# QCD at LHC

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# QCD

Premio Nobel 2004!

- an unbroken Yang-Mills gauge field theory featuring asymptotic freedom $\longrightarrow$ confinement
- in non-perturbative regime (low $Q^2$) many approaches: lattice, Regge theory, $\chi$ PT, large $N_c$, HQET
- in perturbative regime (high $Q^2$) QCD is a precision toolkit for exploring Higgs & BSM physics
- LEP was an electroweak machine
- Tevatron & LHC are QCD machines

# Precision QCD

Precise determination of

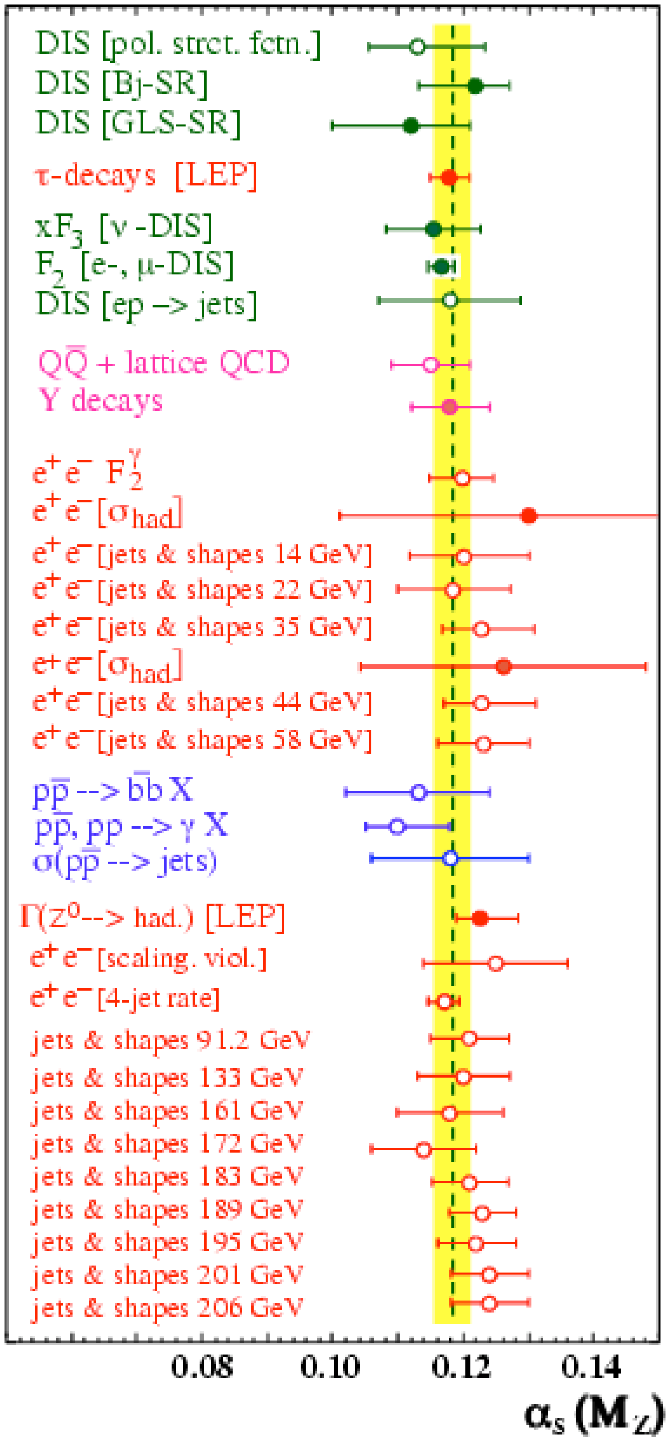
- strong coupling constant $\alpha_s$
- parton distributions
- electroweak parameters
- LHC parton luminosity

Precise prediction for

- Higgs production
- new physics processes
- their backgrounds

# Summary of $\alpha_S(M_Z)$

S. Bethke hep-ex/0407021




filled symbols are NNLO results

world average of $\alpha_S(M_Z)$
using $\overline{\mathrm{MS}}$ and NNLO results only

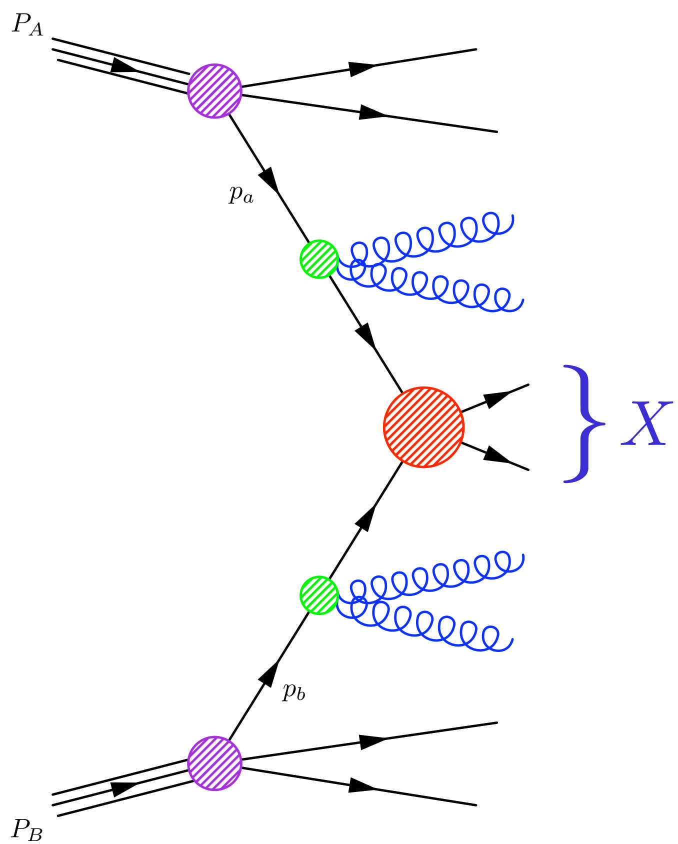
$$\alpha_S(M_Z) = 0.1182 \pm 0.0027$$

(cf. 2002 $\alpha_S(M_Z) = 0.1183 \pm 0.0027$
outcome almost identical
because new entries wrt 2002
- LEP jet shape observables and
4-jet rate, and HERA jet rates
and shape variables - are NLO )

# Strong interactions at high $Q^2$

- Parton model
- Perturbative QCD
  - factorisation
  - universality of IR behaviour
  - cancellation of IR singularities
  - IR safe observables: inclusive rates
    - jets
    - event shapes

# Factorisation




is the separation between
the short- and the long-range interactions

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{a/A}(x_1,\mu_F^2)\, f_{b/B}(x_2,\mu_F^2)$$
$$\times\ \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_F^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

$$X = W, Z, H, Q\bar{Q}, \text{high-}E_T\text{jets}, \ldots$$

$\hat{\sigma}$ is known as a fixed-order expansion in $\alpha_S$

$$\hat{\sigma} = C\alpha_S^n(1 + c_1\alpha_S + c_2\alpha_S^2 + \ldots)$$

$c_1 =$ NLO $\quad c_2 =$ NNLO

or as an all-order resummation

$$\hat{\sigma} = C\alpha_S^n[1 + (c_{11}L + c_{10})\alpha_S + (c_{22}L^2 + c_{21}L + c_{20})\alpha_S^2 + \ldots]$$

where $L = \ln(M/q_T), \ln(1-x), \ln(1/x), \ln(1-T), \ldots$

$c_{11}, c_{22} =$ LL $\quad c_{10}, c_{21} =$ NLL $\quad c_{20} =$ NNLL

# Evolution

- factorisation scale $\mu_F$ is arbitrary
  - cross section cannot depend on $\mu_F$

$$\mu_F \frac{d\sigma}{d\mu_F} = 0$$

implies DGLAP equations

V. Gribov L. Lipatov; Y. Dokshitzer
G. Altarelli G. Parisi

$$\mu_F \frac{df_a(x,\mu_F^2)}{d\mu_F} = P_{ab}(x,\alpha_S(\mu_F^2)) \otimes f_b(x,\mu_F^2) + \mathcal{O}(\frac{1}{Q^2})$$

$$\mu_F \frac{d\hat{\sigma}_{ab}(Q^2/\mu_F^2,\alpha_S(\mu_F^2))}{d\mu_F} = -P_{ac}(x,\alpha_S(\mu_F^2)) \otimes \hat{\sigma}_{cb}(Q^2/\mu_F^2,\alpha_S(\mu_F^2)) + \mathcal{O}(\frac{1}{Q^2})$$

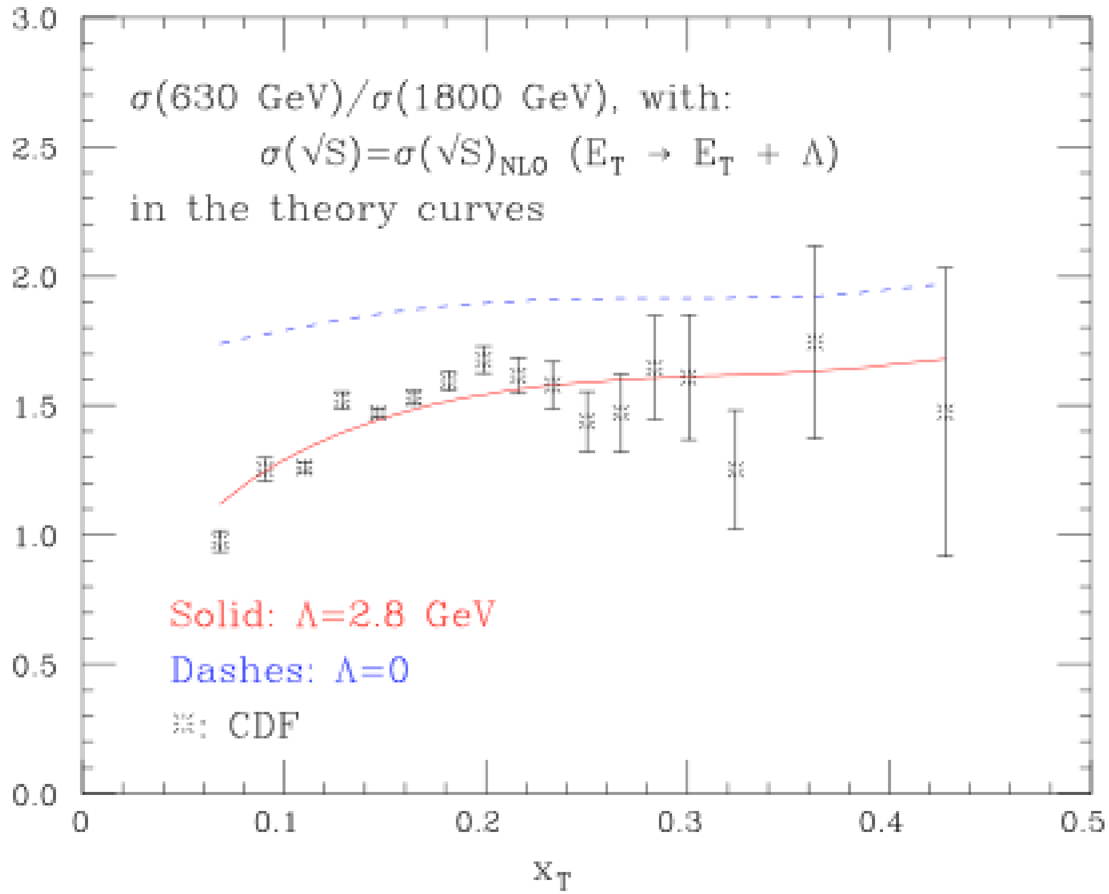
- $P_{ab}(x,\alpha_S(\mu_F^2))$ is calculable in pQCD

# Factorisation-breaking contributions

- underlying event (see Rick Field's studies at CDF)
- power corrections
  - MC's and theory modelling of power corrections laid out and tested at LEP where they provide an accurate determination of $\alpha_S$
    models still need be tested in hadron collisions
    (see e.g. Tevatron studies at different $\sqrt{s}$ ) ⇨
- double-parton scattering
  - observed by Tevatron CDF in the inclusive sample
    $p\bar{p} \rightarrow \gamma + 3$ jets
    potentially important at LHC $\sigma_D \propto \sigma_S^2$
- diffractive events ⇨

# Power corrections at Tevatron

Ratio of inclusive jet cross sections at 630 and 1800 GeV



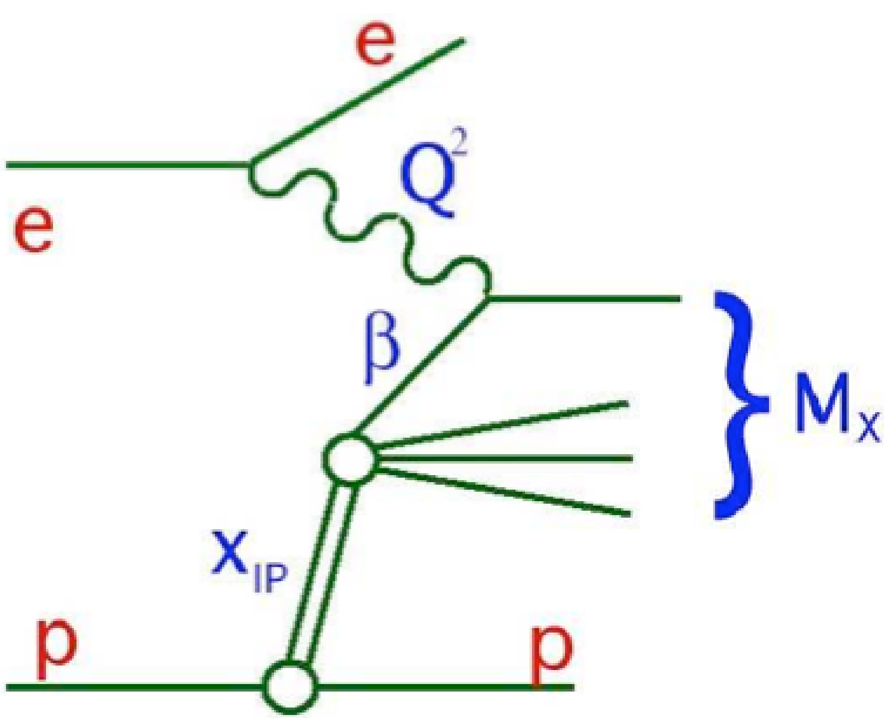

M.L. Mangano
KITP collider conf 2004

Bjorken-scaling variable

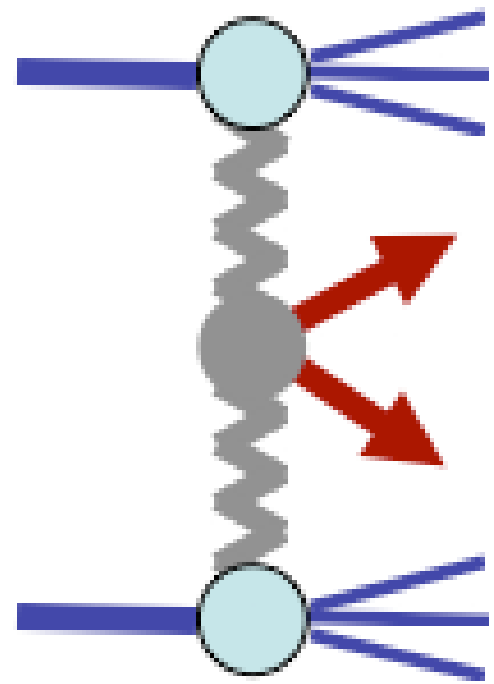
$$x_T = \frac{2E_T}{\sqrt{s}}$$

- In the ratio the dependence on the pdf's cancels
- dashes: theory prediction with no power corrections
- solid: best fit to data with free power-correction parameter $\Lambda$ in the theory

# Factorisation in diffraction ??




diffraction in DIS



double pomeron exchange in $p\bar{p}$

- no proof of factorisation in diffractive events
- data do not support it

# 3 complementary approaches to $\hat{\sigma}$

| | matrix-elem MC's | fixed-order x-sect | shower MC's |
|---|---|---|---|
| final-state description | hard-parton jets. Describes geometry, correlations, ... | limited access to final-state structure | full information available at the hadron level |
| higher-order effects: loop corrections | hard to implement: must introduce negative probabilities | straightforward to implement (when available) | included as vertex corrections (Sudakov FF's) |
| higher-order effects: hard emissions | included, up to high orders (multijets) | straightforward to implement (when available) | approximate, incomplete phase space at large angles |
| resummation of large logs | ? | feasible (when available) | unitarity implementation (i.e. correct shapes but not total rates) |

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# Matrix-element MonteCarlo generators

- efficient multi-parton generation: up to 2 ⇨ 9 jets subprocesses
  - ALPGEN M.L.Mangano M. Moretti F. Piccinini R. Pittau A. Polosa 2002
  - MADGRAPH/MADEVENT W.F. Long F. Maltoni T. Stelzer 1994/2003
  - COMPHEP A. Pukhov et al. 1999
  - GRACE/GR@PPA T. Ishikawa et al. K. Sato et al. 1992/2001
  - HELAC C. Papadopoulos et al. 2000
- merged with parton showers
  - all of the above, merged with HERWIG or PYTHIA
  - SHERPA F. Krauss et al. 2003

⇨ talk di Frixione

# Shower MonteCarlo generators

- HERWIG B. Webber et al. 1992

  being re-written as a C++ code (HERWIG++)

- PYTHIA T. Sjostrand 1994

and more

- CKKW S. Catani F. Krauss R. Kuhn B. Webber 2001

  a procedure to interface parton subprocesses with
  a different number of final states to parton showers

- MC@NLO S. Frixione B. Webber 2002

  a procedure to interface NLO computations to shower MC's

⇒ talk di Frixione

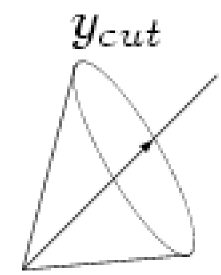
# NLO features

- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: $\mu_R, \mu_F$
  - predictive normalisation of observables
    - first step toward precision measurements
    - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC's: MC@NLO

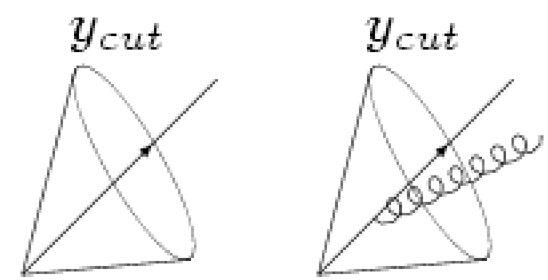
# Jet structure

the jet non-trivial structure shows up first at NLO

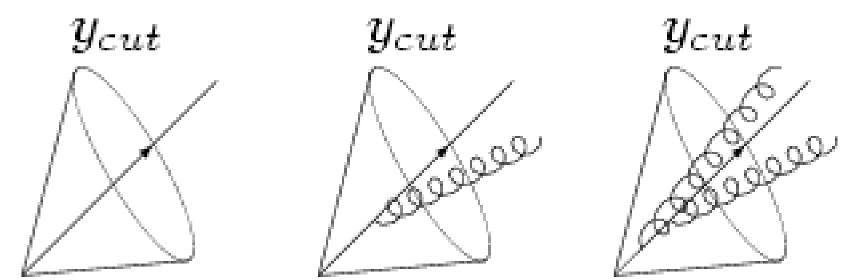
leading order




NLO




NNLO



# Somebody's wishlist

Dear Santa Claus,

I'd like to have the following cross sections at NLO

Run II Monte Carlo Workshop, April 2001

| Single boson | Diboson | Triboson | Heavy flavour |
|---|---|---|---|
| $W + \leq 5j$ | $WW + \leq 5j$ | $WWW + \leq 3j$ | $t\bar{t} + \leq 3j$ |
| $W + b\bar{b} + \leq 3j$ | $WW + b\bar{b} + \leq 3j$ | $WWW + b\bar{b} + \leq 3j$ | $t\bar{t} + \gamma + \leq 2j$ |
| $W + c\bar{c} + \leq 3j$ | $WW + c\bar{c} + \leq 3j$ | $WWW + \gamma\gamma + \leq 3j$ | $t\bar{t} + W + \leq 2j$ |
| $Z + \leq 5j$ | $ZZ + \leq 5j$ | $Z\gamma\gamma + \leq 3j$ | $t\bar{t} + Z + \leq 2j$ |
| $Z + b\bar{b} + \leq 3j$ | $ZZ + b\bar{b} + \leq 3j$ | $WZZ + \leq 3j$ | $t\bar{t} + H + \leq 2j$ |
| $Z + c\bar{c} + \leq 3j$ | $ZZ + c\bar{c} + \leq 3j$ | $ZZZ + \leq 3j$ | $t\bar{b} + \leq 2j$ |
| $\gamma + \leq 5j$ | $\gamma\gamma + \leq 5j$ | | $b\bar{b} + \leq 3j$ |
| $\gamma + b\bar{b} + \leq 3j$ | $\gamma\gamma + b\bar{b} + \leq 3j$ | | |
| $\gamma + c\bar{c} + \leq 3j$ | $\gamma\gamma + c\bar{c} + \leq 3j$ | | |
| | $WZ + \leq 5j$ | | |
| | $WZ + b\bar{b} + \leq 3j$ | | |
| | $WZ + c\bar{c} + \leq 3j$ | | |
| | $W\gamma + \leq 3j$ | | |
| | $Z\gamma + \leq 3j$ | | |

# NLO history

- $e^+e^- \to 3$ jets — K. Ellis, D. Ross, A. Terrano 1981
  $e^+e^- \to 4$ jets — Z. Bern et al., N. Glover et al., Z. Nagy Z.Trocsanyi 1996-97
- $pp \to 1, 2$ jets — K. Ellis J. Sexton 1986, W. Giele N. Glover D. Kosower 1993
  $pp \to 3$ jets — Z. Bern et al., Z. Kunszt et al. 1993-1995, Z. Nagy 2001
- $pp \to V + 1$ jet — W. Giele N. Glover & D. Kosower1993
  $pp \to V + 2$ jet — Bern et al., Glover et al. 1996-97, K. Ellis & Campbell 2003
- $pp \to Vb\bar{b}$ — K. Ellis & J. Campbell 2003
  $pp \to Vb\bar{b} + 1$ jet — ??
- $pp \to VV$ — Ohnemus & Owens, Baur et al.1991-96, Dixon et al. 2000
  $pp \to VV + 1$ jet — ??
- $pp \to \gamma\gamma$ — B. Bailey et al 1992, T. Binoth et al 1999
  $pp \to \gamma\gamma + 1$ jet — Z. Bern et al. 1994, V. Del Duca et al. 2003
- $pp \to Q\bar{Q}$ — Dawson K. Ellis Nason 1989, Mangano Nason Ridolfi 1992
  $pp \to Q\bar{Q} + 1$ jet — A. Brandenburg et al. 2005 ?

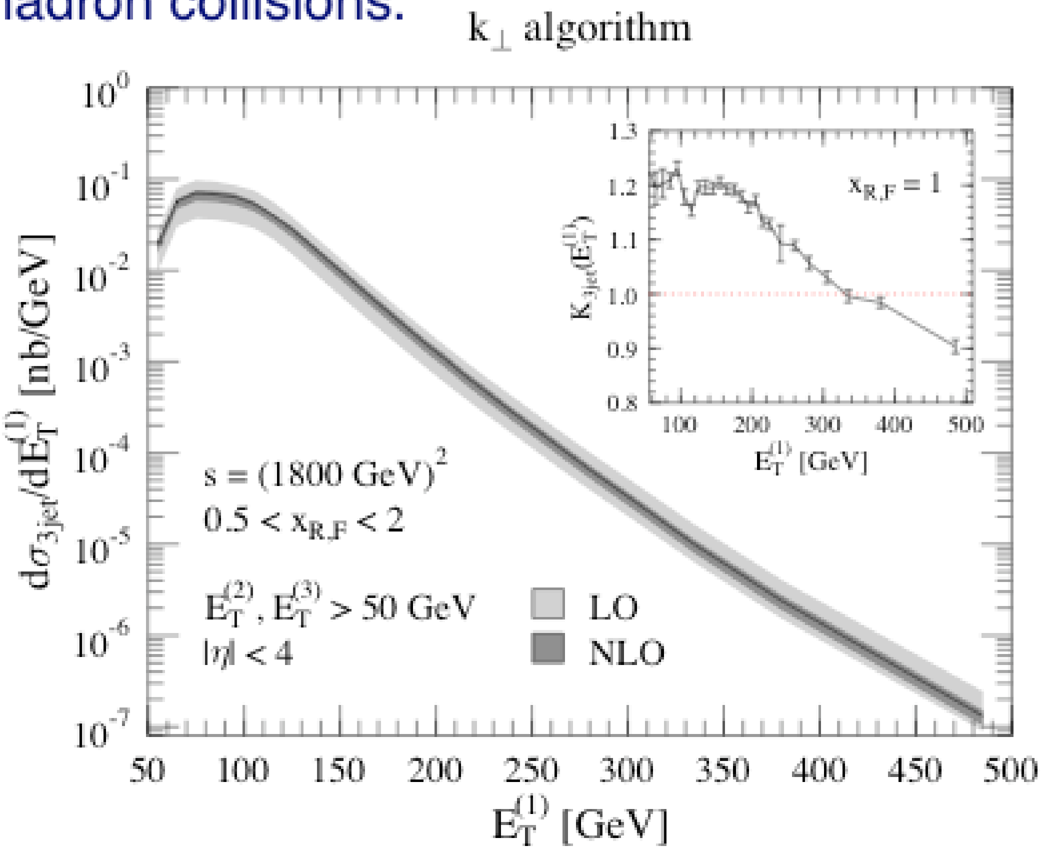
# *NLOJET++*

Author(s): Z. Nagy
http://www.ippp.dur.ac.uk/~nagyz/nlo++.html
Multi-purpose C++ library for calculating jet cross-sections in $e^+e^-$ annihilation, DIS and hadron-hadron collisions.

$e^+e^- \longrightarrow \leq 4$ jets

$ep \longrightarrow (\leq 3+1)$ jets

$p\bar{p} \longrightarrow \leq 3$ jets




hep-ph/0110315

# *MCFM*

Author(s): JC, R. K. Ellis

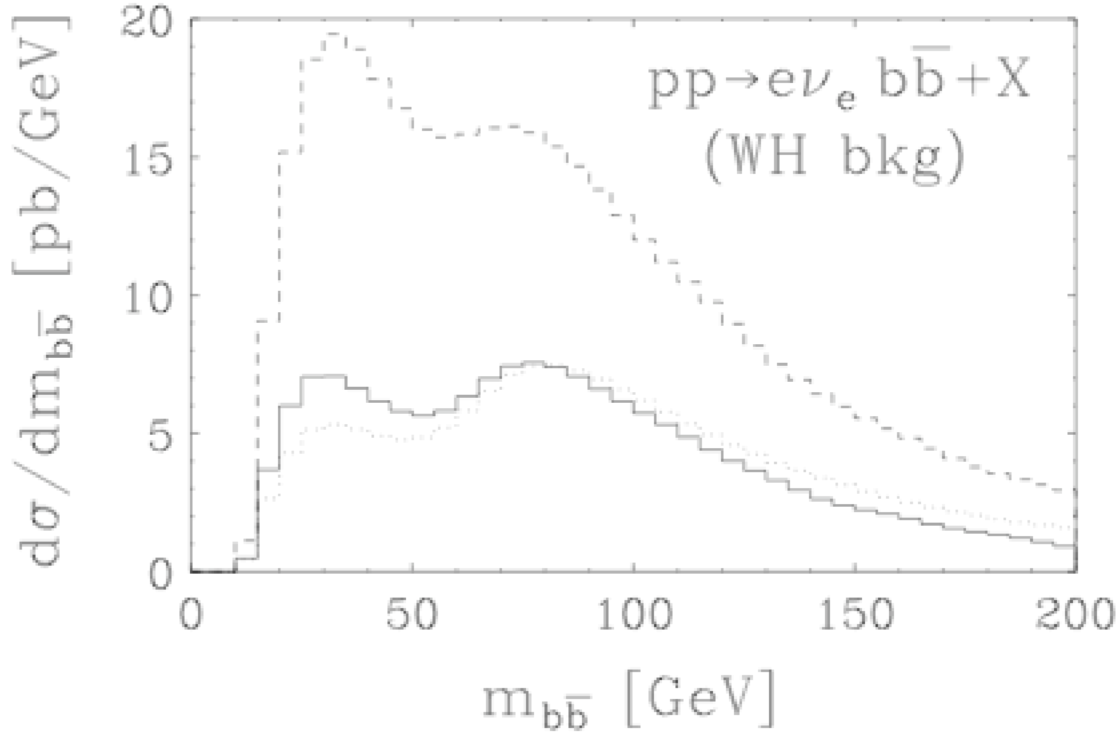
http://mcfm.fnal.gov

Fortran package for calculating a number of processes involving vector bosons, Higgs, jets and heavy quarks at hadron colliders.

$p\bar{p} \longrightarrow V+ \leq 2$ jets

$p\bar{p} \longrightarrow V + b\bar{b}$

with $V = W, Z$.




hep-ph/0308195

# AYLEN/EMILIA

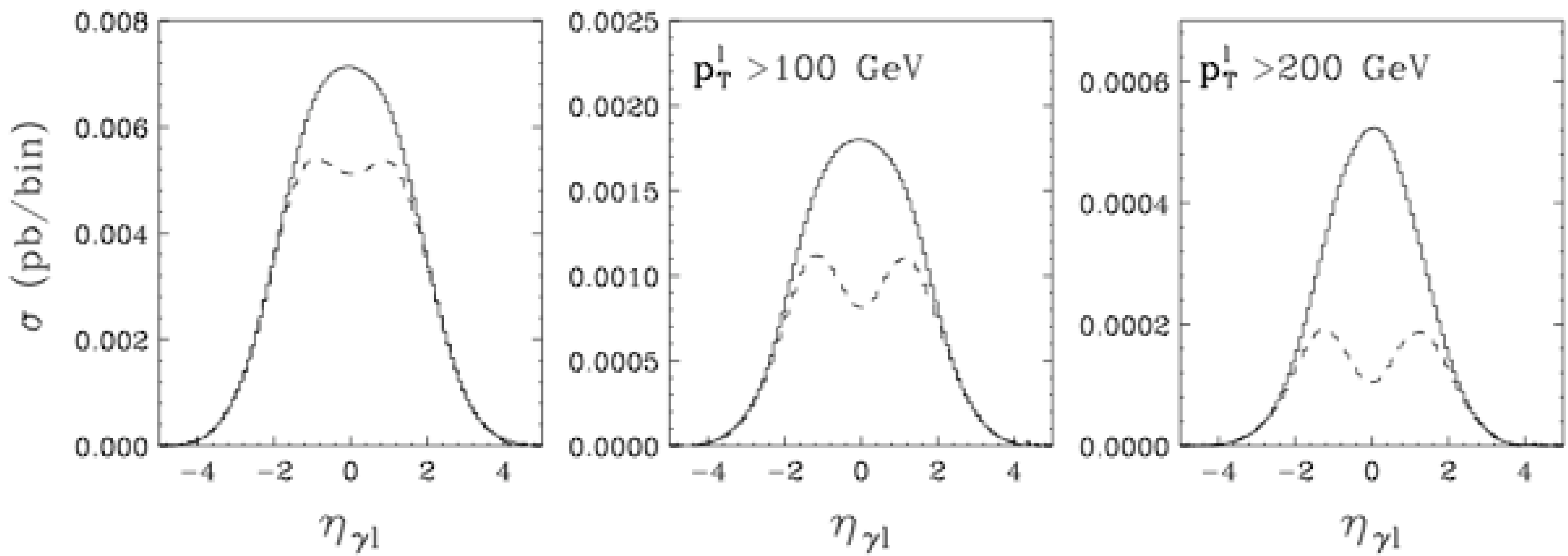
Author(s): L. Dixon, Z. Kunszt, A.Signer, D. de Florian

http://www.itp.phys.ethz.ch/staff/dflorian/codes.html

Fortran implementation of gauge boson pair production at hadron colliders, including full spin and decay angle correlations.

$p\bar{p} \longrightarrow VV'$ and $p\bar{p} \longrightarrow V\gamma$ with $V, V' = W, Z$

Anomalous triple gauge boson couplings at the LHC:




hep-ph/0002138

# DIPHOX/EPHOX

Author(s): P. Aurenche, T.Binoth, M. Fontannaz, J. Ph. Guillet, G. Heinrich, E. Pilon, M. Werlen

http://wwwlapp.in2p3.fr/lapth/PHOX_FAMILY/main.html

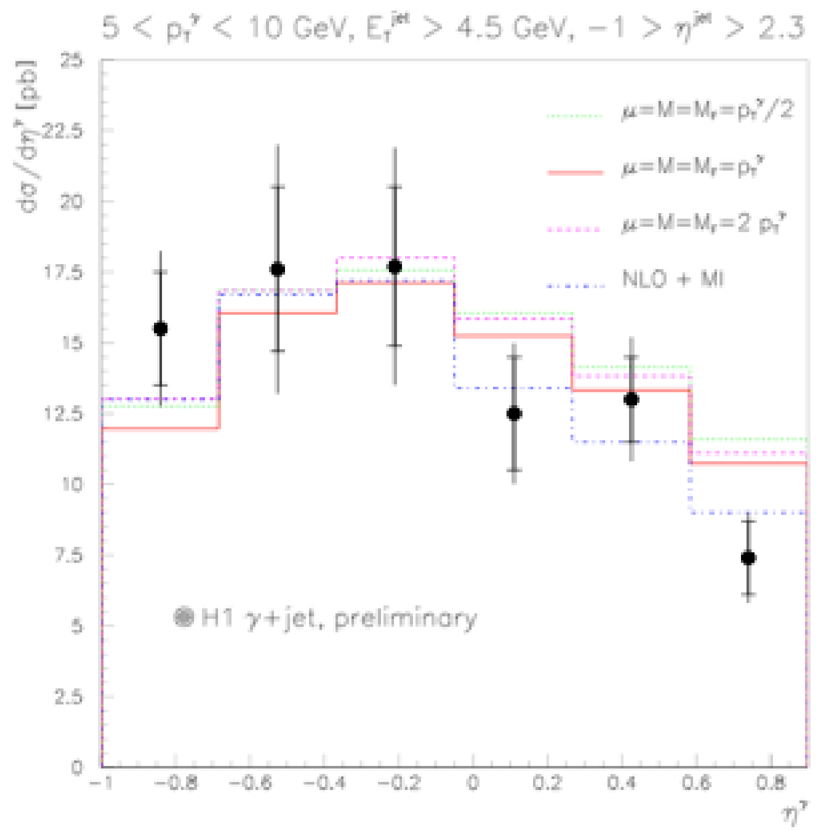
Fortran code to compute processes involving photons, hadrons and jets in DIS and hadron colliders.

$$p\bar{p} \longrightarrow \gamma + \leq 1 \text{ jet}$$

$$p\bar{p} \longrightarrow \gamma\gamma$$

$$\gamma p \longrightarrow \gamma + \text{jet}$$

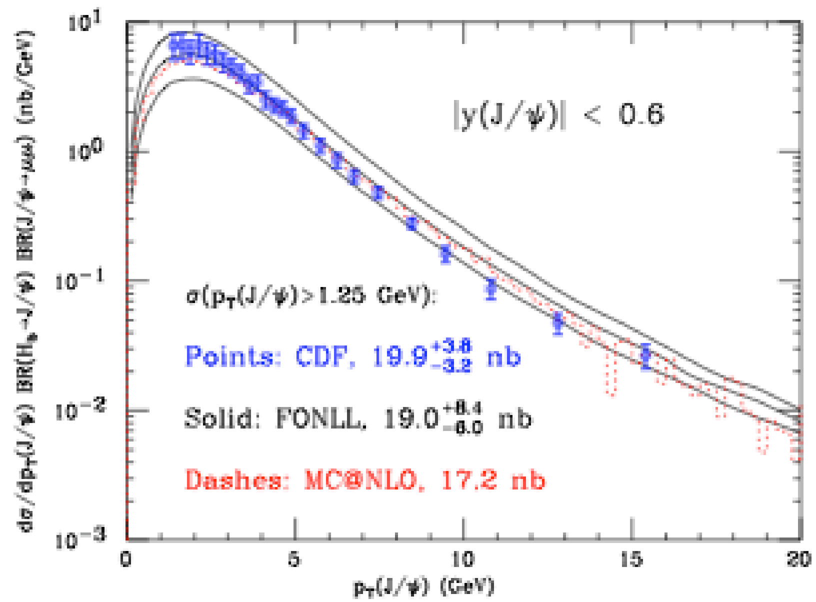
Preliminary H1 data, hep-ph/0312070.



# *Heavy quark production*

Author(s): M. L. Mangano, P. Nason and G. Ridolfi
`http://www.ge.infn.it/~ridolfi/hvqlibx.tgz`
Fortran code for the calculation of heavy quark cross-sections and distributions in a fully differential manner

- Based on the more inclusive calculations of Dawson et al, Beenakker et al.
- Does not include multiple gluon radiation, $\log(p_T/m_b)$ (FONLL)
  Cacciari et al., hep-ph/9803400
- These are the same matrix elements that are incorporated into MC@NLO
  Frixione et al., hep-ph/0305252

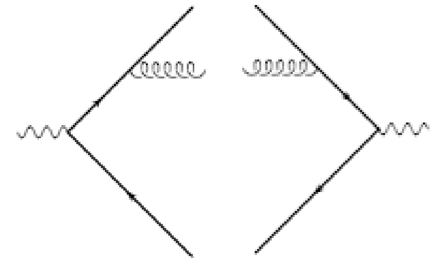



hep-ph/0312132

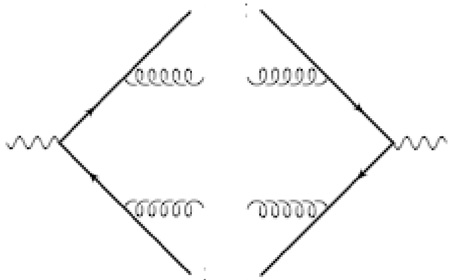
# NLO assembly kit

$e^+e^- \to 3$ jets

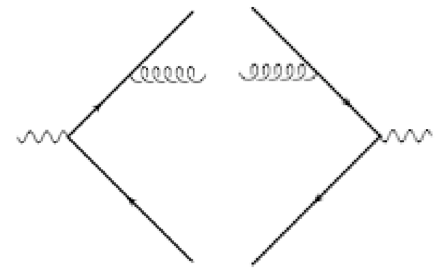
**leading order** $|\mathcal{M}_n^{tree}|^2$



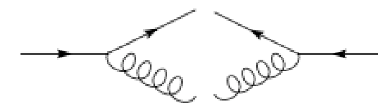
**NLO real**



IR



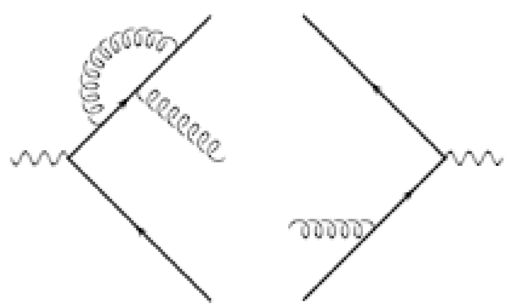
$\otimes$



$$|\mathcal{M}_{n+1}^{tree}|^2 \quad \to \quad |\mathcal{M}_n^{tree}|^2 \quad + \quad \int dPS |P_{split}|^2$$

$$= \quad -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)$$

**NLO virtual**



$d = 4 - 2\epsilon$

$$\int d^d l \; 2(\mathcal{M}_n^{loop})^* \mathcal{M}_n^{tree} \quad = \quad \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) |\mathcal{M}_n^{tree}|^2 + fin.$$

# NLO production rates

## Process-independent procedure devised in 1992-96

Giele Glover & Kosower; Frixione Kunszt & Signer, Catani & Seymour

slicing subtraction

$$\hat{\sigma} = \sigma^{\rm LO} + \sigma^{\rm NLO} = \int_n d\sigma^B + \sigma^{\rm NLO}$$

$$\sigma^{\rm NLO} = \int_{n+1} d\sigma^R + \int_n d\sigma^V$$

the 2 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

$$\sigma^{\rm NLO} = \int_{n+1} \left[ (d\sigma^R)_{\epsilon=0} - (d\sigma^A)_{\epsilon=0} \right] + \int_n \left( d\sigma^V + \int_1 d\sigma^A \right)_{\epsilon=0}$$

the 2 terms on the rhs are finite in d=4

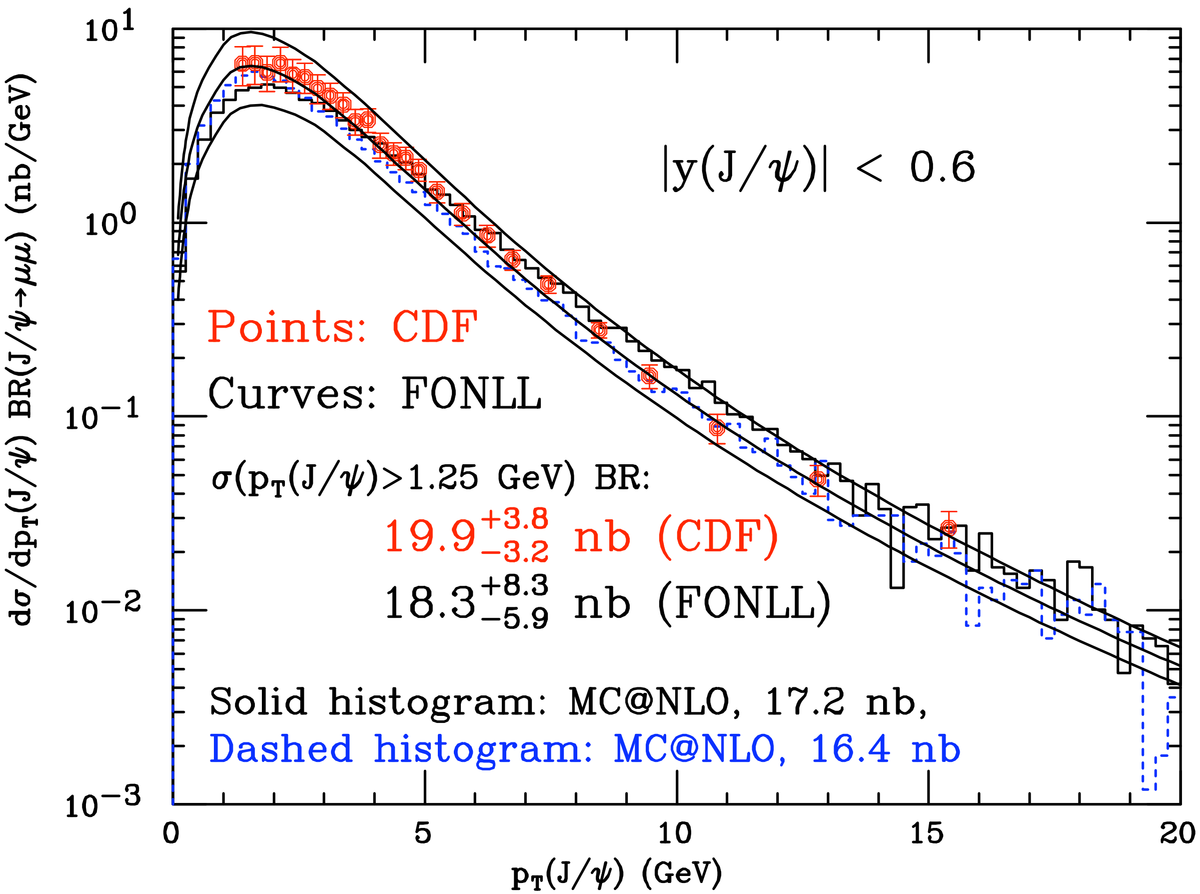
# NLO complications

- loop integrals are involved and process-dependent
- more particles ⇒ many scales ⇒ lenghty analytic expressions
  - even though it is known how to compute loop integrals with $2 \to n$ particles no integrals with $n > 3\ (4)$ have been computed analytically (numerically)
- no numeric methods yet for hadron collisions
  - counterterms are subtracted analytically

# Is NLO enough to describe data ?

$b$ cross section in $p\bar{p}$ collisions at 1.96 TeV

$$d\sigma(p\bar{p} \to H_b X, H_b \to J/\psi\ X)/dp_T(J/\psi)$$



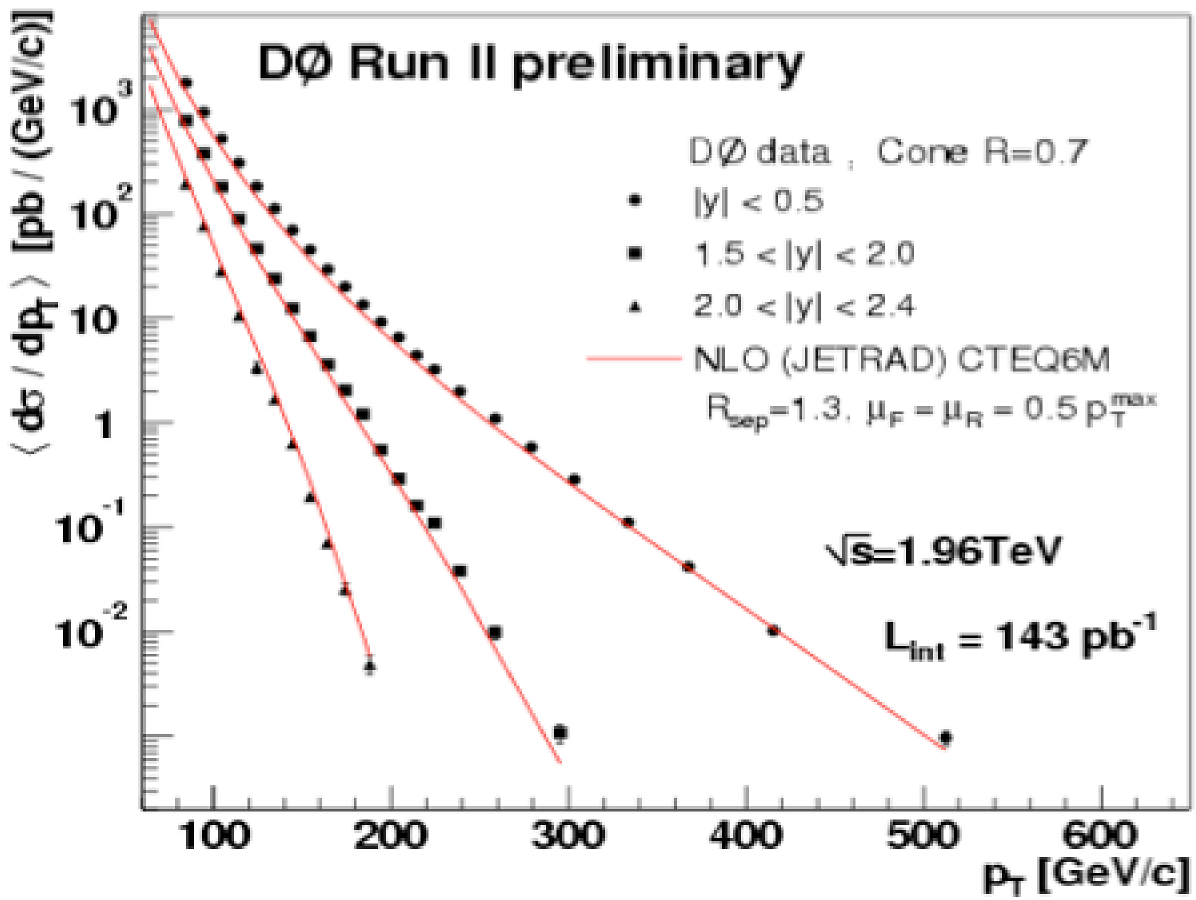

## NLO + NLL

perfect agreement with data (with use of updated FF's by Cacciari & Nason)

Cacciari, Frixione, Mangano, Nason, Ridolfi 2003

# Is NLO enough to describe data ?

## Inclusive jet $p_T$ cross section at Tevatron

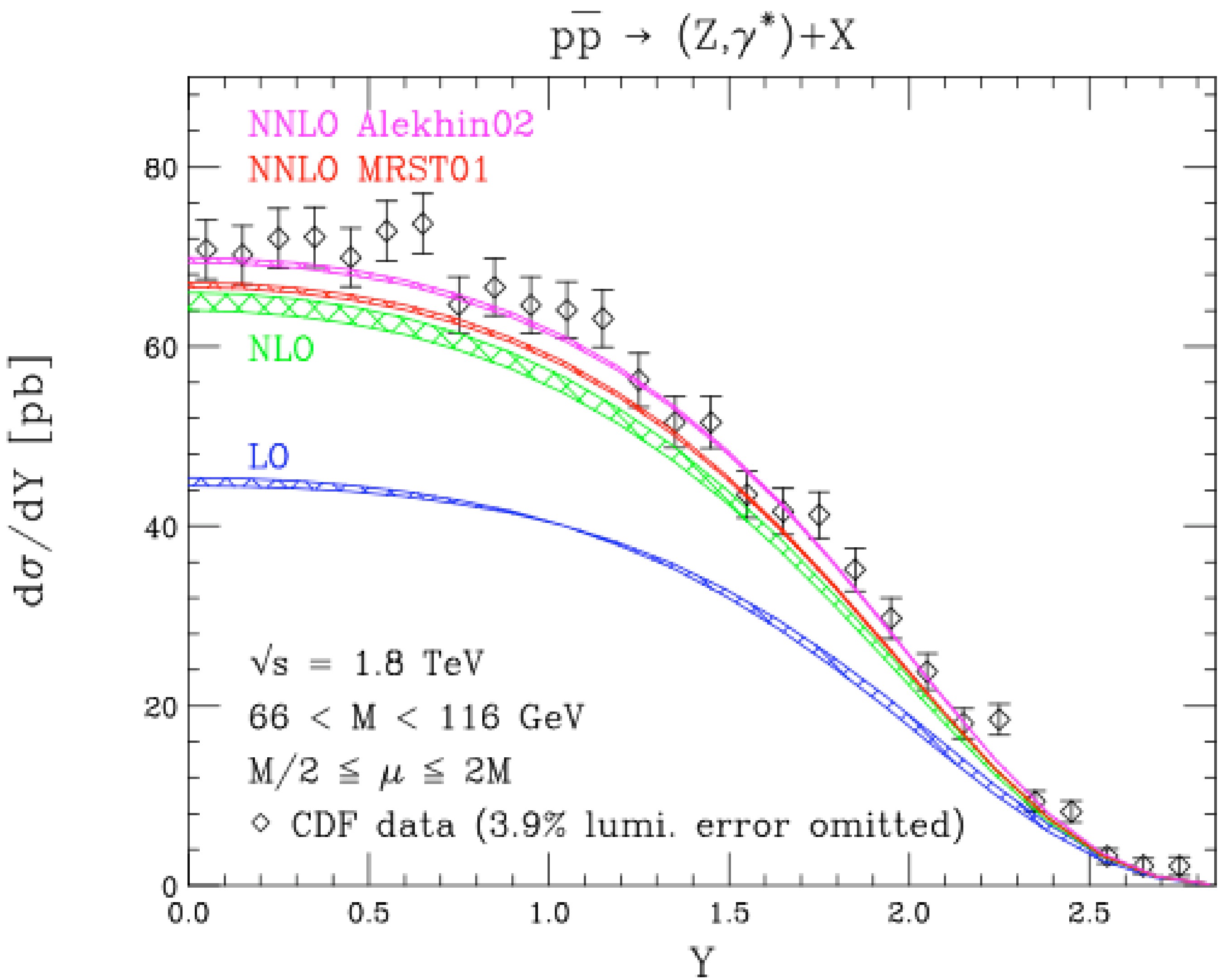



good agreement between NLO and data over several orders of magnitude

constrains the gluon distribution at high $x$

# Is NLO enough to describe data ?

di-lepton rapidity distribution for $(Z, \gamma^*)$ production vs. Tevatron Run I data




LO and NLO curves are for the MRST PDF set

no spin correlations

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# Is NLO enough to describe data ?

## Drell-Yan $W$ cross section at LHC with leptonic decay of the $W$

Cuts A $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 20$ GeV, $p_T^{(\nu)} > 20$ GeV

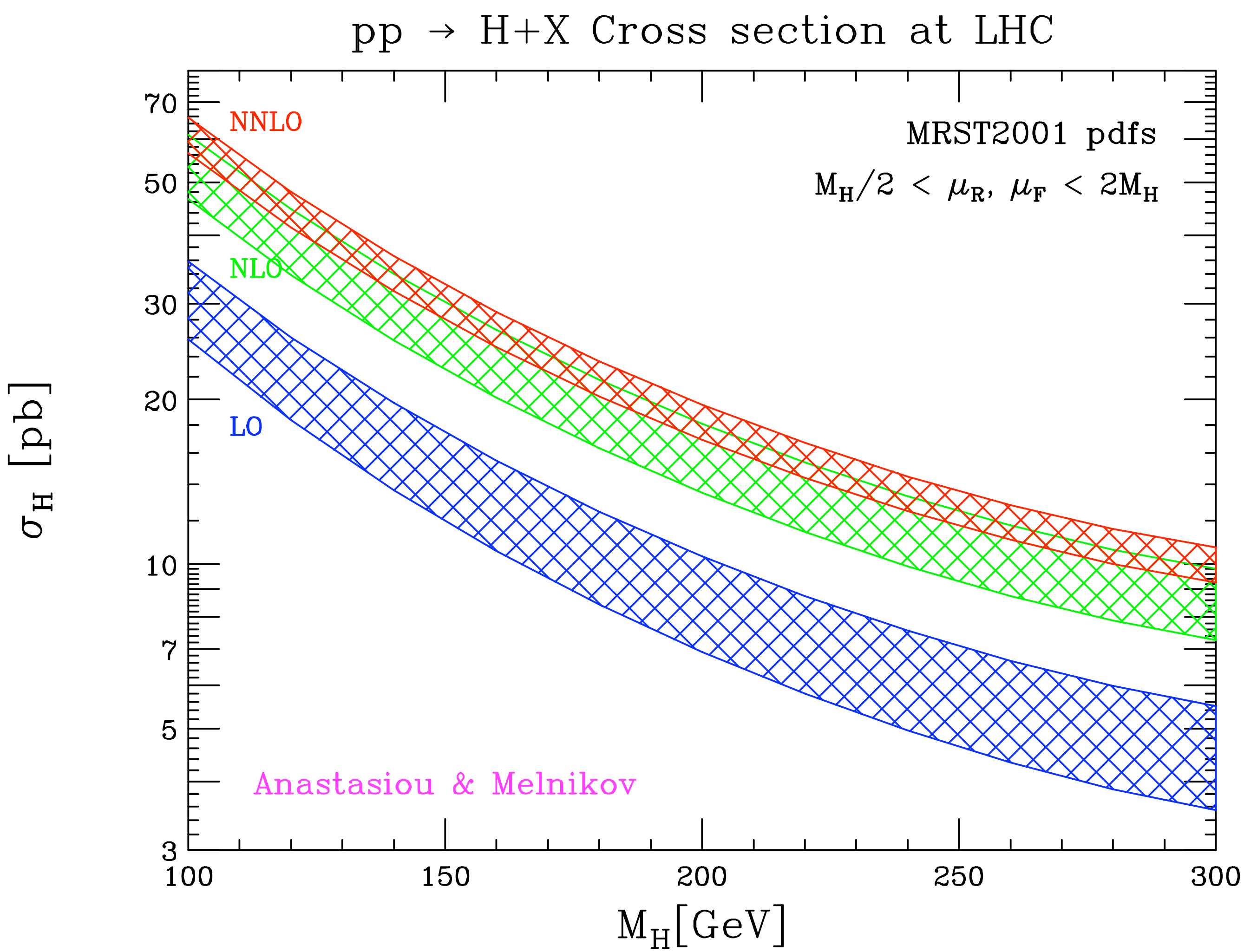
Cuts B $\longrightarrow |\eta^{(e)}| < 2.5,\ p_T^{(e)} > 40$ GeV, $p_T^{(\nu)} > 20$ GeV

| | LO | | LO+HW | NLO | | MC@NLO |
|---|---|---|---|---|---|---|
| Cuts A | 0.5249 | $\xrightarrow{-7.7\%}$ | 0.4843 | 0.4771 | $\xrightarrow{+1.5\%}$ | 0.4845 |
| | $\downarrow$5.4% | | | $\downarrow$7.0% | | $\downarrow$6.3% |
| Cuts A, no spin | 0.5535 | | | 0.5104 | | 0.5151 |
| Cuts B | 0.0585 | $\xrightarrow{+208\%}$ | 0.1218 | 0.1292 | $\xrightarrow{+2.9\%}$ | 0.1329 |
| | $\downarrow$29% | | | $\downarrow$16% | | $\downarrow$18% |
| Cuts B, no spin | 0.0752 | | | 0.1504 | | 0.1570 |

- $|\mathrm{MC@NLO} - \mathrm{NLO}| = \mathcal{O}(2\%)$ S. Frixione M.L. Mangano 2004
- NNLO useless without spin correlations
- Precisely evaluated Drell-Yan $W, Z$ cross sections could be used as ``standard candles'' to measure the parton luminosity at LHC

# Is NLO enough to describe data ?

## Total cross section for inclusive Higgs production at LHC




contour bands are

lower

$\mu_R = 2M_H \quad \mu_F = M_H/2$

upper

$\mu_R = M_H/2 \quad \mu_F = 2M_H$

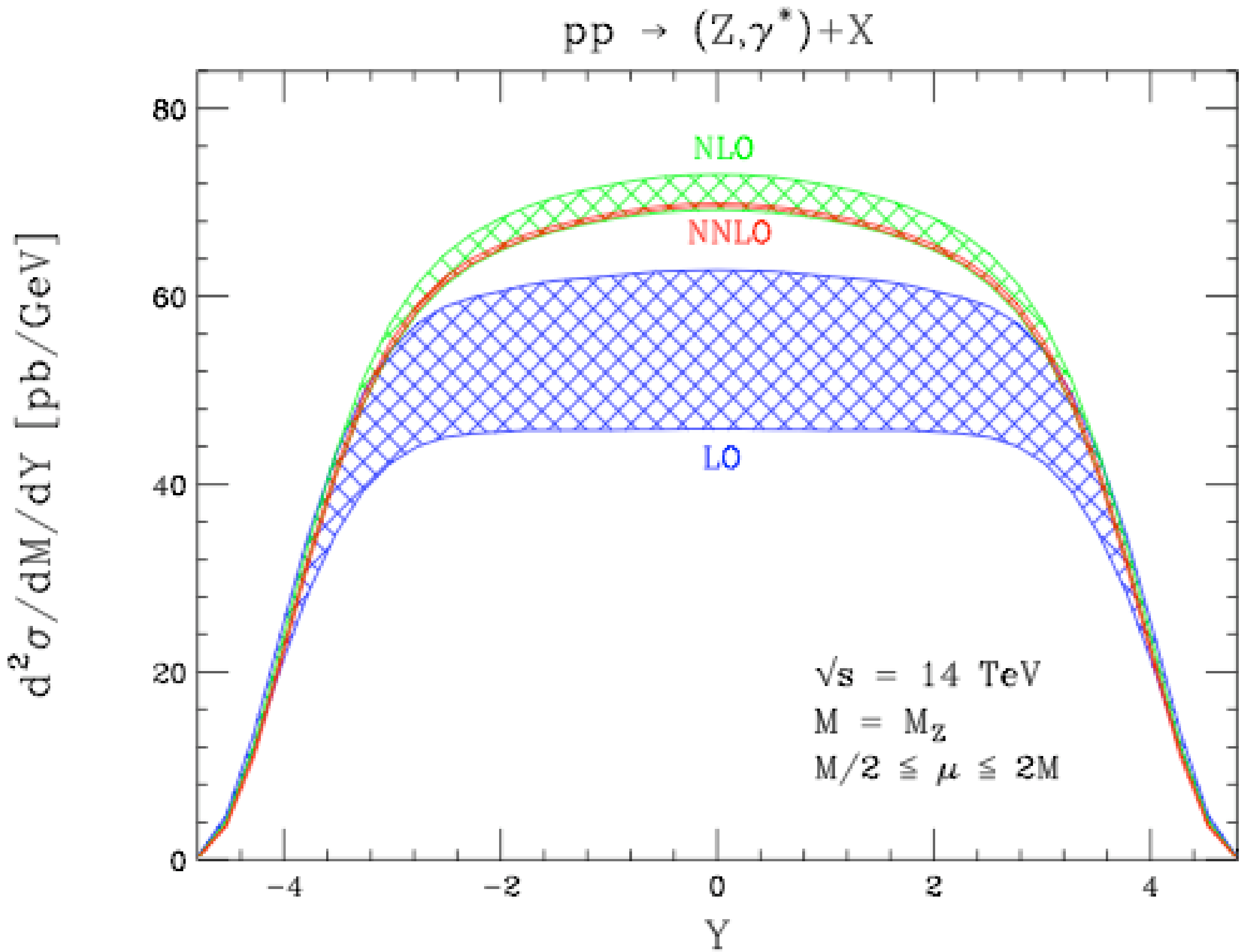
scale uncertainty is about $10\%$

NNLO prediction stabilises the perturbative series

# NNLO state of the art

- Drell-Yan $W, Z$ production
  - total cross section — Hamberg, van Neerven, Matsuura 1990; Harlander, Kilgore 2002
  - rapidity distribution — Anastasiou et al. 2003
- Higgs production
  - total cross section — Harlander, Kilgore; Anastasiou, Melnikov 2002
  - fully differential cross section — Anastasiou, Melnikov, Petriello 2004
- $e^+e^- \to 3$ jets
  - the $C_F^2$ term — the Gehrmanns, Glover 2004

# Drell-Yan $Z$ production at LHC




Rapidity distribution for an on-shell $Z$ boson

- $30\%(15\%)$ NLO increase wrt to LO at central Y's (at large Y's)
  NNLO decreases NLO by $1-2\%$
- scale variation: $\approx 30\%$ at LO; $\approx 6\%$ at NLO; less than $1\%$ at NNLO

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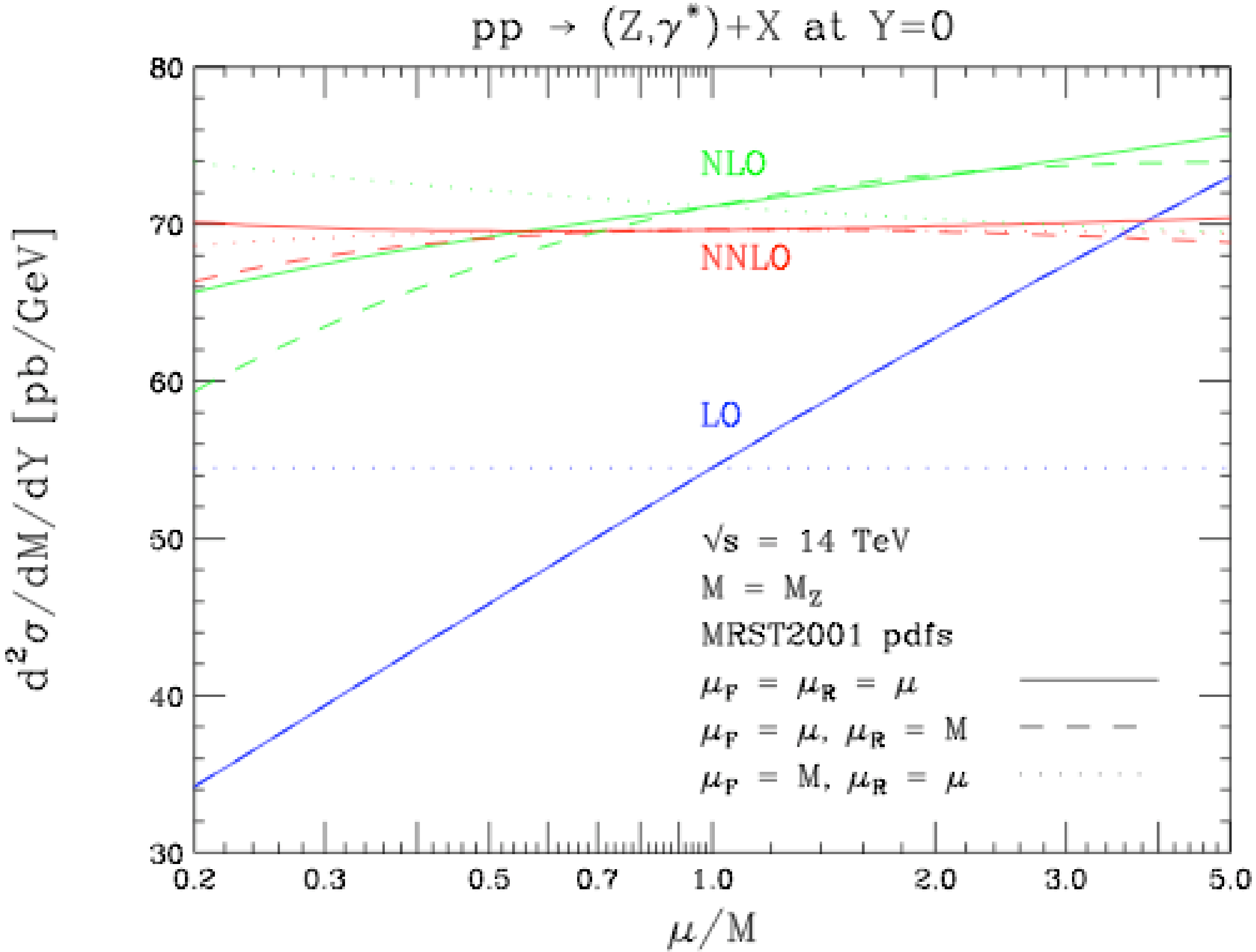
# Scale variations in Drell-Yan $Z$ production




- solid: vary $\mu_R$ and $\mu_F$ together
- dashed: vary $\mu_F$ only
- dotted: vary $\mu_R$ only

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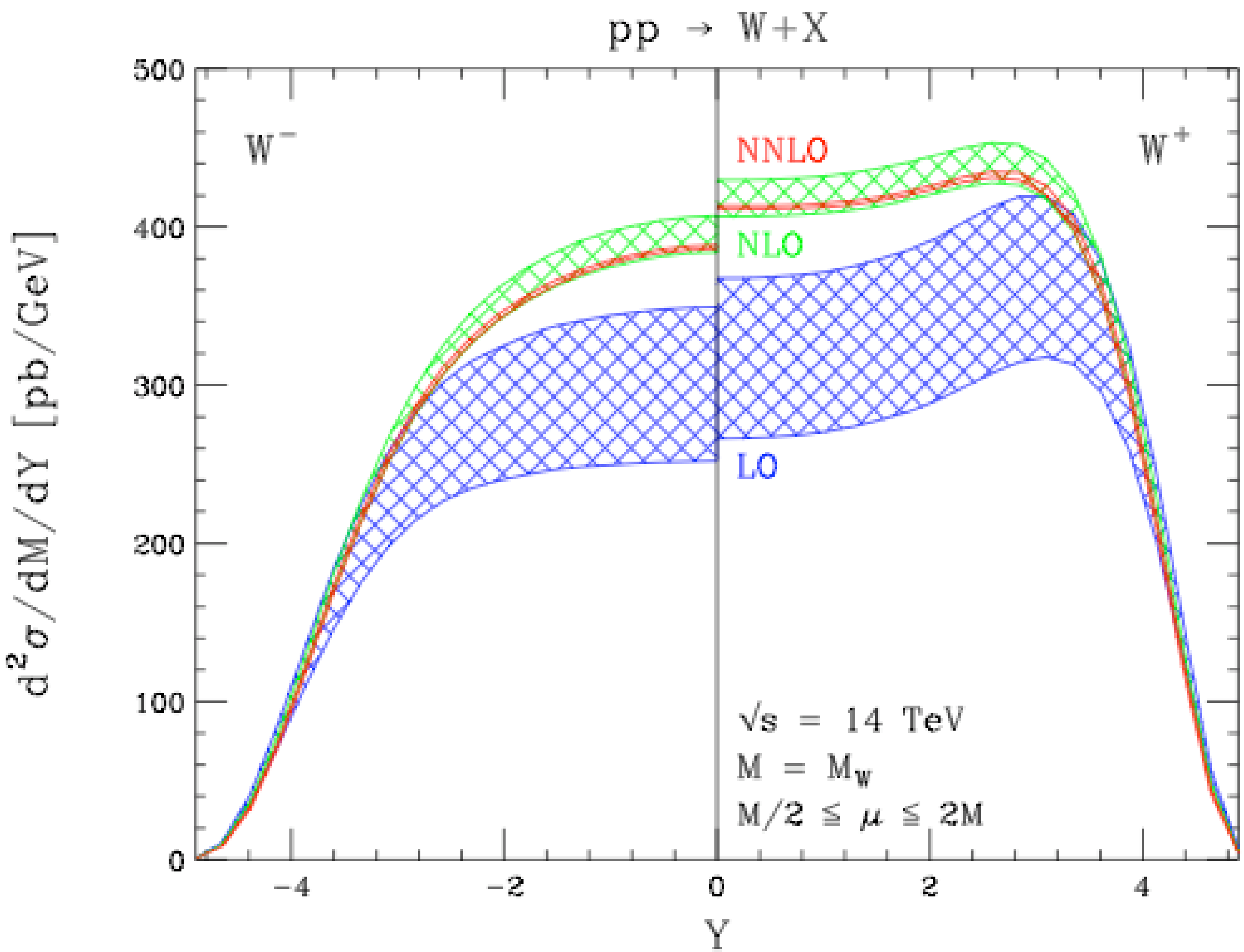
# Drell-Yan $W$ production at LHC




Rapidity distribution for an on-shell $W^-$ boson (left) $W^+$ boson (right)

- distributions are symmetric in $Y$
- NNLO scale variations are $1\%(3\%)$ at central (large) $Y$

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# Higgs production at LHC

a fully differential cross section:
bin-integrated rapidity distribution, with a jet veto

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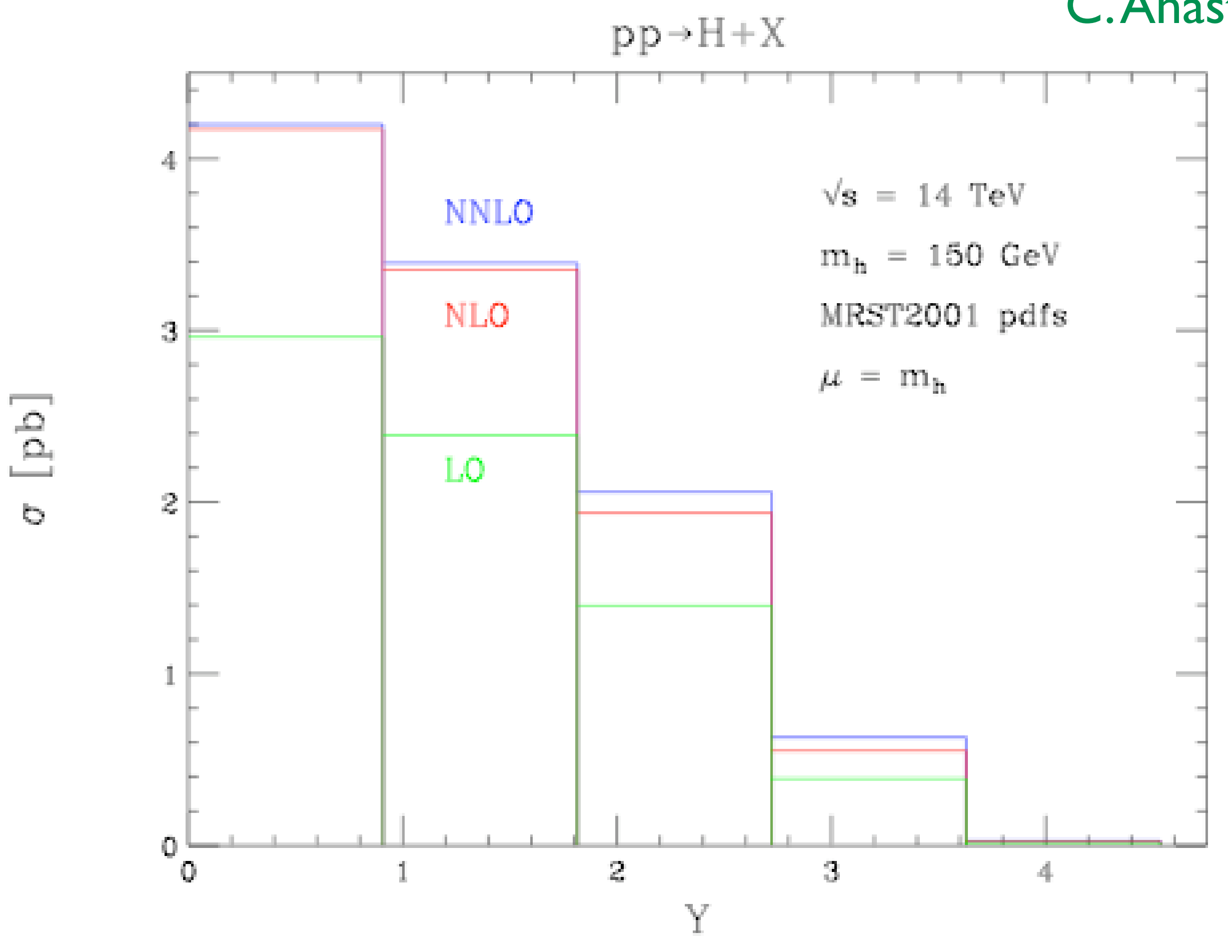



jet veto: require

$R = 0.4$

$|\mathbf{p}_T^j| < p_T^{veto} = 40 \text{ GeV}$

for 2 partons

$R_{12}^2 = (\eta_1 - \eta_2)^2 + (\phi_1 - \phi_2)^2$

if $R_{12} > R$

$|\mathbf{p}_T^1|, |\mathbf{p}_T^2| < p_T^{veto}$

if $R_{12} < R$

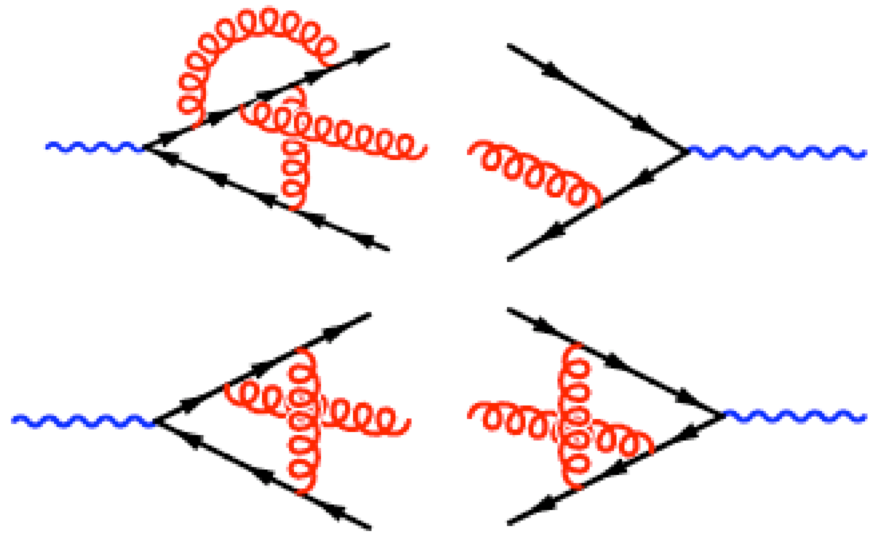
$|\mathbf{p}_T^1 + \mathbf{p}_T^2| < p_T^{veto}$

- $M_H = 150 \text{ GeV}$ (jet veto relevant in the $H \to W^+W^-$ decay channel)
- K factor is much smaller for the vetoed x-sect than for the inclusive one: average $|\mathbf{p}_T^j|$ increases from NLO to NNLO: less x-sect passes the veto

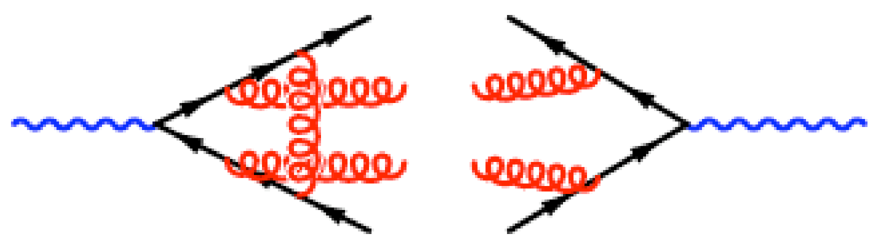
# NNLO assembly kit

$e^+e^- \to 3$ jets

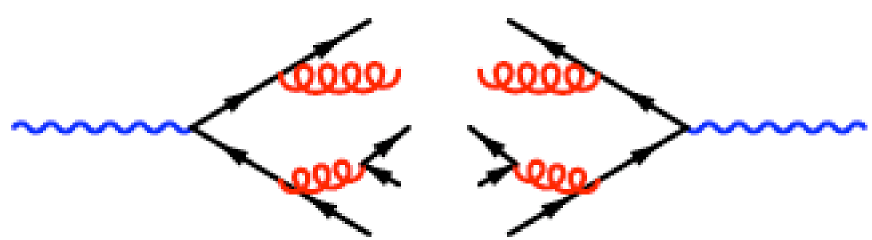
double virtual



real-virtual



double real



# Two-loop matrix elements

- two-jet production $qq' \to qq',\ q\bar{q} \to q\bar{q},\ q\bar{q} \to gg,\ gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

- photon-pair production $q\bar{q} \to \gamma\gamma,\ gg \to \gamma\gamma$

  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
  Z. Bern A. De Freitas L. Dixon 2002

- $e^+e^- \to 3$ jets $\gamma^* \to q\bar{q}g$

  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

- $V + 1$ jet production $q\bar{q} \to Vg$

  T. Gehrmann E. Remiddi 2002

- Drell-Yan $V$ production $q\bar{q} \to V$

  R. Hamberg W. van Neerven T. Matsuura 1991

- Higgs production $gg \to H$ (in the $m_t \to \infty$ limit)

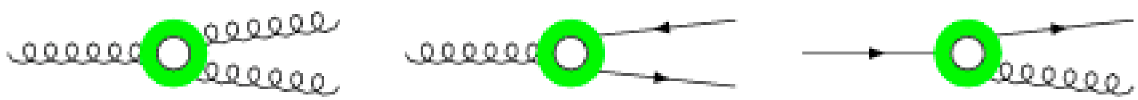
  R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

# NNLO cross sections

- universal IR structure ⟹ process-independent procedure
- universal collinear and soft currents
  - 3-parton tree splitting functions

J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

  - 2-parton one-loop splitting functions

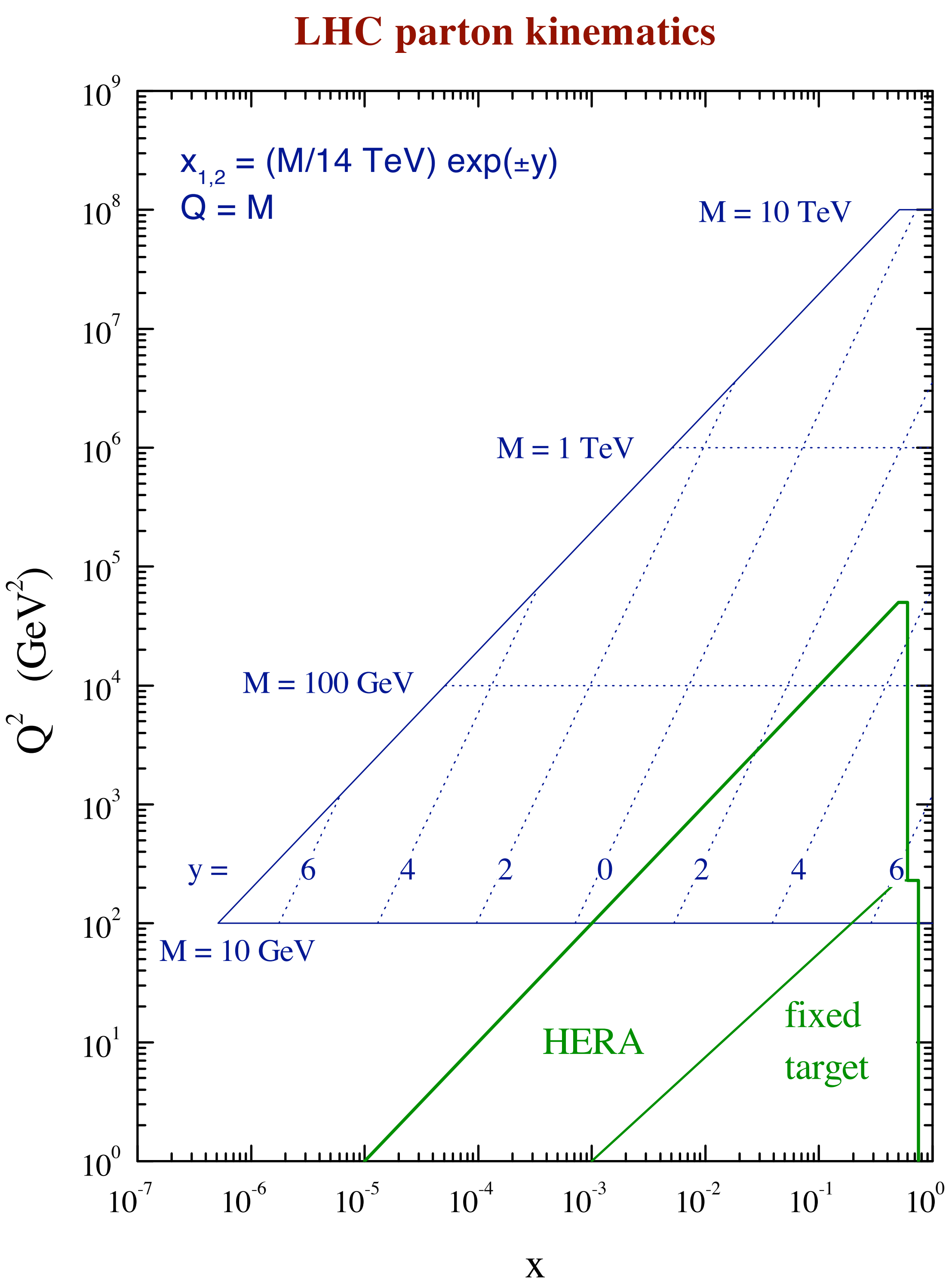


Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; D. Kosower 2003

- universal subtraction counterterms
  - several ideas and works in progress
    but so far not yet completely figured out

D. Kosower; S. Weinzierl; the Gehrmanns & G. Heinrich 2003

J. Stirling



LHC parton kinematics
$x_{1,2}$ = (M/14 TeV) exp(±y)
Q = M
M = 10 TeV
M = 1 TeV
M = 100 GeV
M = 10 GeV
y = 6 4 2 0 2 4 6
HERA
fixed
target
$Q^2$ (GeV$^2$)
x
$10^0$
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
$10^6$
$10^7$
$10^8$
$10^9$
$10^{-7}$
$10^{-6}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
$10^0$

# Parton distribution functions (PDF)

- factorisation for the structure functions (e.g. $F_2^{ep},\ F_L^{ep}$ )

$$\mathcal{F}_i(x,\mu_F^2) = C_{ij}\otimes q_j + C_{ig}\otimes g$$

with the convolution $$[a\otimes b](x) \equiv \int_x^1 \frac{dy}{y}\, a(y)\, b\left(\frac{x}{y}\right)$$

$C_{ij},\ C_{ig}$ coefficient functions

$q_i(x,\mu_F^2)\quad g(x,\mu_F^2)$ PDF's

- DGLAP evolution equations

$$\frac{d}{d\ln\mu_F^2}\begin{pmatrix} q_i \\ g \end{pmatrix} = \begin{pmatrix} P_{\mathrm{q_i q_j}} & P_{\mathrm{q_j g}} \\ P_{\mathrm{g q_j}} & P_{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} q_j \\ g \end{pmatrix}$$

- perturbative series $P_{ij} \approx \alpha_s P_{ij}^{(0)} + \alpha_s^2 P_{ij}^{(1)} + \alpha_s^3 P_{ij}^{(2)}$

- anomalous dimension $\gamma_{ij}(N) = -\int_0^1 dx\, x^{N-1}\, P_{ij}(x)$

# PDF's

- general structure of the quark-quark splitting functions

$$P_{q_i q_k} = P_{\bar{q}_i \bar{q}_k} = \delta_{ik} P^{v}_{qq} + P^{s}_{qq}$$
$$P_{q_i \bar{q}_k} = P_{\bar{q}_i q_k} = \delta_{ik} P^{v}_{q\bar{q}} + P^{s}_{q\bar{q}}$$

- non-singlet
  - flavour asymmetry

$$q^{\pm}_{\mathrm{ns},ik} = q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k) \quad \Longleftarrow \quad P^{\pm}_{\mathrm{ns}} = P^{v}_{qq} \pm P^{v}_{q\bar{q}}$$

  - sum of valence distributions of all flavours

$$q^{v}_{\mathrm{ns}} = \sum_{r=1}^{n_f} (q_r - \bar{q}_r) \quad \Longleftarrow \quad P^{v}_{\mathrm{ns}} = P^{v}_{qq} - P^{v}_{q\bar{q}} + n_f (P^{s}_{qq} - P^{s}_{q\bar{q}})$$

- singlet

$$q_s = \sum_{i=1}^{n_f} (q_i + \bar{q}_i) \quad \Longleftarrow \quad \frac{d}{d \ln \mu_F^2} \begin{pmatrix} q_s \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_s \\ g \end{pmatrix}$$

with

$$P_{qq} = P^{+}_{\mathrm{ns}} + n_f (P^{s}_{qq} + P^{s}_{\bar{q}q})$$
$$P_{qg} = n_f P_{q_i g} \, , \quad P_{gq} = P_{g q_i}$$

# PDF history

- leading order (or one-loop)
  anomalous dim/splitting functions — Gross Wilczek 1973; Altarelli Parisi 1977

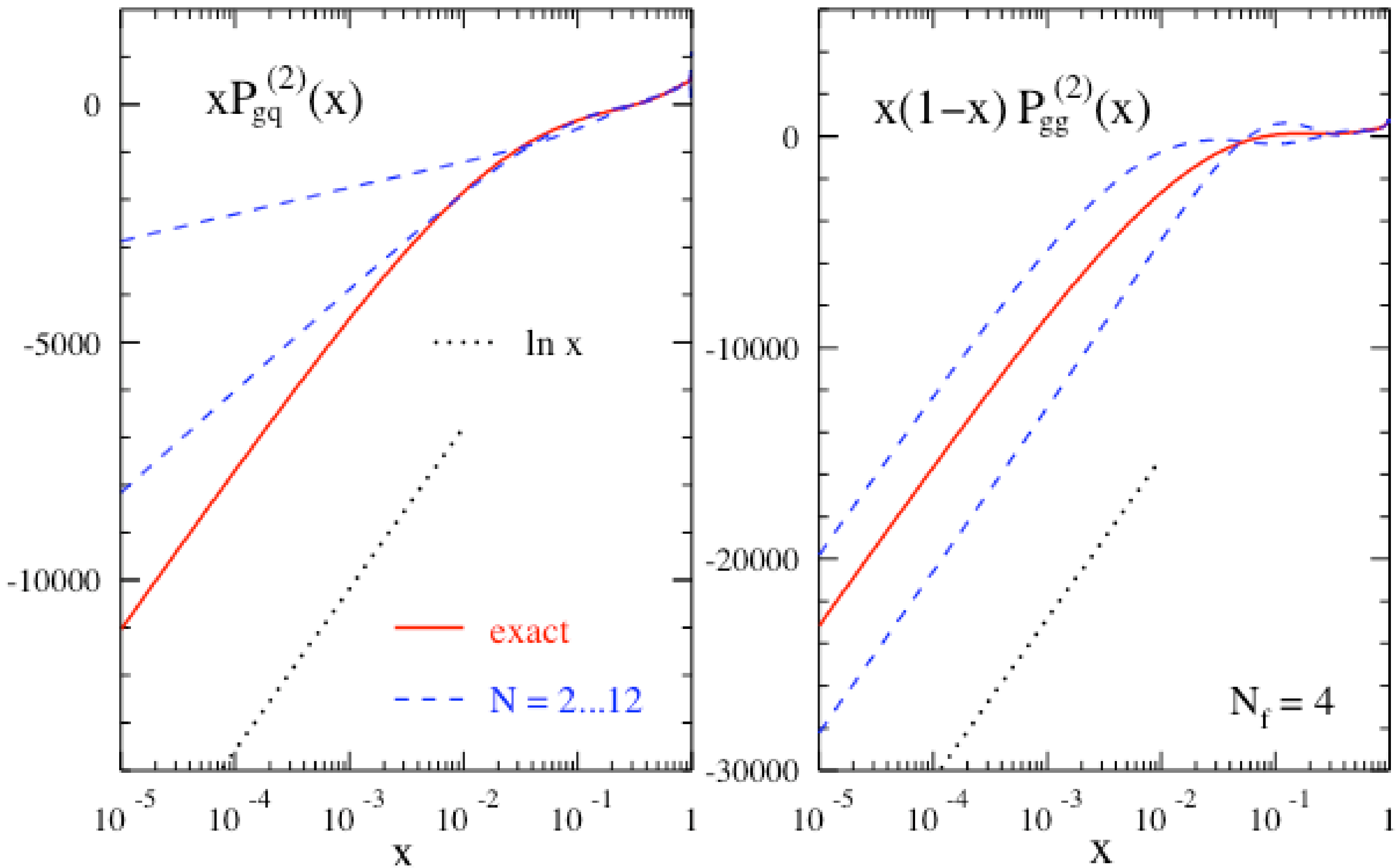
- NLO (or two-loop)
  $F_2, F_L$ — Bardeen Buras Duke Muta 1978
  anomalous dim/splitting functions — Curci Furmanski Petronzio 1980

- NNLO (or three-loop)
  $F_2, F_L$ — Zijlstra van Neerven 1992; Moch Vermaseren 1999
  anomalous dim/splitting functions — Moch Vermaseren Vogt 2004

  - the calculation of the three-loop anomalous dimension is the toughest calculation ever performed in perturbative QCD!
    - one-loop $\gamma_{ij}^{(0)}/P_{ij}^{(0)}$ ⇒ 18 Feynman diagrams
    - two-loop $\gamma_{ij}^{(1)}/P_{ij}^{(1)}$ ⇒ 350 Feynman diagrams
    - three-loop $\gamma_{ij}^{(2)}/P_{ij}^{(2)}$ ⇒ 9607 Feynman diagrams

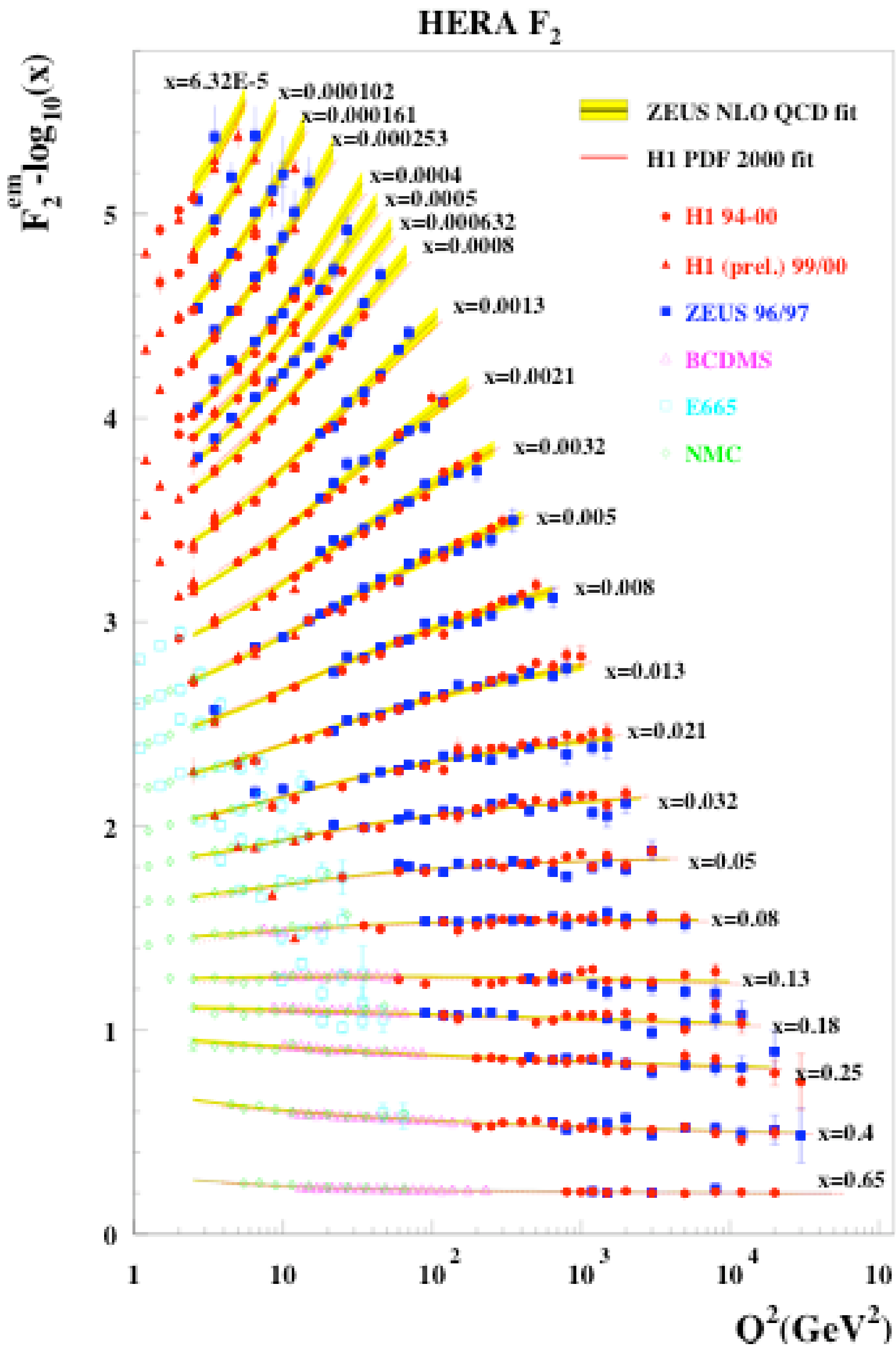
    20 man-year-equivalents, $10^6$ lines of dedicated algebra code

# Numerical examples




- exact NNLO results, estimates from fixed moments and leading small-$x$ term

# HERA $F_2$




Bjorken-scaling violations

H1, ZEUS: ongoing fits for PDF's; so far NNLO not included

# PDF global fits

J. Stirling, KITP collider conf 2004

## global fits

MRST: Martin Roberts Stirling Thorne

CTEQ: Pumplin et al.

Alekhin (DIS data only)

## method

Perform fit by minimising $\chi^2$ to all data, including both statistical and systematic errors

Start evolution at some $Q_0^2$, where PDF's are parametrised with functional form, e.g.

$xf(x,Q_0^2) = (1-x)^\eta(1+\epsilon x^{0.5}+\gamma x)x^\delta$

Cut data at $Q^2 > Q^2_{\rm min}$ and at $W^2 > W^2_{\rm min}$ to avoid higher twist contamination

Allow $\bar{u} \neq \bar{d}$ as implied by E866 Drell-Yan asymmetry data

## accuracy

NLO evolution
and fixed moments of NNLO

H1, ZEUS $F_2^{e^+p}(x,Q^2)$, $F_2^{e^-p}(x,Q^2)$

BCDMS $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

NMC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2), (F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$

SLAC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

E665 $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$

CCFR $F_2^{\nu(\bar\nu)p}(x,Q^2), F_3^{\nu(\bar\nu)p}(x,Q^2)$

$\rightarrow$ $q$, $\bar{q}$ at all $x$ and $g$ at medium, small $x$

H1, ZEUS $F_{2,c}^{e^+p}(x,Q^2) \rightarrow c$

E605, E772, E866 Drell-Yan $pN \rightarrow \mu\bar{\mu} + X \rightarrow \bar{q}\ (g)$

E866 Drell-Yan p,n asymmetry $\rightarrow \bar{u}, \bar{d}$

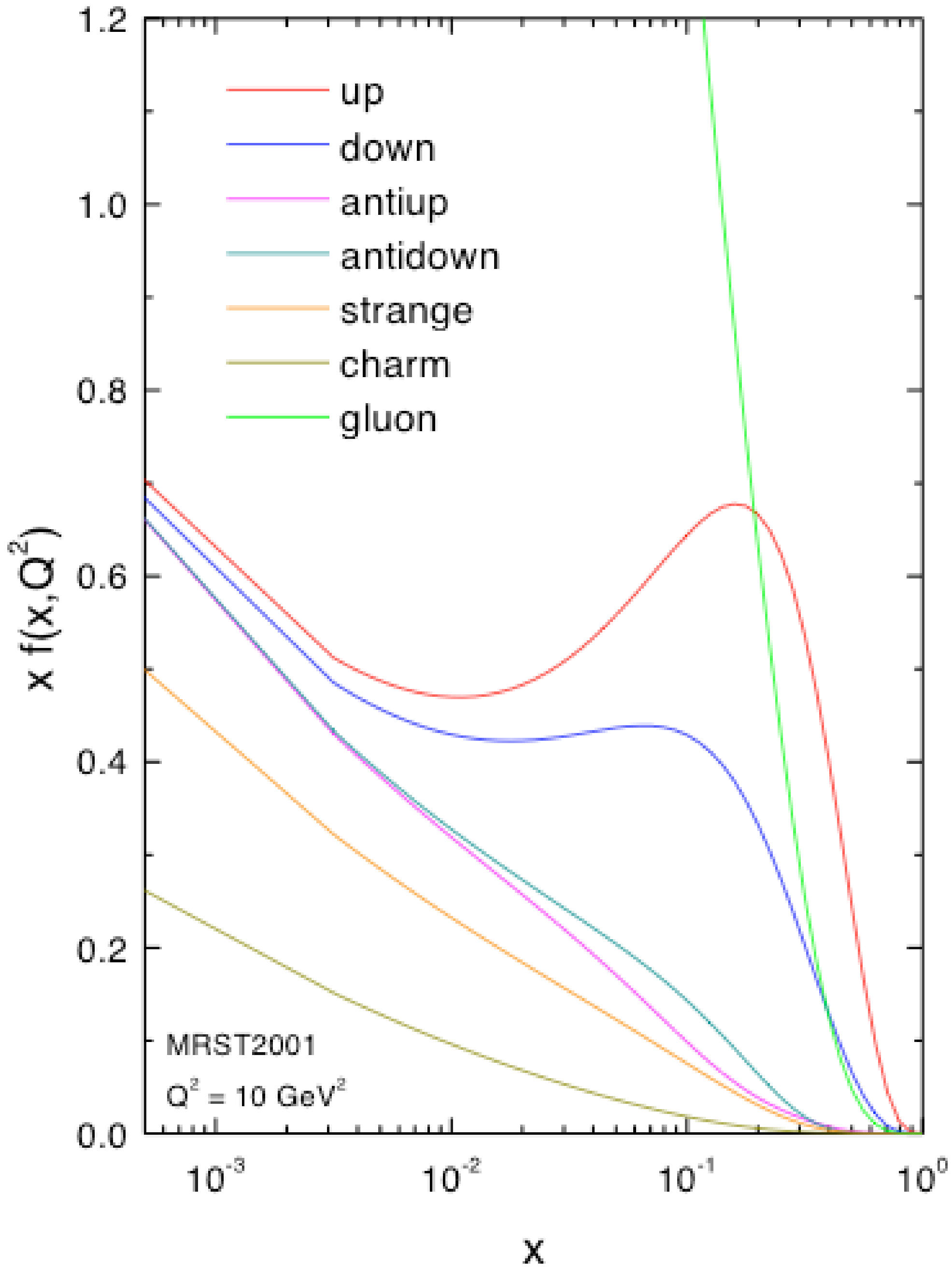
CDF W rapidity asymmetry $\rightarrow u/d$ ratio at high $x$

CDF, D0 Inclusive jet data $\rightarrow g$ at high $x$

CCFR, NuTeV Dimuon data constrains strange sea $s$, $\bar{s}$

- no prompt photon data included in the fits

# MRST 2001 PDF's



# PDF uncertainties

- direct effect on Tevatron & LHC cross section predictions
- various approaches being used, most notably
  - Hessian (error matrix) approach (H1, ZEUS, CTEQ, Alekhin)

    $$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j}(a_i - a_i^{(0)})H_{ij}(a_j - a_j^{(0)})$$

    $H$ is related to the covariance matrix of the parameters $C_{ij}(a) = \Delta\chi^2(H^{-1})_{ij}$

    diagonalise $H_{ij}$ and define PDF sets $S_i^{\pm}$ displaced along the eigenvector direction by $\Delta\chi^2 = \sum_i z_i^2$. Then uncertainty on physical quantity is given by

    $$(\Delta F)^2 = \frac{1}{2}\sum_i \left(F(S_i^{(+)}) - F(S_i^{(-)})\right)^2$$

  - Lagrange multiplier method (CTEQ, MRST)

    perform fit while constraining value of some physical quantity $F$. Minimise

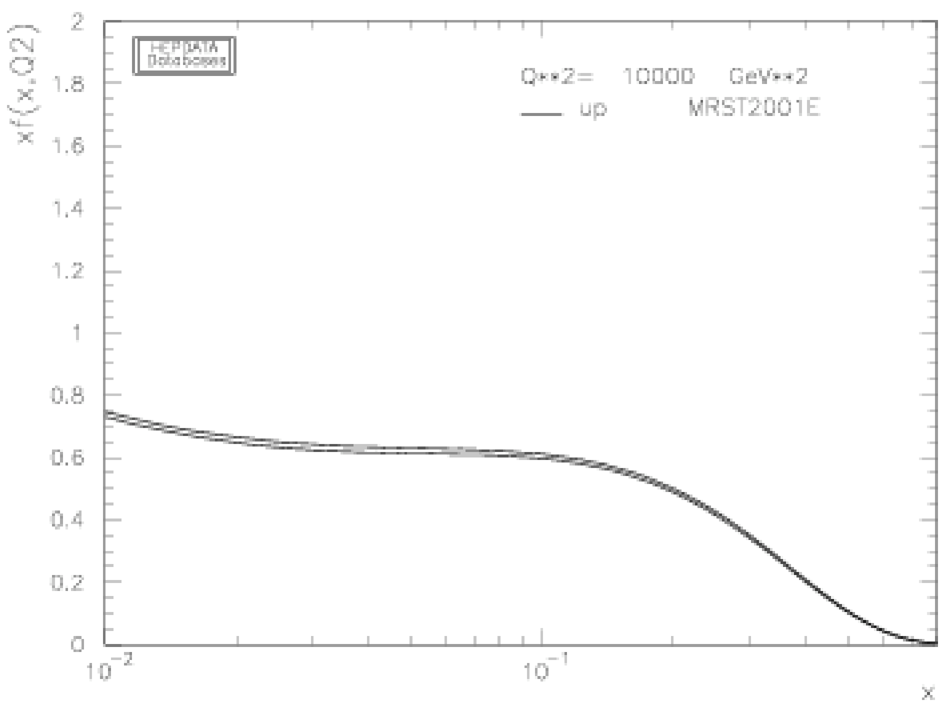
    $$\Psi(\lambda, a) = \chi^2_{\rm global}(a) + \lambda F(a)$$

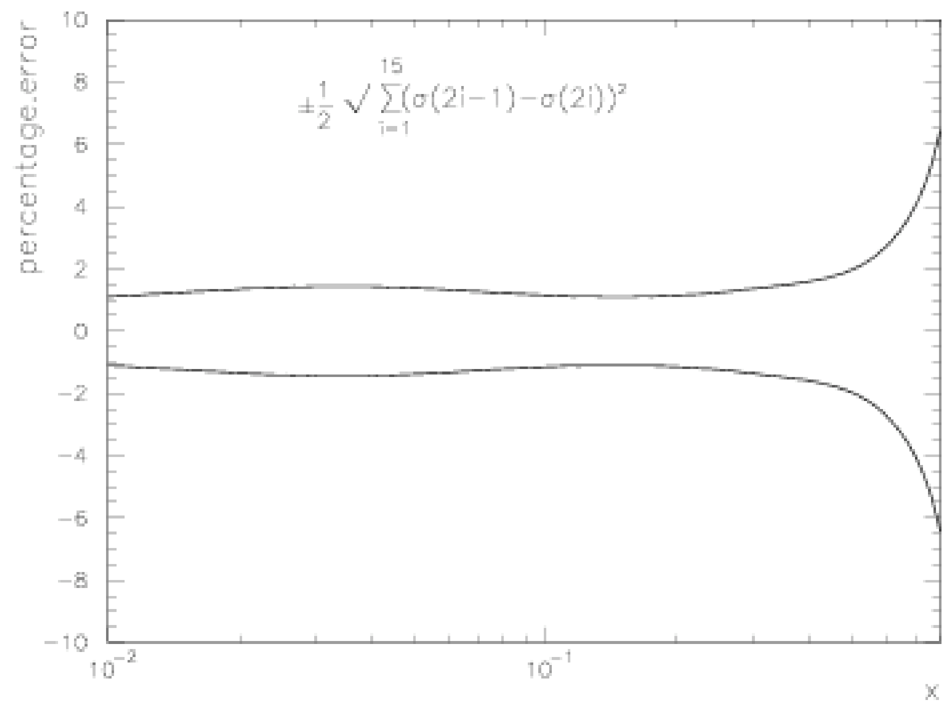
    for various values of $\lambda$ and parton parameters $\{a\}$. Gives set of best fits for particular values of parameter $F(a)$. Uncertainty then determined by deciding allowed range of $\Delta\chi^2$. Can also see which data sets in global fit most directly influenced by variation in $F(a)$

want to know more ? see e.g. R. Thorne et al hep-ex/0205233

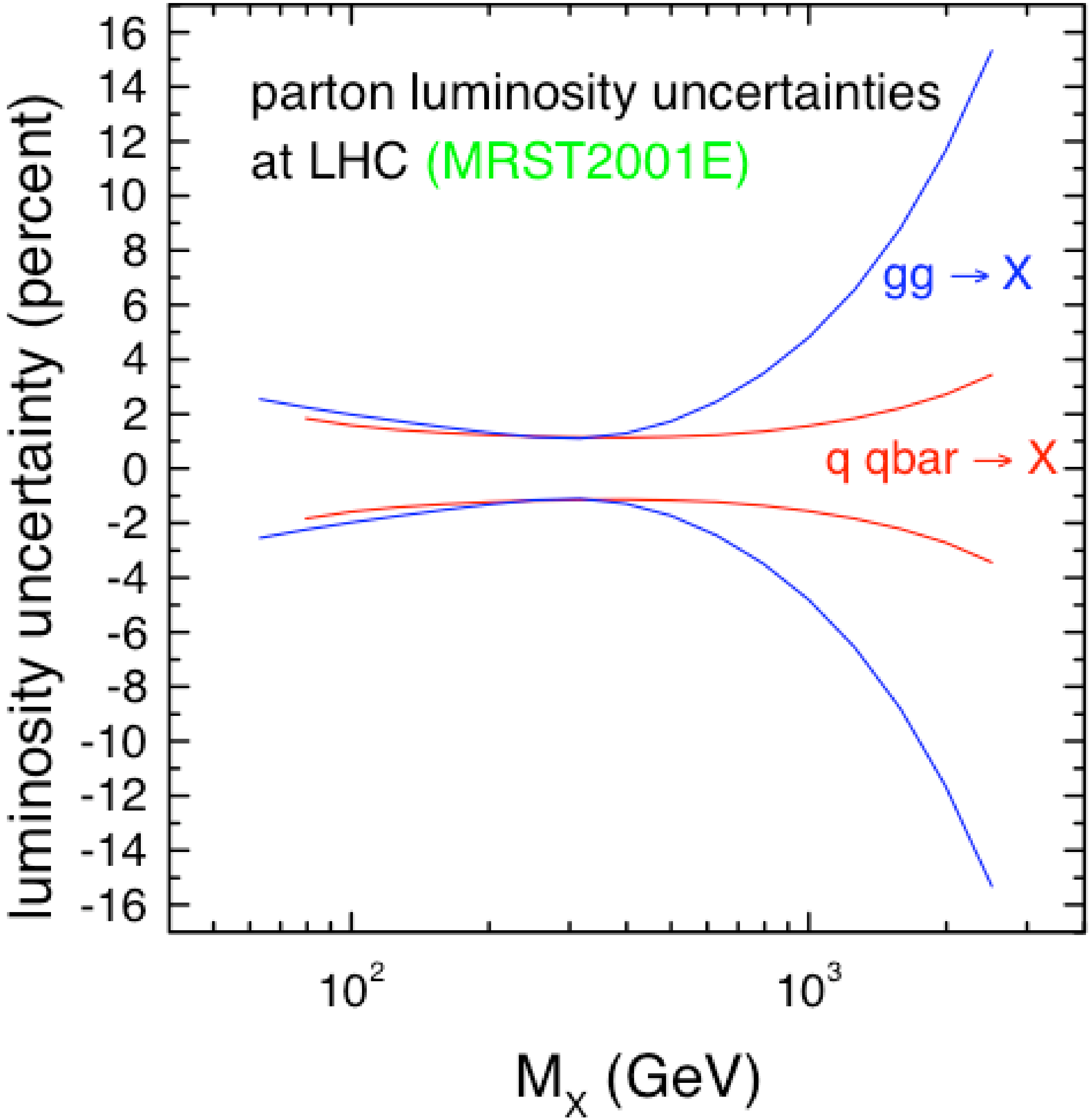
# Error on up distribution at $Q^2 = 10000 \text{ GeV}^2$

from MRST2001E (see hep-ph/0211080)



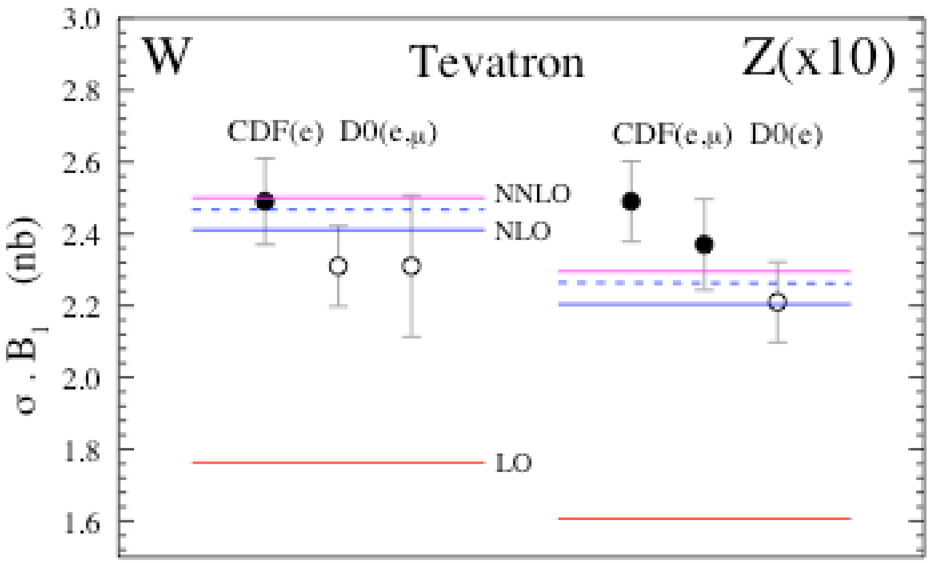



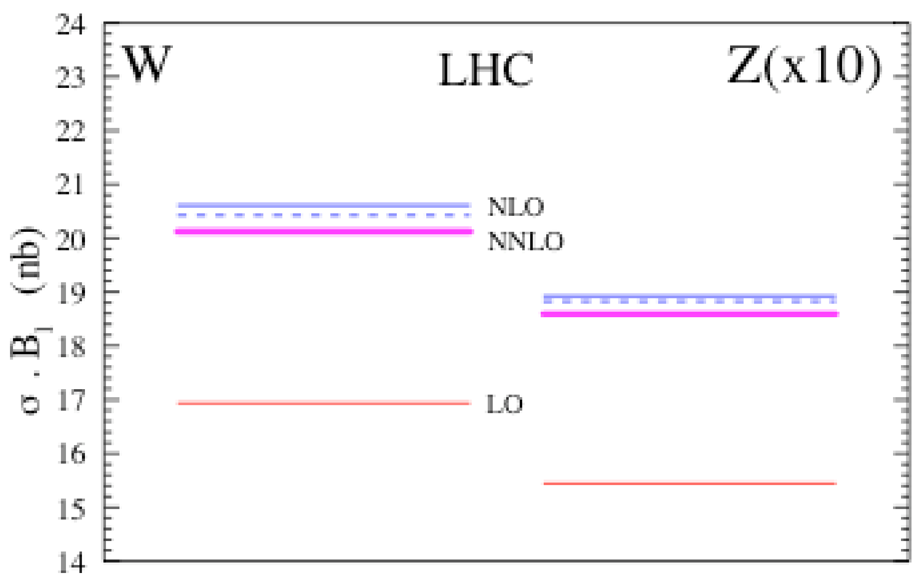

- Hessian method used
- error is about 2%




- for $q\bar{q}$ (relevant for Drell-Yan production)
- for $gg$ (relevant for Higgs production)

# $W, Z$ total cross sections



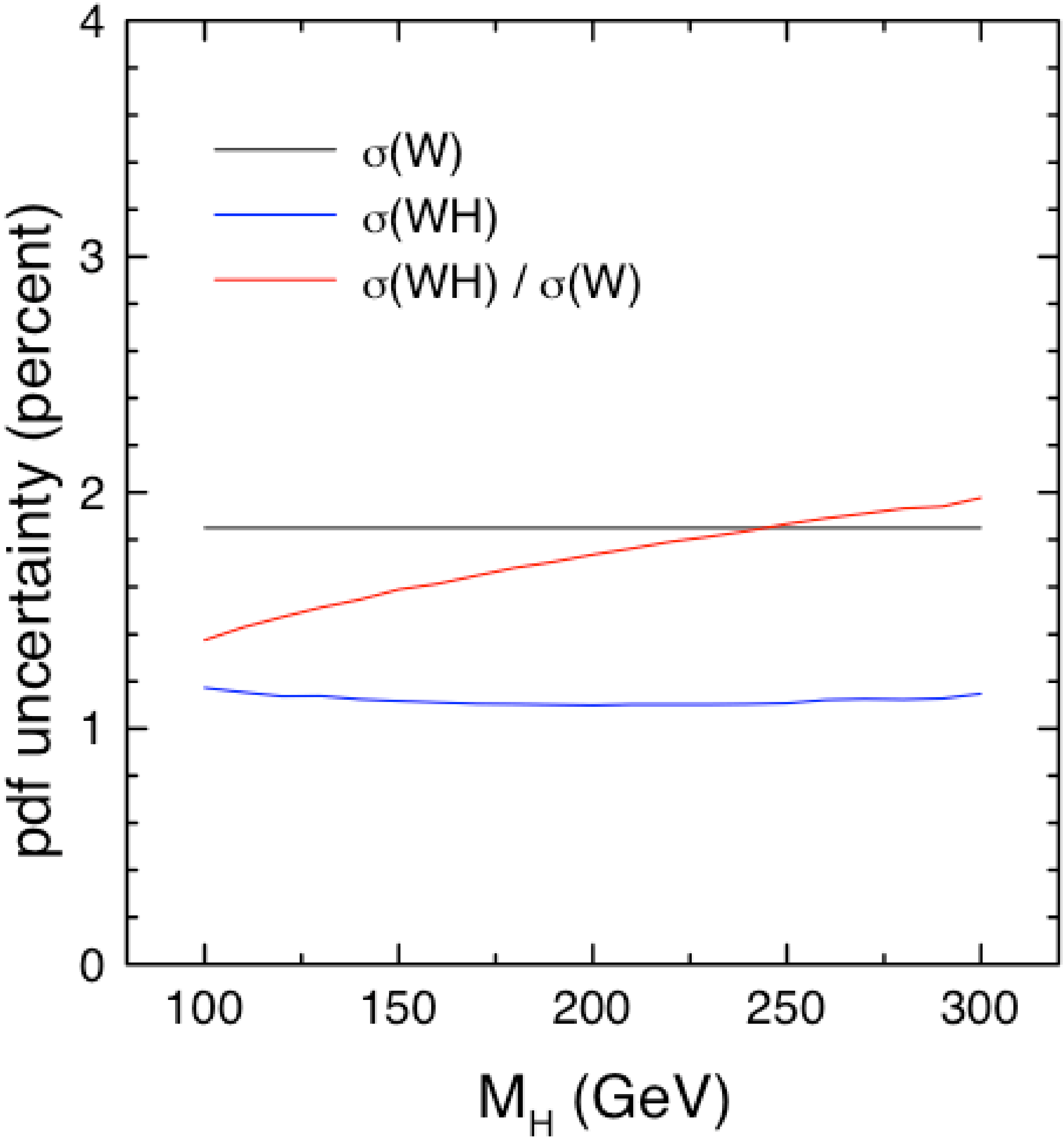




- MRST2001
- NNLO: only few fixed moments
- current best (MRST) estimate

  $\delta\sigma_{W,Z}^{\mathrm{NNLO}}(\text{total pdf}) = \pm 4\%$

  (expt. pdf error is 2%)
- larger uncertainty in the NLO prediction, because of problems at small $x$ in the global fit to DIS data and because large rapidity $W, Z$'s sample small $x$

# PDF uncertainty on $W, WH$ cross sections at LHC

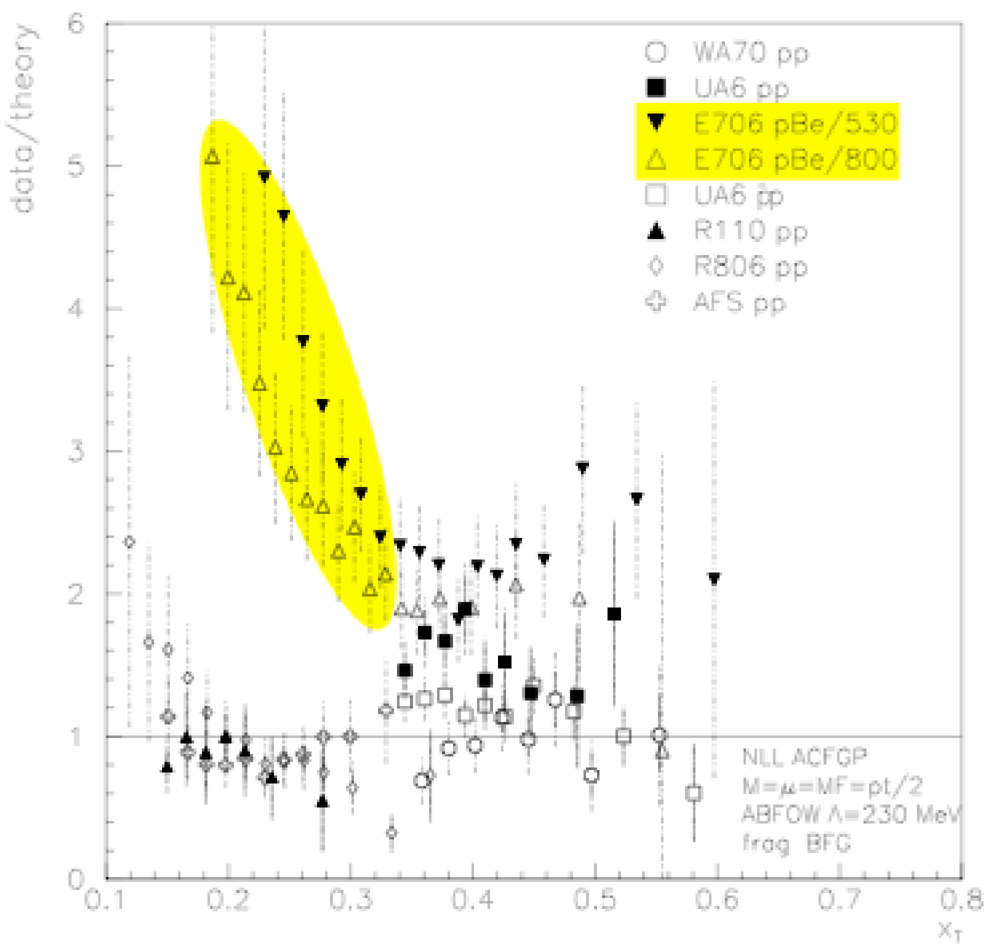



- MRST2001E
- use $\sigma(W), \sigma(Z)$ as ``standard candles'', i.e. to calibrate other cross sections, e.g. $\sigma(WH)$
- $\sigma(WH)$ more precisely predicted because it samples quark PDF's at higher $x$ than $\sigma(W)$

# Hinc sunt photones

## Photons at fixed-target experiments

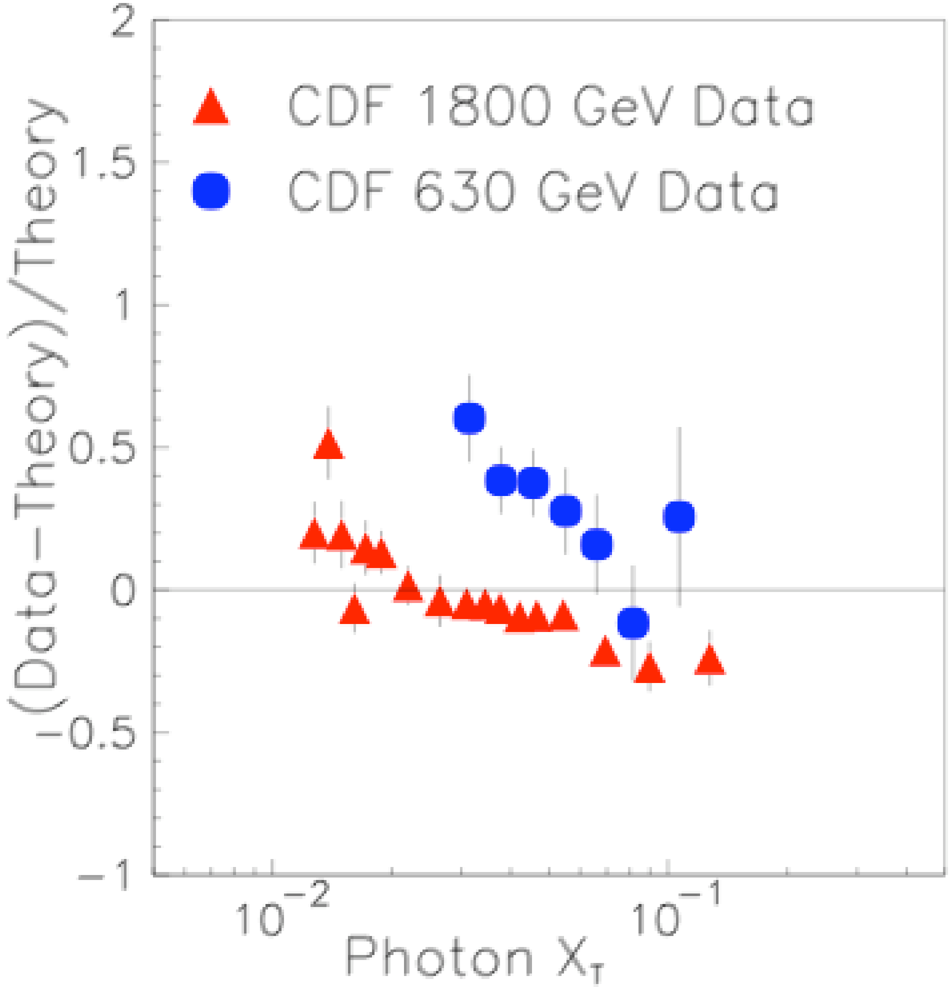
- probe the gluon distribution at high $x$
  at $\sqrt{s} = 1800 \text{ GeV}, \quad p_{Tjet} = 180 \text{ GeV}$ ➡ $x_T = 0.2$

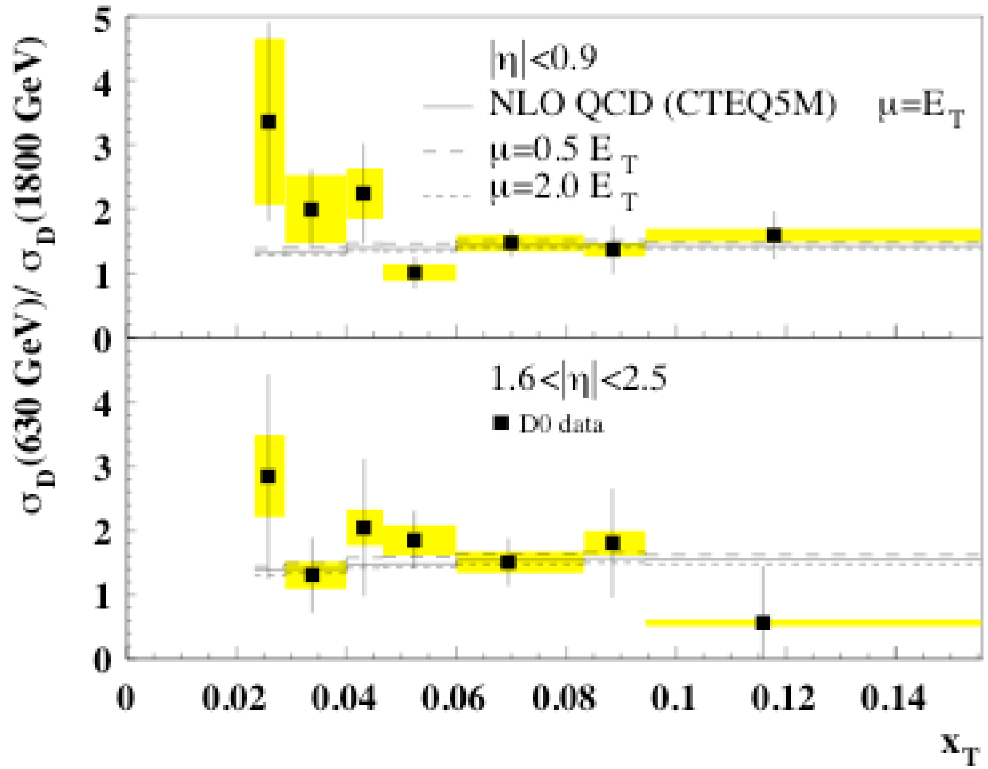



P. Aurenche et al. 1998

- data are not consistent with theory, and (even more worrisome) are not consistent with each other
- currently they are not used in PDF fits

# Photons at the Tevatron at 1800 GeV and 630 GeV

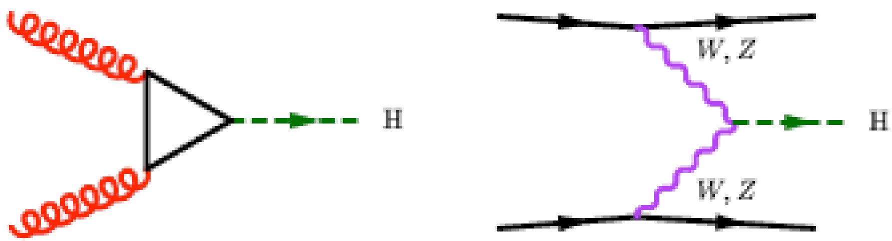



- data are not consistent with theory (but D0 is better off than CDF)
- Problems ? TH: Narrow isolation cones used by experiments

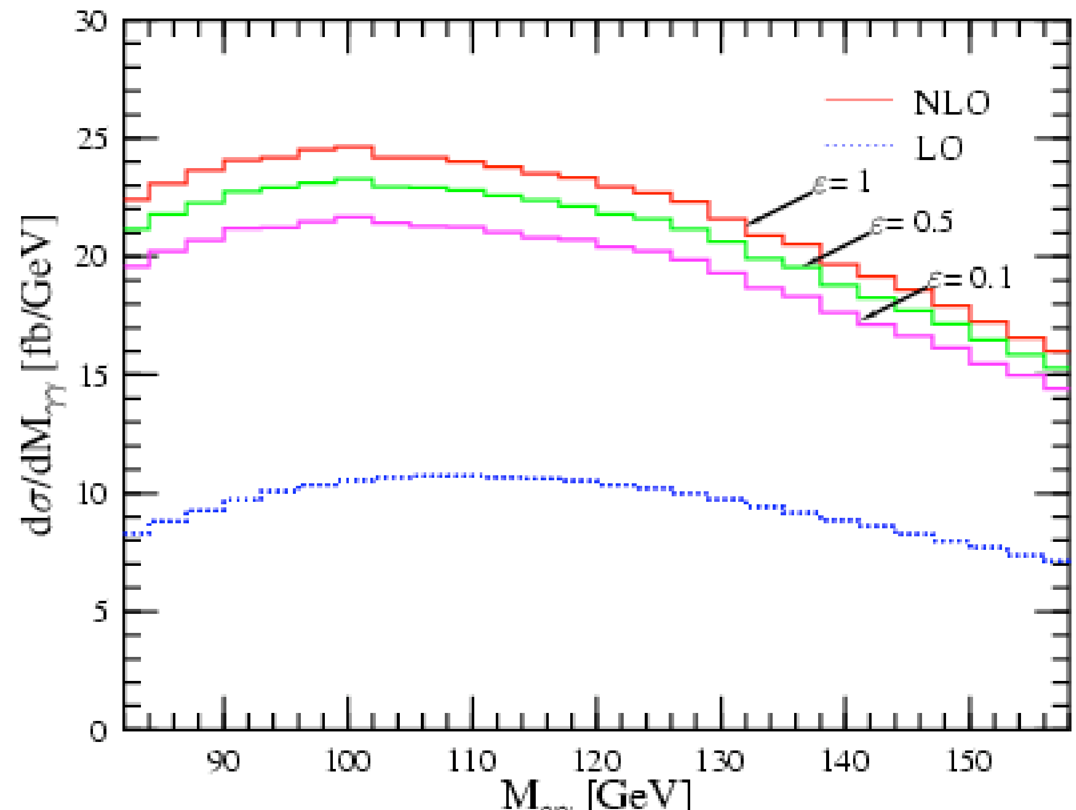
# Photons as a background to Higgs searches

- Higgs production



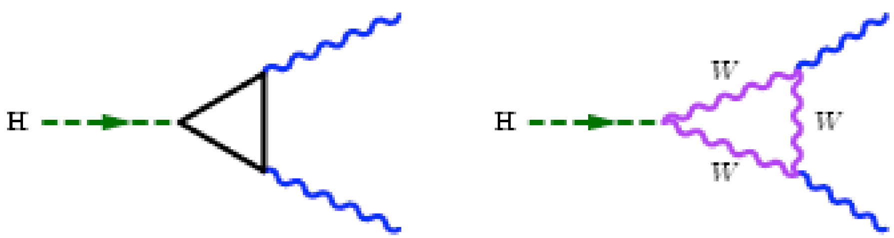

$pp \to \gamma\gamma + \text{jet}$ at LHC

di-photon invariant mass distribution




F. Maltoni Z. Nagy Z. Trocsanyi VDD 2003

- Higgs decay




- Di-photon decay important in the low-mass Higgs searches
- isolation cone $R_\gamma = 0.4$
  hadronic energy allowed inside cone is $E_{T,\max} = \epsilon \, p_{T\gamma}$
- used Frixione's photon isolation criterion (avoids use of fragmentation functions)

$$E_T \leq E_{T,max} \left( \frac{1 - \cos r}{1 - \cos R_\gamma} \right)^n$$

- $K$ factor is $> 2$

# Conclusions

- QCD is an extensively developed and tested gauge theory
- a lot of progress in the last 4-5 years in
  - MonteCarlo generators
  - NLO cross sections with one more jet
  - NNLO computations
- better and better approximations of signal and background for Higgs and New Physics
- new formal developments (I didn't discuss): QCD as a string theory in twistor space E. Witten 2003
  - novel ways of computing (analytically) tree multi-parton matrix elements and (N=4) loop matrix elements F. Cachazo P. Svrcek E. Witten 2004