)$ not fitted



- calculation at $(\mu_\ell^*, \mu_h^*) \sim (40, 90)$ MeV.
- cut-off effects: 'M1': $\sim$ 7% 'M2': $\sim$ 15%

B_K @ (physical point & CL)

- Two methods:
  - Use q-masses: need to know $\mu_{u/d}$ and μ_s at CL
    (i) interpolate B_K to μ_s; obtain $B_K(\mu_\ell, \mu_s)$
    (ii) extrapolate to $\mu_{u/d} \rightarrow B_K(\mu_{u/d}, \mu_s)$
  - Use pseudoscalar masses:
    (i) interpolate B_K to a number of reference pseudoscalar masses M^2_{hh}
    (ii) extrapolate $B_K(M^2_{\ell\ell}, M^2_{hh})$ to the physical point M^2_π
    (iii) interpolate to $M^2_{ss} = 2M^2_K - M^2_\pi \rightarrow B_K(M^2_\pi, M^2_{ss})$
- Combined chiral + continuum fits

B_K @ (physical point & CL)

- $SU(2)$ Chiral fit formula

  (C.Allton et al, 2008; Sharpe & Yang, 1996)

$$B_K(\mu_{u/d},\mu_h) = B_\chi(\mu_h)\left[1 + b(\mu_h)\frac{2B_0}{f_0^2}\mu_\ell - \frac{2B_0\mu_\ell}{32\pi^2 f_0^2}\log\frac{2B_0\mu_\ell}{\Lambda_\chi^2}\right]$$

or

$$B_K(M_\pi^2, M_{hh}^2) = B'_\chi(M_{hh}^2)\left[1 + b'(M_{hh}^2)\frac{M_{\ell\ell}^2}{f_0^2} - \frac{M_{\ell\ell}^2}{32\pi^2 f_0^2}\log\frac{M_{\ell\ell}^2}{\Lambda_\chi^2}\right]$$

- Fit the discretization effects

scale inputs for the physical point

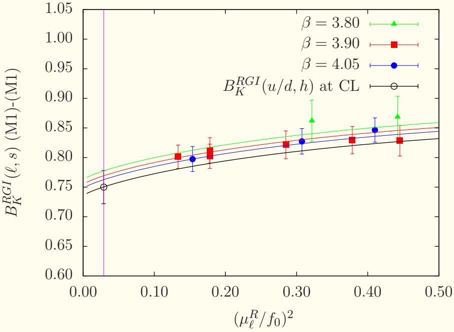
- Use r_0 or f_0 for setting the scale

  (calculated by a scaling analysis in the light sector)

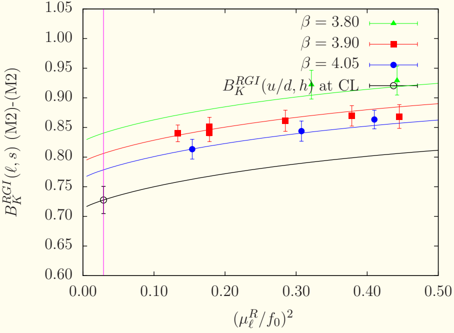
$\Longrightarrow$ Verify the *non* dependence (in practice) of B_K ($\leftarrow$ dimensionless quantity)

Fits wrt q-mass (using scale from f_0)

RI-MOM RCs with $\mathcal{O}(a^2p^2)$ fitted

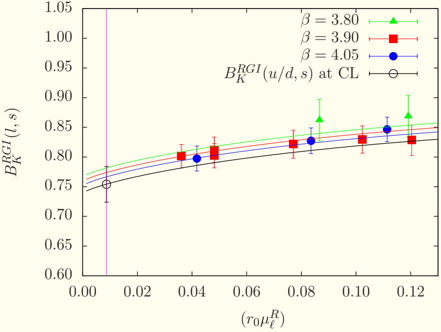


RI-MOM RCs with $\mathcal{O}(a^2p^2)$ not fitted

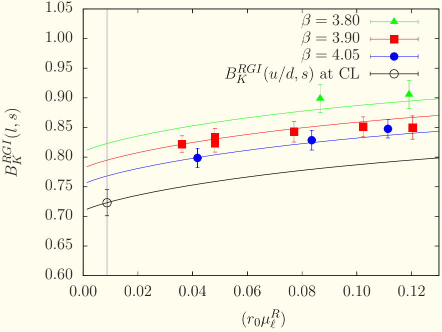


Fits wrt q-mass (using scale from r_0)

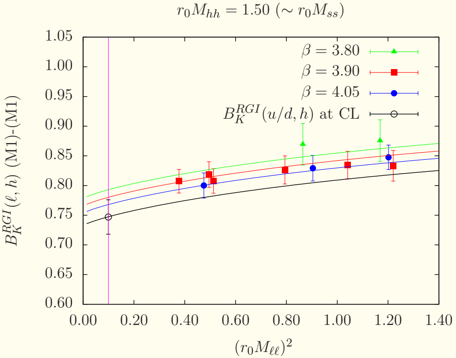
RI-MOM RCs with $\mathcal{O}(a^2p^2)$ fitted

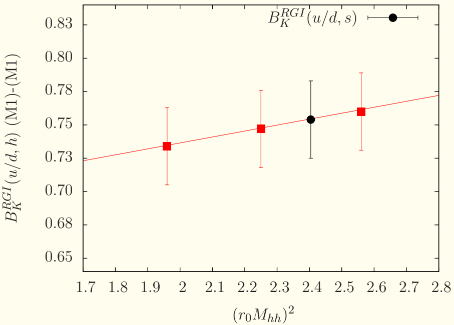


RI-MOM RCs with $\mathcal{O}(a^2p^2)$ not fitted



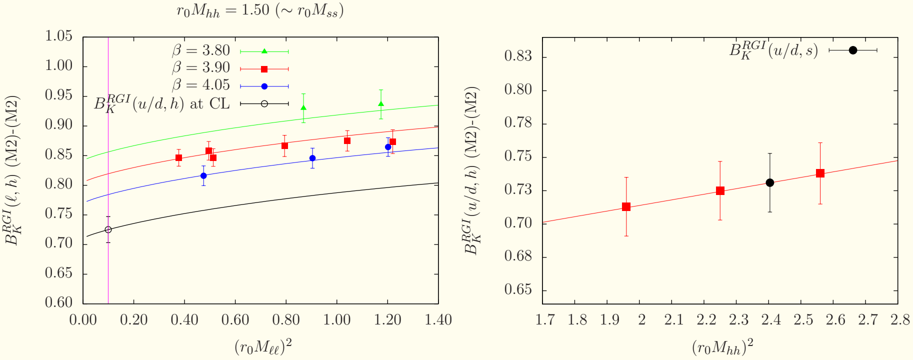
Fits wrt pseudoscalar masses and RI-MOM RCs with $\mathcal{O}(a^2p^2)$ fitted





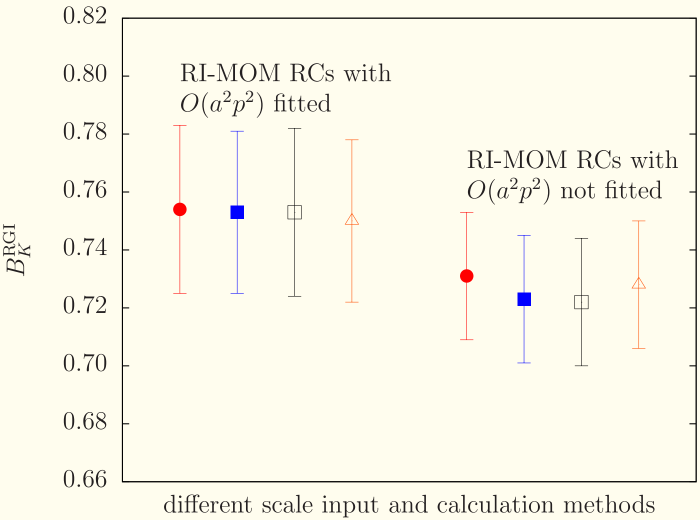
physical point at: $M_{hh}^2 \equiv M_{ss}^2 = 2M_K^2 - M_\pi^2$

Fits wrt pseudoscalar masses and RI-MOM RCs with $\mathcal{O}(a^2p^2)$ not fitted



physical point at: $M_{hh}^2 \equiv M_{ss}^2 = 2M_K^2 - M_\pi^2$

B_K @ (physical point & CL)



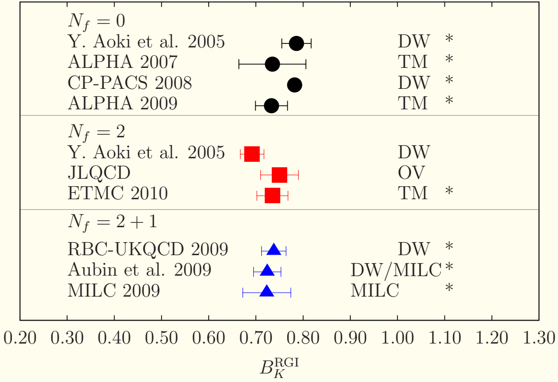
B_K @ (physical point & CL): Result

$$B_K^{\rm RGI} = 0.733(29)(16)[33]$$

- 1st error : (B_K-bare + Fit + RCs + scale input) uncertainty
- 2nd error : due to the difference (up to $O(a^2)$) of the RI-MOM RCs.

An alternative counting for the error budget:

- (B_K-bare + Fit) uncertainty $\sim 2\%$
- (RCs + scale input) uncertainty $\sim 2.5\%$
- "systematic" uncertainty due to RI-MOM RCs $\sim 2\%$



(∗): Continuum Limit result

- No significant dependence on N_f dynamical quark flavours!

$K^0 - \bar{K}^0$ oscillation Beyond SM

in collaboration with

V. Bertone, R. Frezzotti, V. Gimenez, V. Lubicz, G. Martinelli, F. Mescia, GC. Rossi, S. Simula, M. Papinutto, A. Vladikas

Including $\Delta S = 2$ Supersymmetric Operators

The effective Hamiltonian takes the form:

$$\mathcal{H}_{\rm eff}^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu)\mathcal{O}_i + \sum_{i=1}^{3} \tilde{C}_i(\mu)\tilde{\mathcal{O}}_i$$

- In SM case only $\mathcal{O}_1$ contributes.

$$\begin{aligned}
\mathcal{O}_1 &= [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1-\gamma_5)d^b] & \tilde{\mathcal{O}}_1 &= [\bar{s}^a\gamma_\mu(1+\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b] \\
\mathcal{O}_2 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b] & \tilde{\mathcal{O}}_2 &= [\bar{s}^a(1+\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b] \\
\mathcal{O}_3 &= [\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1-\gamma_5)d^a] & \tilde{\mathcal{O}}_3 &= [\bar{s}^a(1+\gamma_5)d^b][\bar{s}^b(1+\gamma_5)d^a] \\
\mathcal{O}_4 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b] \\
\mathcal{O}_5 &= [\bar{s}^a(1-\gamma_5)d^b][\bar{s}^b(1-\gamma_5)d^a]
\end{aligned}$$

(Gabrielli et al. 1996; Bagger et al. 1997; Ciuchini et al. 1997,1998)

- Parity-even parts of $\mathcal{O}_i$ and $\tilde{\mathcal{O}}_i'$ coincide.

Lattice basis

- Parity even operators

$$
\begin{aligned}
O^{VV} &= (\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) \\
O^{AA} &= (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d) \\
O^{PP} &= (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \\
O^{SS} &= (\bar{s}d)(\bar{s}d) \\
O^{TT} &= (\bar{s}\sigma_{\mu\nu} d)(\bar{s}\sigma_{\mu\nu} d)
\end{aligned}
$$

- Lattice basis

$$
\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix} = \begin{pmatrix} O^{VV} + O^{AA} \\ O^{VV} - O^{AA} \\ O^{SS} - O^{PP} \\ O^{SS} + O^{PP} \\ O^{TT} \end{pmatrix}
$$

Ss basis

Through Fierz transformation

$$\begin{aligned}
\mathcal{O}_1 &= (O^{VV} + O^{AA}) \\
\mathcal{O}_2 &= (O^{SS} + O^{PP}) \\
\mathcal{O}_3 &= (O^{SS} + O^{PP} - O^{TT})(-\frac{1}{2}) \\
\mathcal{O}_4 &= (O^{SS} - O^{PP}) \\
\mathcal{O}_5 &= (O^{VV} - O^{AA})(-\frac{1}{2})
\end{aligned}$$

- OS-setup brings to a continuum-like renormalisation pattern

$$\begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \\ \mathcal{O}_5 \end{pmatrix}_{\mathrm{REN}} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \\ \mathcal{O}_5 \end{pmatrix}$$

(Donini et al. 1999)

B_i

$$\langle \bar{K}^0|\mathcal{O}_1(\mu)|K^0\rangle = B_1(\mu)\,(8/3)m_K^2 f_K^2 = B_K(\mu)\,(8/3)m_K^2 f_K^2$$

$$\langle \bar{K}^0|\mathcal{O}_2(\mu)|K^0\rangle = B_2(\mu)\left[\frac{m_K^2 f_K}{m_s(\mu)+m_d(\mu)}\right]^2(-5/3)$$

$$\langle \bar{K}^0|\mathcal{O}_3(\mu)|K^0\rangle = B_3(\mu)\left[\frac{m_K^2 f_K}{m_s(\mu)+m_d(\mu)}\right]^2(1/3)$$

$$\langle \bar{K}^0|\mathcal{O}_4(\mu)|K^0\rangle = B_4(\mu)\left[\frac{m_K^2 f_K}{m_s(\mu)+m_d(\mu)}\right]^2(2)$$

$$\langle \bar{K}^0|\mathcal{O}_5(\mu)|K^0\rangle = B_5(\mu)\left[\frac{m_K^2 f_K}{m_s(\mu)+m_d(\mu)}\right]^2(2/3)$$

R_i

- Avoid systematic errors coming from the quark masses:

construct appropriate ratios

$$R_i = \left(\frac{f_K^2}{m_K^2}\right)_{\rm exp} \left[\left(\frac{m_K}{f_K}\right)_{\rm tm} \left(\frac{m_K}{f_K}\right)_{\rm OS} \frac{\langle \bar{K}^0 | \mathcal{O}_i(\mu) | K^0 \rangle}{\langle \bar{K}^0 | \mathcal{O}_1(\mu) | K^0 \rangle} \right] \quad i = 2, \ldots, 5$$

- Then

$$\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle \propto C_1(\mu) \langle \bar{K}^0 | \mathcal{O}_1(\mu) | K^0 \rangle \left(1 + \sum_{2,\ldots,5} \frac{C_i(\mu)}{C_1(\mu)} R_i \right)$$

(Donini et al. 2000)

- Up to now, *only*, quenched results published.

(Allton et al. 1998; Donini et al. 2000; Babich et al. 2006; Nakamura et al. 2006.)

RCs matrix

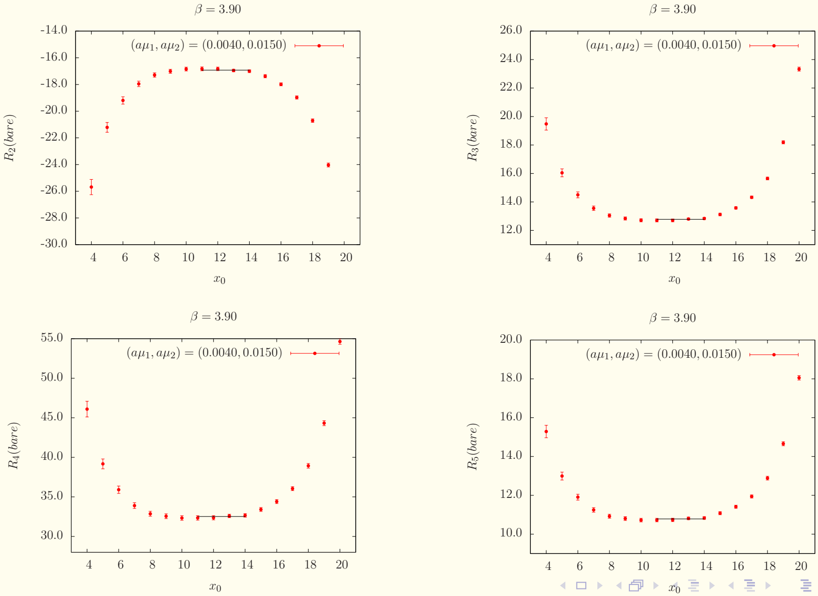
- **RCs matrix in the lattice basis**

$$Z^{\overline{\rm MS}-2{\rm GeV}}(\beta = 3.80) = \begin{pmatrix} 0.420(15) & 0 & 0 & 0 & 0 \\ 0 & 0.488(14) & 0.201(09) & 0 & 0 \\ 0 & 0.024(02) & 0.248(12) & 0 & 0 \\ 0 & 0 & 0 & 0.290(10) & -0.012(02) \\ 0 & 0 & 0 & -0.178(09) & 0.572(15) \end{pmatrix}$$

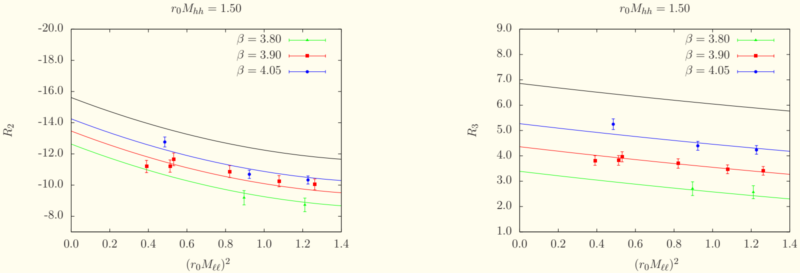
$$Z^{\overline{\rm MS}-2{\rm GeV}}(\beta = 3.90) = \begin{pmatrix} 0.438(08) & 0 & 0 & 0 & 0 \\ 0 & 0.503(08) & 0.200(06) & 0 & 0 \\ 0 & 0.025(02) & 0.271(07) & 0 & 0 \\ 0 & 0 & 0 & 0.313(07) & -0.012(02) \\ 0 & 0 & 0 & -0.206(06) & 0.595(09) \end{pmatrix}$$

$$Z^{\overline{\rm MS}-2{\rm GeV}}(\beta = 4.05) = \begin{pmatrix} 0.491(07) & 0 & 0 & 0 & 0 \\ 0 & 0.550(08) & 0.216(08) & 0 & 0 \\ 0 & 0.026(01) & 0.296(07) & 0 & 0 \\ 0 & 0 & 0 & 0.345(05) & -0.010(01) \\ 0 & 0 & 0 & -0.275(08) & 0.663(11) \end{pmatrix}$$

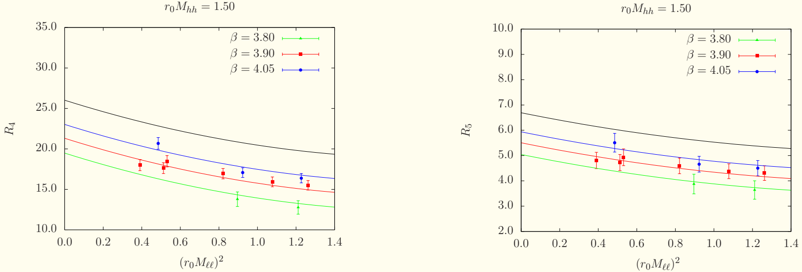
$R_{i=2,\ldots,5}$: signal quality at $\beta = 3.90$



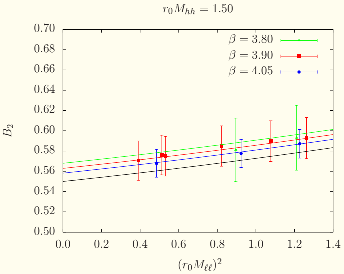
$R_{i=2,\ldots,5}$ for $\beta = 3.80, 3.90, 4.05$

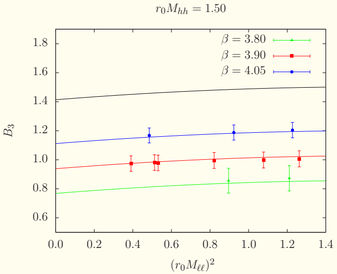


quadratic functions wrt $M^2_{\ell\ell}$ for combined fits + discretization effects

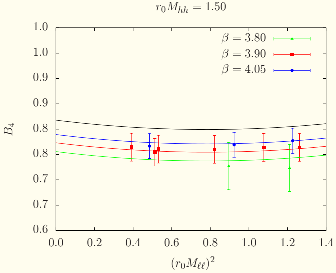


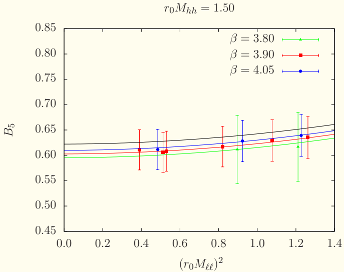
$B_{i=2,\dots,5}$ for $\beta = 3.80, 3.90, 4.05$





quadratic functions wrt $M_{\ell\ell}^2$ for combined fits + discretization effects





Results (preliminary)

fit function wrt $M^2_{\ell\ell}$	i	$\frac{<\mathcal{O}_i>}{<\mathcal{O}_1>}$ from R–method	$\frac{<\mathcal{O}_i>}{<\mathcal{O}_1>}$ from B–method	$B_i^{\overline{\mathrm{MS}}-2\mathrm{GeV}}$
quadratic	2	-14.3(0.8)	-16.6(1.6)	0.520(39)
	3	6.9(0.5)	8.4(0.7)	1.308(83)
	4	25.4(1.7)	30.2(3.2)	0.786(48)
	5	5.4(0.4)	6.5(0.7)	0.504(53)
linear	2	-13.1(0.4)	-16.6(1.2)	0.519(18)
	3	6.3(0.2)	8.4(0.4)	1.304(41)
	4	22.9(0.8)	29.8(2.5)	0.775(24)
	5	4.8(0.2)	6.4(0.5)	0.502(27)

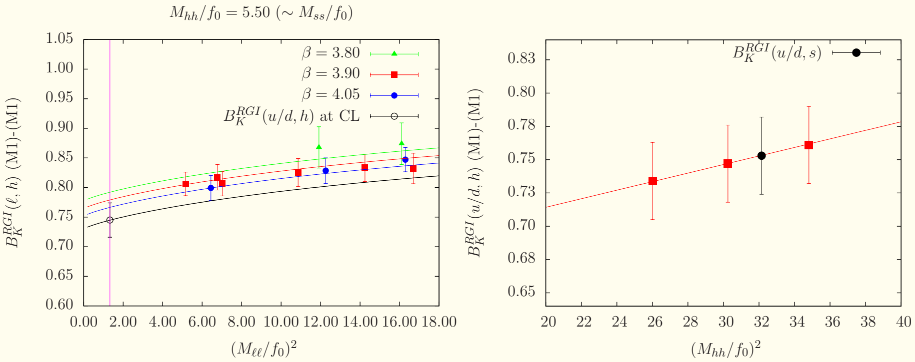
Conclusions

- B_K (SM)
  - $\mathcal{O}(a)$ automatic improvement
  - renormalisation pattern without unwanted mixings
  - ETMC result compatible with B_K-world
  - Total uncertainty $\sim 4\%$
    (apart systematics by the dynamical strange degree of freedom)

- $\bar{K}^0 - K^0$ beyond SM
  - all operators $\mathcal{O}(a)$-improved and continuum-like renormalisation pattern
  - good scaling properties
  - uncertainties of the ratios $\delta R_i \sim 5 - 10\%$

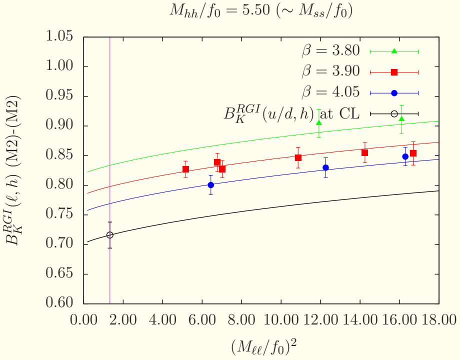
- Calculation with 2+1+1 dynamical flavours already started

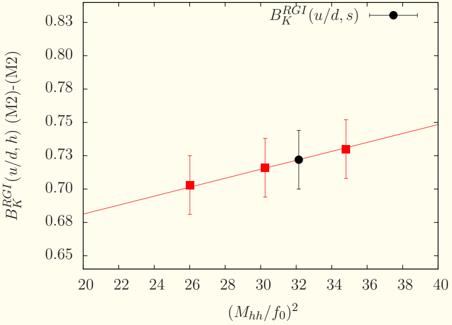
Bckup: Fits wrt pseudoscalar masses and RI-MOM RCs with $\mathcal{O}(a^2p^2)$ fitted - II



physical point at: $M_{hh}^2 \equiv M_{ss}^2 = 2M_K^2 - M_\pi^2$

Bckup: Fits wrt pseudoscalar masses and RI-MOM RCs with $\mathcal{O}(a^2p^2)$ not fitted - II





physical point at: $M_{hh}^2 \equiv M_{ss}^2 = 2M_K^2 - M_\pi^2$