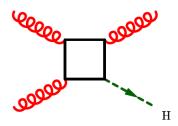
Top and b mass effects in Higgs production at LHC

Vittorio Del Duca INFN LNF

in collaboration with R. Bonciani, H. Frellesvig, M. Hidding, V. Hirschi, F. Moriello, G. Salvatori, G. Somogyi, F. Tramontano

ETH 2 April 2024

Higgs p_T distribution at LHC



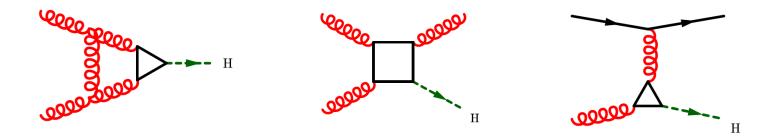
- high- p_T tail of the Higgs p_T distribution is sensitive to the structure of the loop-mediated Higgs-gluon coupling New Physics particles circulating in the loop would modify it
- QCD NLO corrections to the top- and b-quark mass effects on the Higgs p_T distribution, in the on-shell and MSbar mass renormalisation schemes

Higgs production at LHC

In proton collisions, the Higgs boson is produced mostly via gluon fusion
The gluons do not couple directly to the Higgs boson
For matter, the coupling is mediated by a heavy quark loop
The largest contribution comes from the top-quark loop
The production mode is (roughly) proportional to the top Yukawa coupling yt²

QCD NLO corrections are known for top-, b- and charm-quark loops
 (in principle for any heavy quark mass)

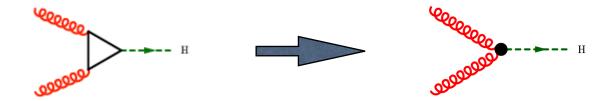
Djouadi Graudenz Spira Zerwas 1991-1995



- QCD NLO corrections are about 100% larger than leading order
- QCD NNLO corrections are known for top- and b-quark loops (in principle for any heavy quark mass)

Higgs Effective Field Theory





all amplitudes are reduced by one loop

$$R_{LO} = \frac{\sigma_{ex:t}^{LO}}{\sigma_{EFT}^{LO}} = 1.063$$

rescaled HEFT (rHEFT) tuned to reproduce the exact (only top) LO σ

in HEFT QCD corrections have been computed at N3LO

Anastasiou Duhr Dulat Herzog Mistlberger 2015 Mistlberger 2018

(in terms of MPLs and elliptic integrals)

when the N3LO computation raised the central value by 3% and featured a 3-4% scale variation

QCD NLO corrections

					$\frac{\sigma_{t+b}}{\sigma_{t+b}} - 1$
σ^{LO}_{EFT}	15.05 pb	σ_{EFT}^{NLO}	34.66 pb		σ_t
$R_{LO}\sigma_{EFT}^{LO}$	16.00 pb	$R_{LO} \sigma_{EFT}^{NLO}$	36.84 pb	$LOO(\alpha_s^2)$	- 6.6 %
$\sigma_{ex;t}^{LO}$	$16.00~\rm pb$	$\sigma^{NLO}_{ex;t}$	$36.60~\rm pb$	NLO O(α_s^2) + O(α_s^3)	- 4.5 %
$\sigma^{LO}_{ex;t+b}$	14.94 pb	$\sigma^{NLO}_{ex;t+b}$	34.96 pb	NLO O(α_s^3)	- 2.8 %
$\sigma^{LO}_{ex;t+b+c}$	14.83 pb	$\sigma^{NLO}_{ex;t+b+c}$	34.77 pb		2.0 70

Anastasiou Duhr Dulat Furlan Gehrmann Herzog Lazopoulos Mistlberger 2016

Higgs production

Handbook 4 of LHC Higgs Cross Sections 2016

including quark-mass effects and QCD-EW interference the cross section is

$$\sigma = 48.58 \,\text{pb}_{-3.27 \,\text{pb} \,(-6.72\%)}^{+2.22 \,\text{pb} \,(+4.56\%)} \,\,\text{(theory)} \pm 1.56 \,\text{pb} \,(3.20\%) \,\,\text{(PDF} + \alpha_s)$$

The breakdown of the cross section

$$48.58 \,\mathrm{pb} = 16.00 \,\mathrm{pb} \quad (+32.9\%) \qquad (\mathrm{LO, \, rEFT}) \\ + 20.84 \,\mathrm{pb} \quad (+42.9\%) \qquad (\mathrm{NLO, \, rEFT}) \\ - 2.05 \,\mathrm{pb} \quad (-4.2\%) \qquad ((t, b, c), \, \mathrm{exact \, NLO}) \\ + 9.56 \,\mathrm{pb} \quad (+19.7\%) \qquad (\mathrm{NNLO, \, rEFT}) \\ + 0.34 \,\mathrm{pb} \quad (+0.2\%) \qquad (\mathrm{NNLO, \, 1}/m_t) \\ + 2.40 \,\mathrm{pb} \quad (+4.9\%) \qquad (\mathrm{EW, \, QCD-EW}) \\ + 1.49 \,\mathrm{pb} \quad (+3.1\%) \qquad (\mathrm{N}^3\mathrm{LO, \, rEFT})$$

Anastasiou Duhr Dulat Furlan Gehrmann Herzog Lazopoulos Mistlberger 2016

Higgs production

Handbook 4 of LHC Higgs Cross Sections 2016

6 sources of uncertainties due to: higher orders truncation of the threshold expansion PDFs

NLO corrections to QCD-EW interference quark mass effects (2: top mass and top-b interference) at NNLO

δ (scale)	δ (trunc)	$\delta(\text{PDF-TH})$	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
+0.10 pb -1.15 pb	±0.18 pb	±0.56 pb	±0.49 pb	±0.40 pb	±0.49 pb
$^{+0.21\%}_{-2.37\%}$	₹0.37%	$\pm 1.16\%$	±1%	±0.83%	±1%

 $\delta(\text{trunc}) = 0.11 \text{ pb}$ Mistlberger 2018

 $\delta(I/m_t) = -0.26\%$ Czakon Harlander Klappert Niggetiedt 2021

 $\delta(t, b) = -4.6\%$ Czakon Eschment Niggetiedt Poncelet Schellenberger 2023

QCD NNLO corrections

Top and b-quark mass corrections at NNLO

Czakon Harlander Klappert Niggetiedt 2021 (top)
Czakon Eschment Niggetiedt Poncelet Schellenberger 2023 (top + b)

Order	$\sigma_{ m HEFT} \; [m pb]$	$(\sigma_t - \sigma_{ ext{HEFT}}) \; [ext{pb}]$	$\sigma_{t ext{-}b ext{-}interference} \; ext{[pb]}$	$\sigma_{t ext{-}b ext{-interference}}/\sigma_{ ext{HEFT}}$ [%]	
$\sqrt{s} = 8 \text{ TeV}$					
$\mathcal{O}(lpha_s^2)$	+7.39	_	-0.895		
LO	$7.39^{+1.98}_{-1.40}$	_	$-0.895^{+0.17}_{-0.24}$	-12	
$\mathcal{O}(\alpha_s^3)$	+9.14	-0.0873	-0.269(2)		
NLO	$16.53^{+3.63}_{-2.73}$	$-0.0873^{+0.030}_{-0.052}$	$-1.16^{+0.10}_{-0.08}$	$-7.0^{+1.0}_{-0.8}$	
$\mathcal{O}(\alpha_s^4)$	+4.19	+0.0523(2)	+0.167(3)		
NNLO	$20.72^{+1.84}_{-2.06}$	$-0.350(2)_{-0.013}^{+0.048}$	$-0.998(4)_{-0.05}^{+0.12}$	$-4.8^{+0.9}_{-0.8}$	
$\sqrt{s} = 13 \text{ TeV}$					
$\mathcal{O}(\alpha_s^2)$	+16.30	_	-1.975		
LO	$16.30^{+4.36}_{-3.10}$	_	$-1.98^{+0.37}_{-0.53}$	-12	
$\mathcal{O}(lpha_s^3)$	+21.14	-0.3029(2)	-0.447(4)		
NLO	$37.44^{+8.42}_{-6.29}$	$-0.3029(2)_{-0.17}^{+0.10}$	$-2.42^{+0.19}_{-0.12}$	$-6.5^{+0.9}_{-0.8}$	
$\mathcal{O}(lpha_s^4)$	+9.72	+0.147(1)	+0.434(8)		
NNLO	$47.16^{+4.21}_{-4.77}$	$-0.158(1)_{-0.03}^{+0.13}$	$-1.99(1)_{-0.15}^{+0.30}$	$-4.2^{+0.9}_{-0.8}$	

	σ_t	σ_{t+b}	$\frac{\sigma_{t+b}}{\sigma_t} - 1$	$\frac{\sigma_{t+b}}{\sigma_{\text{HEFT}}} - 1$
$O(\alpha_s^3)$	20.84 pb	20.39 pb	- 2.14 %	- 3.55 %
$O(\alpha_s^2 + \alpha_s^3)$	37.14 pb	34.71 pb	- 6.5 %	- 7.3 %
$O(\alpha_s^4)$	9.87 pb	10.30 pb	+ 4.3 %	+ 6.0 %
$O(\alpha_s^2 + \alpha_s^3 + \alpha_s^4)$	47.00 pb	45.01 pb	- 4.2 %	- 4.6 %

- top-b interference has a larger effect than top-quark corrections
- Θ top-b interference as large at $O(\alpha_s^4)$ as it is at $O(\alpha_s^3)$, but with opposite sign (and larger than expected from NLO scale uncertainties)
- for top-quark mass, used $m_t^2/m_{H^2} = 23/12$ (on-shell scheme)

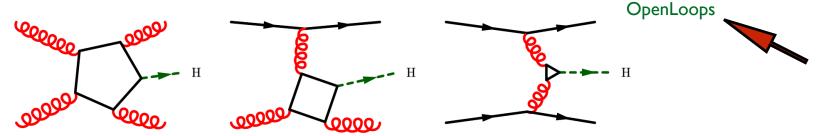
The main obstacle when calculating the total cross section with full top-mass dependence are the two-loop single-emission amplitudes.

Czakon Harlander Klappert Niggetiedt 2021

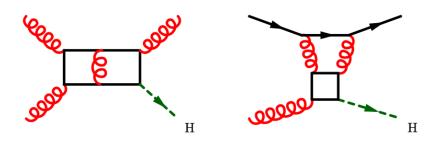
QCD NNLO corrections

Higgs + 4-parton amplitudes at one loop

VDD Kilgore Oleari Schmidt Zeppenfeld 2001 Budge Campbell De Laurentis K. Ellis Seth 2020



Higgs + 3-parton amplitudes at two loops

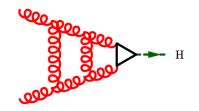


top loop:

Jones Kerner Luisoni 2018 Czakon Harlander Klappert Niggetiedt 2021

Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

gg→Higgs amplitudes at three loops



one scale: one & two top loops
top loop + light-quark loop

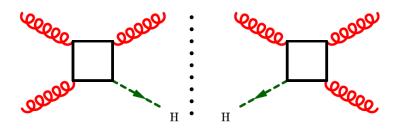
two scales: top loop + b-quark loop

Czakon Niggetiedt 2020 Harlander Prausa Usovitsch 2019

Niggetiedt Usovitsch 2023

Higgs pt distribution at LHC

leading order



K. Ellis Hinchliffe Soldate van der Bij 1988

- high- p_T tail of the Higgs p_T distribution is sensitive to the structure of the loop-mediated Higgs-gluon coupling New Physics particles circulating in the loop would modify it
- Θ in high- p_T regime, clean signature of decay products $(H \to \gamma \gamma)$
- **QCD NLO** corrections
 - for the top-quark, with on-shell scheme

 Jones Kerner Luisoni 2018

 Chen Huss Jones Kerner Lang Lindert Zhang 2021
 - for the top-quark, with on-shell and MSbar schemes for top- and b-quarks (for any heavy quark mass), with MSbar scheme

Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

 Θ HEFT $m_H << 2m_t$ and $p_T << m_t$ Baur Glover 1990

QCD corrections are known at NNLO in HEFT, and yield a 15% increase wrt NLO

Boughezal Caola Melnikov Petriello Schulze 2015 Boughezal Focke Giele Liu Petriello 2015 Chen Cruz-Martinez Gehrmann Glover Jaquier 2016

Higgs p_T distribution at LHC

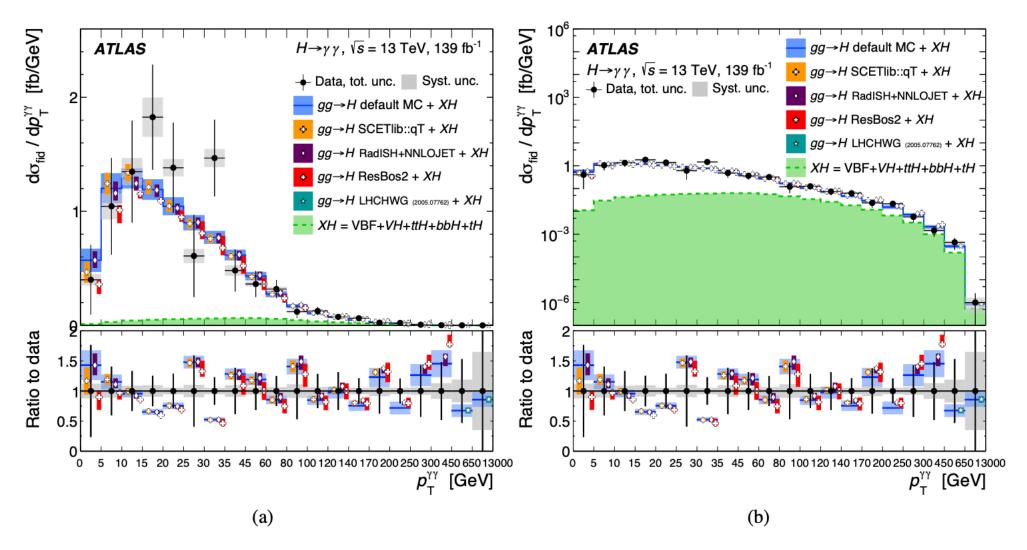
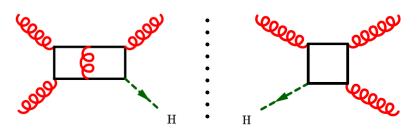


Figure 8: Particle-level fiducial differential cross-sections times branching ratio for the diphoton variable $p_{\rm T}^{\gamma\gamma}$ in (a) linear and (b) logarithmic scale. The measured cross-sections are compared with several predictions changing the

Higgs p_T distribution at NLO



virtual corrections



top-quark loop

Jones Kerner Luisoni 2018 Czakon Harlander Klappert Niggetiedt 2021

top- and b-quark (any heavy quark) loop

Bonciani VDD Frellesvig Henn Moriello V. Smirnov 2016 all above + Hidding Maestri Salvatori 2019

Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

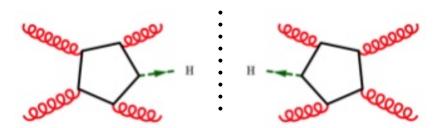
multi-scale problem with complicated analytic structure elliptic iterated integrals appear



in the loop



real corrections

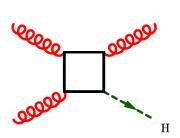


VDD Kilgore Oleari Schmidt Zeppenfeld 2001 Budge Campbell De Laurentis K. Ellis Seth 2020





one-loop amplitudes for Higgs + 3-partons



leading order: up to $O(\varepsilon^2)$

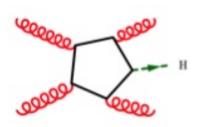
analytic: up to $O(\epsilon^0)$

K. Ellis Hinchliffe Soldate van der Bij 1988

numeric: up to $O(\varepsilon^2)$

(numeric) derivative for mass renormalisation

one-loop amplitudes for Higgs + 4-partons



NLO real corrections: up to $O(\epsilon^0)$

analytic: unitarity-cut methods (taken from MCFM-9.1)

Budge Campbell De Laurentis K. Ellis Seth 2020

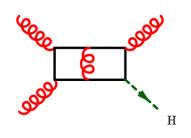
numeric: GoSam & MG5_aMC

run time

analytic: few ms/pt

numeric: O(100) times slower than analytic

two-loop amplitudes for Higgs + 3-partons



NLO virtual corrections

amplitude \rightarrow form factors \rightarrow scalar integrals \rightarrow Master Integrals **IBP**

run time: 5 — 60 min/pt

FIRE-KIRA

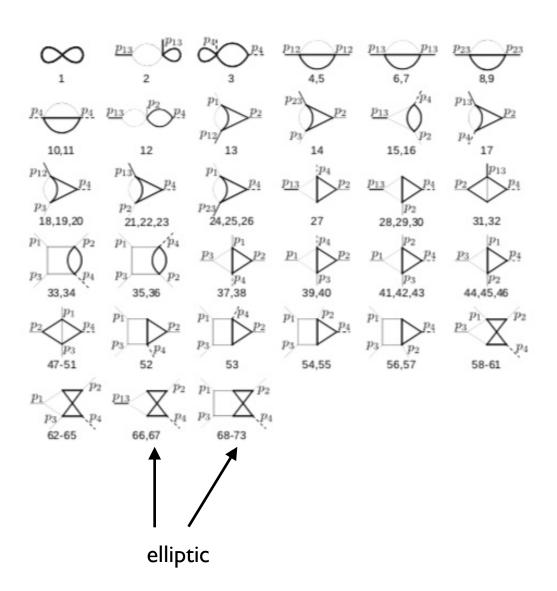
two masses

4 scales, s, t, m_H , $m_t \rightarrow 3$ external parameters

7 seven-propagator integral families

Bonciani VDD Frellesvig Henn Moriello Smirnov 2016 (A, B, C, D) Bonciani VDD Frellesvig Henn Hidding Maestri Moriello Salvatori Smirnov 2019 (F) Frellesvig Hidding Maestri Moriello Salvatori 2019 (G) Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022 (H) elliptic # MIs A: 72 B B: 5 C: 45 D: 17 F: 73 G: 84 H: 12 colour conservation elliptic

Family F: 73 MIs (65 in the polylogarithmic sector, 8 in the elliptic sector) alphabet: 69 independent letters, with 12 independent square roots



Differential Equations

Differential Equation method to solve the MIs

$$\partial_i f(x_n; \varepsilon) = A_i(x_n; \varepsilon) f(x_n; \varepsilon)$$

f: N-vector of MIs, $A_i: N \times N$ matrix, i=1,...,n external parameters

but in some cases &-independent form

$$\partial_i f(x_n; \varepsilon) = \varepsilon A_i(x_n) f(x_n; \varepsilon)$$

Henn 2013

solution in terms of iterated integrals

mass values are floating \rightarrow DEs solved with 3 (top) or 4 (top and b) external parameters

DEs: Series Expansion Method

Take two points $(a_1, ..., a_n)$ and $(b_1, ..., b_n)$ in the *n*-dim parameter space, and parametrise the contour $\gamma(t)$ that connects the two points

$$\gamma(t): t \to \{x_1(t), \dots, x_n(t)\}$$
 $\vec{x}(0) = \vec{a}, \quad \vec{x}(1) = \vec{b}$

and write the differential equation with respect to t.

Then find a solution about a point τ by series expanding the coefficient matrix A and then iteratively integrating it.

The procedure works for both polylogarithmic and elliptic sectors

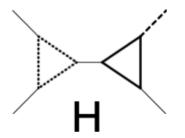
Moriello 2019

- numerical solution of DEs through DiffExp:
 Mathematica implementation of Moriello's series expansion method
 Hidding 2021
- checked with AMFlow Liu Ma Wang 2018

two-loop amplitudes for Higgs + 3-partons: Renormalisation

Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

- oupling constant: 5-flavour running in MSbar
- renormalisation:
 - \bigcirc top Yukawa coupling and top mass in OS scheme (massless b)
 - \bigcirc top Yukawa coupling and top mass in MSbar scheme (massless b)
 - with top and b Yukawa couplings and masses in MSbar scheme



massive b in Higgs-b loop massless b in b loop

alternative: massive b everywhere, but requires 4-flavour running and including $gg \rightarrow Hbb$

two-loop amplitudes for Higgs + 3-partons: validation checks

IR poles

$$\mathcal{M}_{ij,IR}^{(2)} \propto I_{ij}^{(1)}(\{p\},\epsilon)\mathcal{M}_{ij}^{(1)}$$

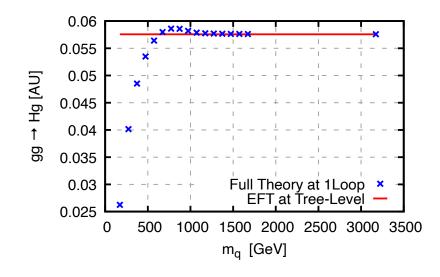
with insertion operators

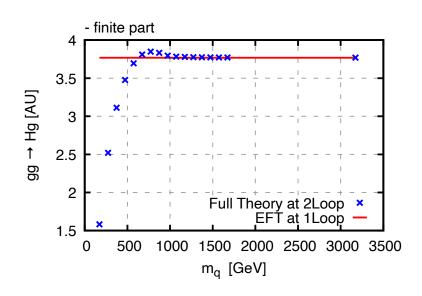
$$I_{gg}^{(1)}(\{p\},\epsilon) = -\frac{\alpha_S}{\pi} \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{N_c}{\epsilon^2} + \frac{\beta_0}{\epsilon}\right) \left[\left(\frac{\mu^2}{-s}\right)^{\epsilon} + \left(\frac{\mu^2}{-t}\right)^{\epsilon} + \left(\frac{\mu^2}{-u}\right)^{\epsilon}\right]$$

$$I_{q\bar{q}}^{(1)}(\{p\},\epsilon) = -\frac{\alpha_S}{2\pi} \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \left\{ -\left(\frac{N_c}{\epsilon^2} + \frac{3N_c}{4\epsilon} + \frac{\beta_0}{2\epsilon}\right) \left[\left(\frac{\mu^2}{-t}\right)^{\epsilon} + \left(\frac{\mu^2}{-u}\right)^{\epsilon}\right] + \frac{1}{N_c} \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right) \left(\frac{\mu^2}{-s}\right)^{\epsilon}\right\}$$

agreement with HEFT limit

$$\mathcal{M} = \mathcal{M}_{HEFT} + \mathcal{O}\left(\frac{1}{M_t}\right)$$

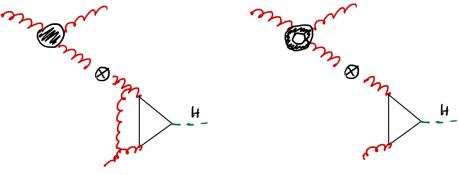




two-loop amplitudes for Higgs + 3-partons: validation checks

soft and collinear limits

(these are checks on real-virtual parts of NNLO cross section, however they are feasible on our two-loop amplitudes)



Aglietti Bonciani Degrassi Vicini 2006

one-loop 2-parton splitting functions

Bern Dixon Dunbar Kosower 1994 Bern Kilgore Schmidt VDD 1998-99 Kosower Uwer 1999

one-loop I-soft-gluon factor

Bern Kilgore Schmidt VDD 1998-99 Catani Grazzini 2000



checked also "two-loop photon correction"

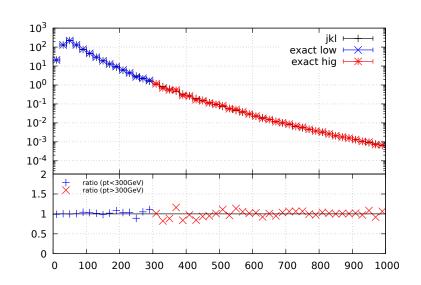
Higgs p_T distribution at NLO: checks with previous results

inclusive p_T distribution ($p_{T,j} > 30$ GeV) with OS mass renormalisation

our result
$$\sigma_{NLO}=14.37\pm0.05\,\mathrm{pb}$$

Chen Huss Jones Kerner Lang Lindert Zhang 2021 (Jones Kerner Luisoni 2018-2021)

$$\sigma_{NLO} = 14.15 \pm 0.07 \,\mathrm{pb}$$



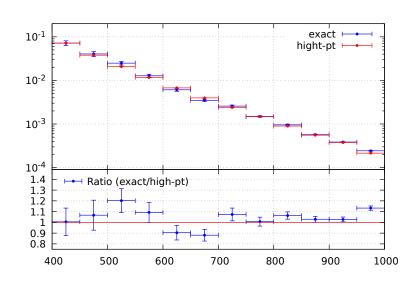
\mathbf{Q} high p_T tail of distribution

checked with approximate high-p_T distribution

Lindert Melnikov Kudashkin Wever 2018

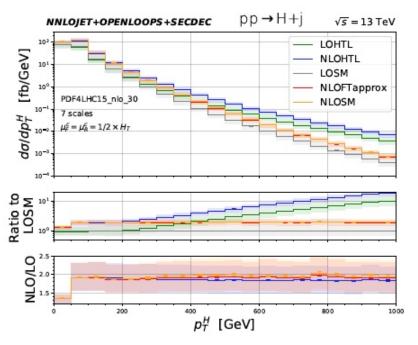
based on approximate high- p_T two-loop amplitudes

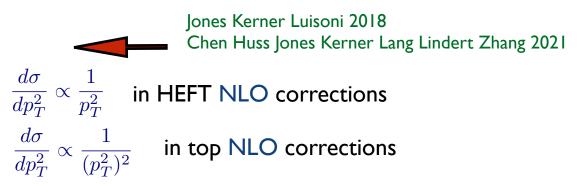
Melnikov Kudashkin Wever 2018



Higgs pt distribution at LHC

QCD NLO corrections for the top-quark (on-shell mass renormalisation)

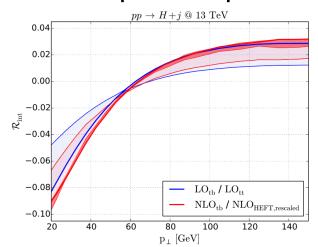




NLO/LO in HEFT and top loop agree to O(10%)

QCD NLO corrections to top-b interference, using top-quark loop in HEFT and b-quark loop in small m_b limit

Lindert Melnikov Tancredi Wever 2017



Higgs p_T distribution at NLO

CoLorFuINLO Somogyi 2009

dual subtraction Prisco Tramontano 2020

evaluated on:

 $3x10^4$ pt for OS top (1.4x10⁴ pt on basic grid, 1.6x10⁴ pt on biased grid) $9x10^4$ pt for MSbar top

 1.8×10^5 pt for MSbar top and b

$$\sqrt{s} = 13 \, \text{TeV}$$

$$m_H = 125.25 \,\mathrm{GeV}$$

$$m_t^{\rm OS}=172.5\,{\rm GeV}$$

$$m_t^{\overline{\mathrm{MS}}}(m_t^{\overline{\mathrm{MS}}}) = 163.4 \,\mathrm{GeV}$$

$$m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}}) = 4.18 \,\mathrm{GeV}$$

$$G_F = 1.16639 \cdot 10^{-5} \,\mathrm{GeV}^{-2}$$

NNPDF40_nlo_as_01180

$$p_{T,j_1} > 20 \,\text{GeV}$$

anti-kt algorithm with R = 0.4

7-pt scale variation about:

$$\mu_R^0 = \mu_F^0 = \frac{H_T}{2} = \frac{1}{2} \left(\sqrt{m_H^2 + p_T^2} + \sum_i |p_{T,i}| \right)$$

inclusive Higgs p_T distribution

QCD NLO corrections Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

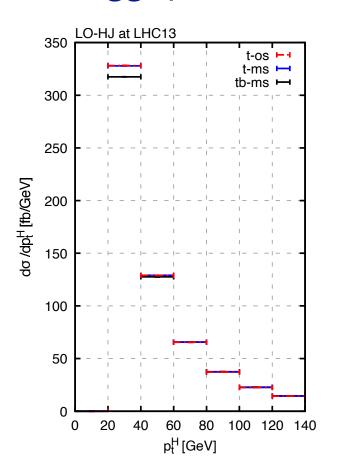
for the top-quark, with on-shell and MSbar schemes for top- and b-quarks with MSbar scheme

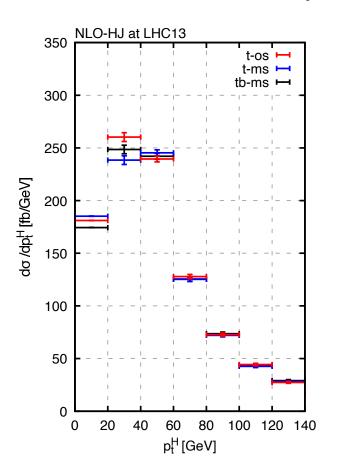
renormalisation of internal masses	$\sigma_{ m LO} \; [m pb]$	$\sigma_{ m NLO}~{ m [pb]}$
$top + bottom - (\overline{MS})$	$12.318^{+4.711}_{-3.117}$	$19.89(8)_{-3.19}^{+2.84}$
$\mathrm{top-}(\overline{\mathrm{MS}})$	$12.538^{+4.822}_{-3.183}$	$19.90(8)^{+2.66}_{-2.85}$
top–(OS)	$12.551^{+4.933}_{-3.244}$	$20.22(8)_{-3.09}^{+3.06}$

- \bigcirc from LO to NLO large k factor and reduction of scale uncertainty, from 30% to 15%
- \bigcirc top-b interference is a negative correction at $O(\alpha_s^3)$ but positive at $O(\alpha_s^4)$
- effect of top mass renormalisation utterly negligible at LO but 15 times bigger at NLO

$$rac{\sigma_{t({
m OS})}}{\sigma_{t({
m \overline{MS}})}} - 1 = \left\{ egin{array}{ll} {
m 0.1\% \ at \ NLO} \end{array}
ight.$$

Higgs p_T distribution at low-intermediate p_T

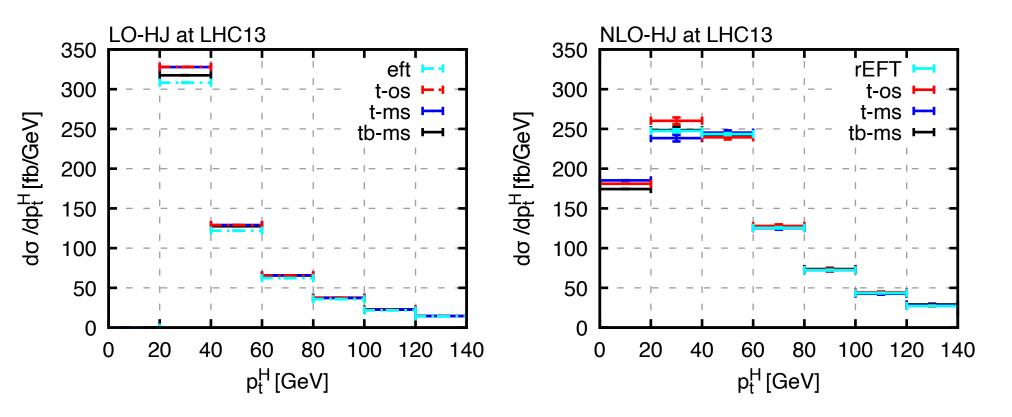




20-40 GeV bin 260⁺¹⁶-83 fb/GeV 249⁺²¹-65 fb/GeV 238⁺²⁷-98 fb/GeV

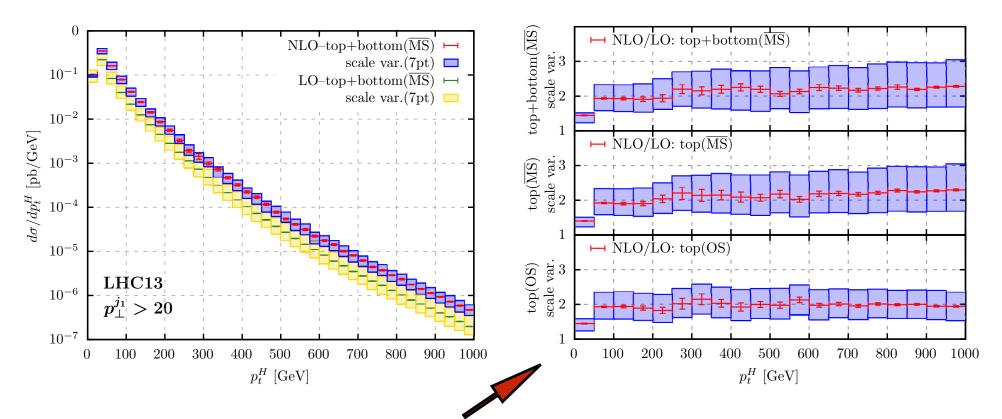
- at LO no events below 20 GeV since $p_{T,i} > 20$ GeV
- \bigcirc at LO no appreciable difference between t(OS) and t(MSbar)
- at NLO sizeable shape distortion in the lowest bins
- scale uncertainty bands (not shown) are much larger than differences

Higgs p_T distribution at low-intermediate p_T



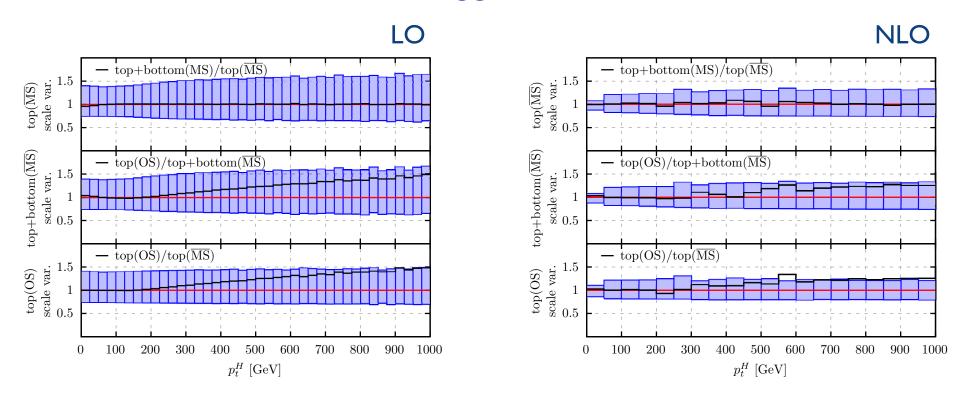
at NLO agreement between exact and rHEFT in the low-middle p_T range HEFT $m_H << 2m_t$ and $m_b << p_T << m_t$

Higgs p_T distribution at LHC



- scale uncertainty bands = ratio of bands at NLO over central value at LO
- \bigcirc k factor almost always larger than 2 for MSbar, and about 2 for OS

Ratios of Higgs p_T distributions



- from LO to NLO, reduction of scale uncertainty and of mass renormalisation scheme dependence
- except in the lowest bins, no appreciable difference between t+b(MSbar) and t(MSbar) The b quark, and thus top-b interference, is negligible, except at low end of p_T range
- p_T distribution for t(MSbar) falls off faster than same for t(OS) as p_T increases because μ_R increases with p_T and so $m_t^{\overline{MS}}(\mu_R)$ decreases
- mass renormalisation scheme difference between t(MSbar) and t(OS) is same size as scale uncertainty at high end of p_T range, both at LO and NLO

Conclusions

- we computed the Higgs p_T distribution at NLO in QCD including for the first time top and b quarks and the MSbar mass scheme
- computation has excellent numerical stability
- Θ b quark, and thus top-b interference, is negligible, except at low end of p_T range, where it affects the shape of the distribution
- \bigcirc in the intermediate to high p_T range, use of top quark only is warranted, but sizeable dependence on mass renormalisation scheme