A subtraction scheme for cross sections at NNLO

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HP^2 Zürich 8 September 2006

NLO features

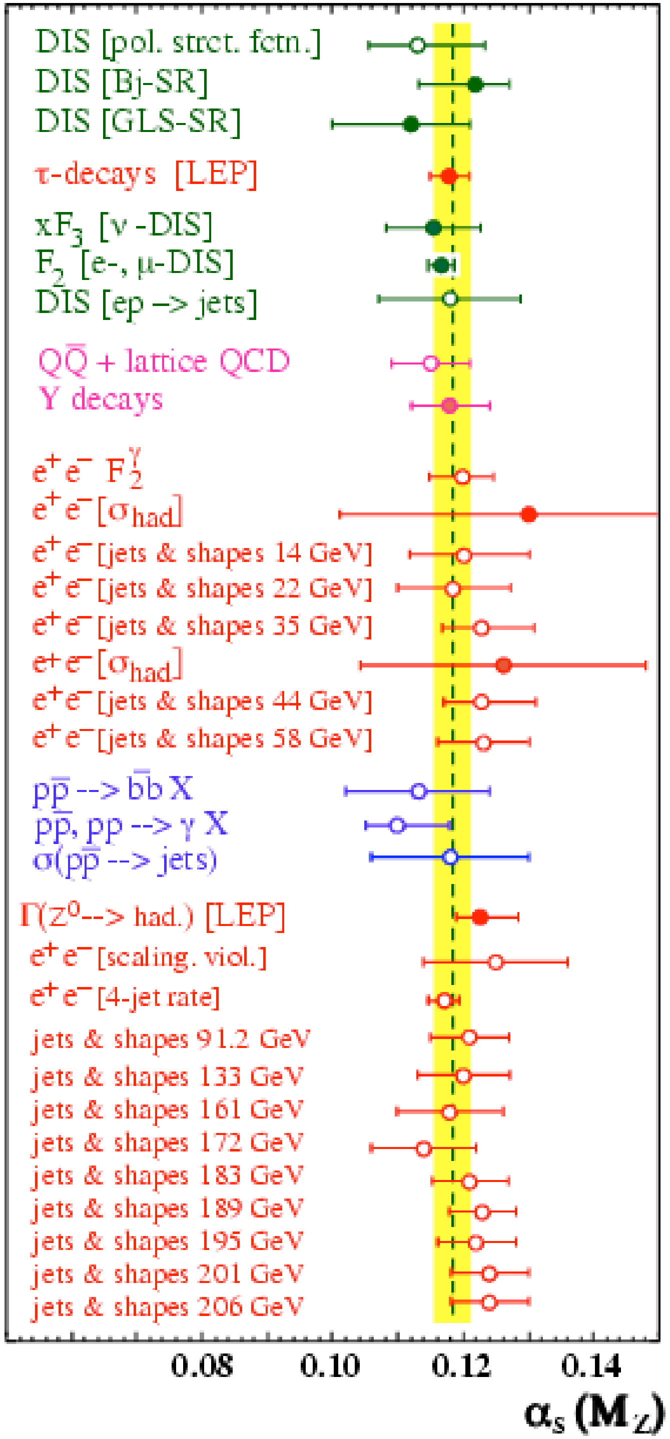
- Jet structure: final-state collinear radiation
- PDF evolution: initial-state collinear radiation
- Opening of new channels
- Reduced sensitivity to fictitious input scales: μ_R, μ_F
  - predictive normalisation of observables
    - first step toward precision measurements
    - accurate estimate of signal and background for Higgs and new physics
- Matching with parton-shower MC's: MC@NLO

NNLO corrections may be relevant if

- the main source of uncertainty in extracting info from data is due to NLO theory: α_S measurements
- NLO corrections are large:
  Higgs production from gluon fusion in hadron collisions
- NLO uncertainty bands are too large to test theory vs. data: b production in hadron collisions
- NLO is effectively leading order:
  energy distributions in jet cones

Summary of $\alpha_S(M_Z)$

S. Bethke hep-ex/0407021



filled symbols are NNLO results

world average of $\alpha_S(M_Z)$
using $\overline{\mathrm{MS}}$ and NNLO results only

$$\alpha_S(M_Z) = 0.1182 \pm 0.0027$$

(cf. 2002 $\alpha_S(M_Z) = 0.1183 \pm 0.0027$
outcome almost identical
because new entries wrt 2002
- LEP jet shape observables and
4-jet rate, and HERA jet rates
and shape variables - are NLO)

NNLO state of the art

- Drell-Yan W, Z production
  - total cross section — Hamberg, van Neerven, Matsuura 1990; Harlander, Kilgore 2002
  - fully differential cross section — Melnikov, Petriello 2006
- Higgs production
  - total cross section — Harlander, Kilgore; Anastasiou, Melnikov 2002; Ravindran, Smith, van Neerven 2003
  - fully differential cross section — Anastasiou, Melnikov, Petriello 2004
- $e^+e^- \to 3$ jets
  - almost complete — De Ridder, Gehrmann, Glover 2004-6

NNLO cross sections

- Analytic integration

  Hamberg, van Neerven, Matsuura 1990
  Anastasiou Dixon Melnikov Petriello 2003

  ⇧ first method

  ⇨ flexible enough to include a limited class of acceptance cuts by modelling cuts as ``propagators''

- Sector decomposition

  Denner Roth 1996; Binoth Heinrich 2000
  Anastasiou, Melnikov, Petriello 2004

  ⇧ flexible enough to include any acceptance cuts

  ⇧ cancellation of divergences is performed numerically

  ⇨ can it handle many final-state partons ?

- Subtraction

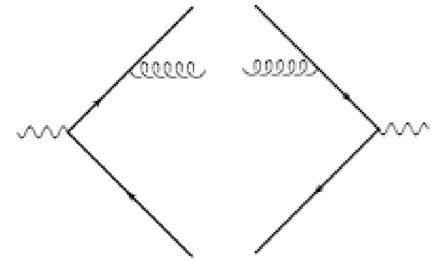
  ⇧ process independent

  ⇨ cancellation of divergences is analytic can it be automatised ?

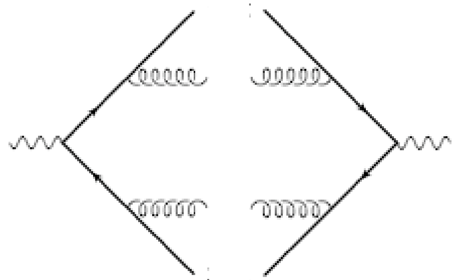
NLO assembly kit

$e^+e^- \to 3$ jets

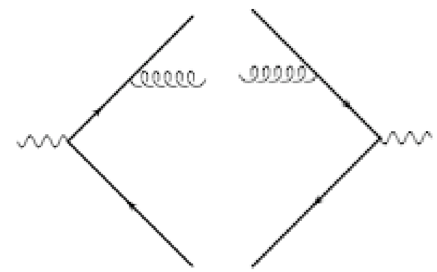
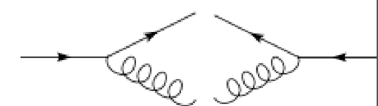
leading order $|\mathcal{M}_n^{tree}|^2$



NLO real



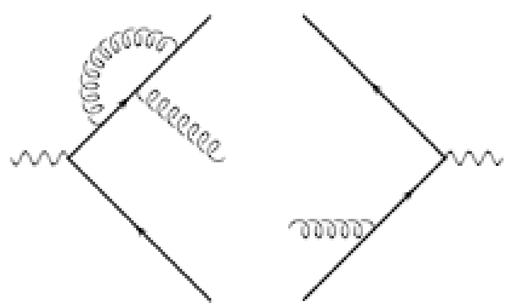
IR

 $\otimes$ 

$$|\mathcal{M}_{n+1}^{\text{tree}}|^2 \quad \to \quad |\mathcal{M}_n^{\text{tree}}|^2 \quad \times \quad \int dPS |P_{\text{split}}|^2$$

$$= \quad -\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right)$$

NLO virtual



$d = 4 - 2\epsilon$

$$\int d^d l \; 2(\mathcal{M}_n^{loop})^* \mathcal{M}_n^{tree} \quad = \quad \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) |\mathcal{M}_n^{tree}|^2 + fin.$$

NLO production rates

Process-independent procedure devised in the 90's

- slicing — Giele Glover & Kosower
- subtraction — Frixione Kunszt & Signer; Nagy & Trocsanyi
  - dipole — Catani & Seymour
  - antenna — Kosower; Campbell Cullen & Glover

$$\sigma = \sigma^{\mathrm{LO}} + \sigma^{\mathrm{NLO}} = \int_m d\sigma_m^B \; J_m + \sigma^{\mathrm{NLO}}$$

$$\sigma^{\mathrm{NLO}} = \int_{m+1} d\sigma_{m+1}^{\mathrm{R}} J_{m+1} + \int_m d\sigma_m^{\mathrm{V}} J_m$$

the 2 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[d\sigma_{m+1}^{\mathrm{R}} J_{m+1} - d\sigma_{m+1}^{\mathrm{R,A}} J_m \right] + \int_m \left[d\sigma_m^{\mathrm{V}} + \int_1 d\sigma_{m+1}^{\mathrm{R,A}} \right] J_m$$

the 2 terms on the rhs are finite in d=4

NLO IR limits

collinear operator

$$\mathrm{C}_{ir}|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2 \propto \frac{1}{s_{ir}}\langle\mathcal{M}_{m+1}(0)(p_{ir},\ldots)|\hat{P}^{(0)}_{f_i f_r}|\mathcal{M}_{m+1}(0)(p_{ir},\ldots)\rangle$$

soft operator

$$\mathrm{S}_r|\mathcal{M}^{(0)}_{m+2}(p_r,\ldots)|^2 \propto \frac{s_{ik}}{s_{ir}s_{rk}}\langle\mathcal{M}_{m+1}(0)(\ldots)|T_i\cdot T_k|\mathcal{M}_{m+1}(0)(\ldots)\rangle$$

counterterm

$$\sum_r\left(\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir}+\mathrm{S}_r\right)|\mathcal{M}^{(0)}_{m+2}(p_i,p_r,\ldots)|^2$$

performs double subtraction in overlapping regions

NLO overlapping divergences

$\mathrm{C}_{ir}\mathrm{S}_r$ can be used to cancel double subtraction

$$\mathrm{C}_{ir}\left(\mathrm{S}_r - \mathrm{C}_{ir}\mathrm{S}_r\right)|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

$$\mathrm{S}_r\left(\mathrm{C}_{ir} - \mathrm{C}_{ir}\mathrm{S}_r\right)|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

the NLO counterterm

$$\mathrm{A}_1|\mathcal{M}^{(0)}_{m+2}|^2 = \sum_r\left[\sum_{i\neq r}\frac{1}{2}\mathrm{C}_{ir} + \left(\mathrm{S}_r - \sum_{i\neq r}\mathrm{C}_{ir}\mathrm{S}_r\right)\right]|\mathcal{M}^{(0)}_{m+2}(p_i, p_r, \ldots)|^2$$

- has the same singular behaviour as SME, and is free of double subtractions

$$\mathrm{C}_{ir}\left(1 - \mathrm{A}_1\right)|\mathcal{M}^{(0)}_{m+1}|^2 = 0 \qquad \mathrm{S}_r\left(1 - \mathrm{A}_1\right)|\mathcal{M}^{(0)}_{m+1}|^2 = 0$$

- contains spurious singularities when parton $s \neq r$ becomes unresolved, but they are screened by J_m

Collinear mapping

$$\tilde{p}_{ir}^{\mu} = \frac{1}{1-\alpha_{ir}}\left(p_i^{\mu} + p_r^{\mu} - \alpha_{ir} Q^{\mu}\right), \qquad \tilde{p}_n^{\mu} = \frac{1}{1-\alpha_{ir}} p_n^{\mu}, \qquad n \neq i, r$$

$$\alpha_{ir} = \frac{1}{2}\left[y_{(ir)Q} - \sqrt{y_{(ir)Q}^2 - 4 y_{ir}}\,\right] \qquad y_{ir} = \frac{2 p_i \cdot p_r}{Q^2}$$

momentum is conserved $\quad \tilde{p}_{ir}^{\mu} + \sum_n \tilde{p}_n^{\mu} = p_i^{\mu} + p_r^{\mu} + \sum_n p_n^{\mu}$

Collinear mapping

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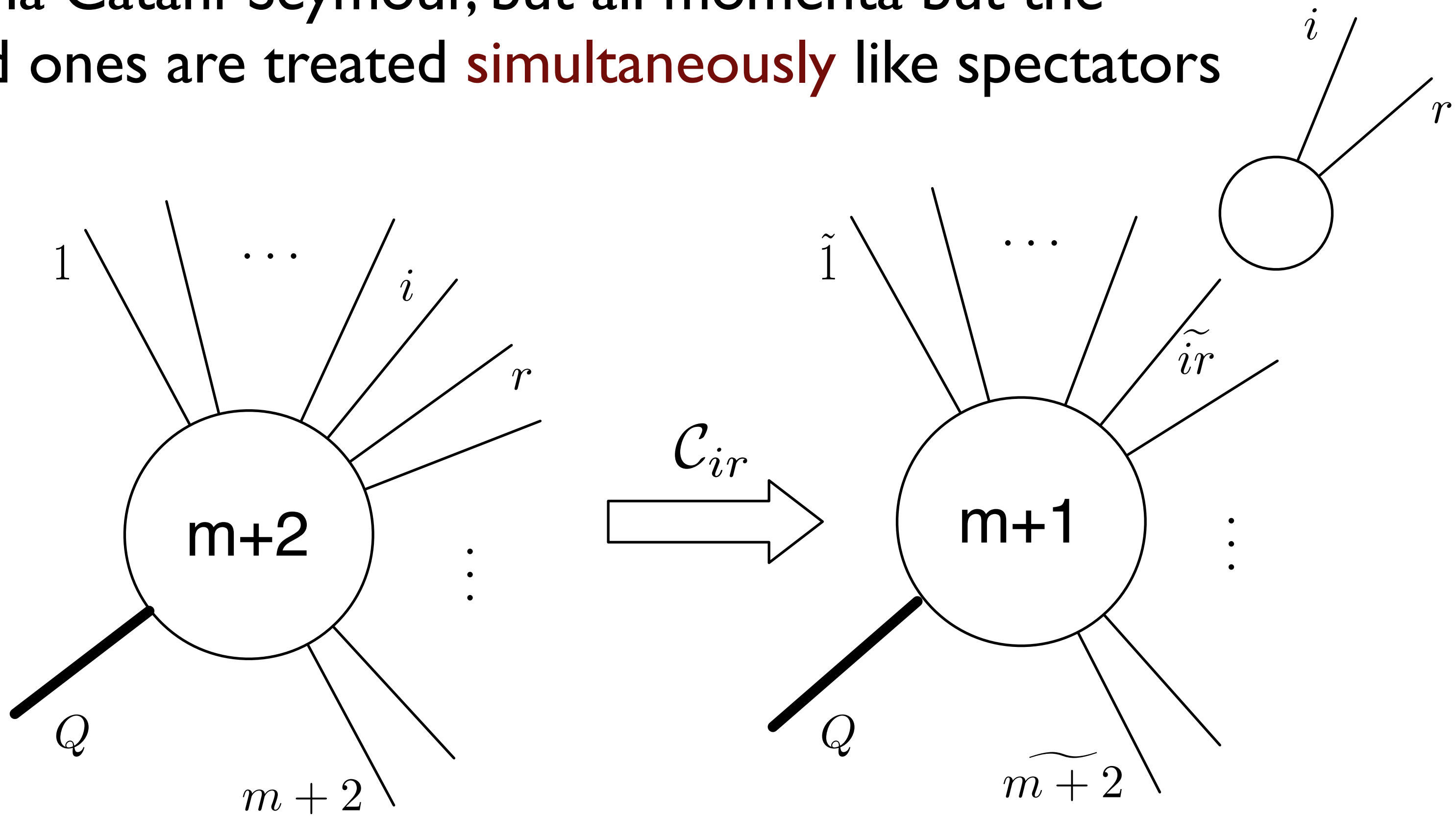
momentum is conserved $\tilde{p}^\mu_{ir} + \sum_n \tilde{p}^\mu_n = p^\mu_i + p^\mu_r + \sum_n p^\mu_n$

mapping à la Catani-Seymour, but all momenta but the unresolved ones are treated simultaneously like spectators



Collinear mapping

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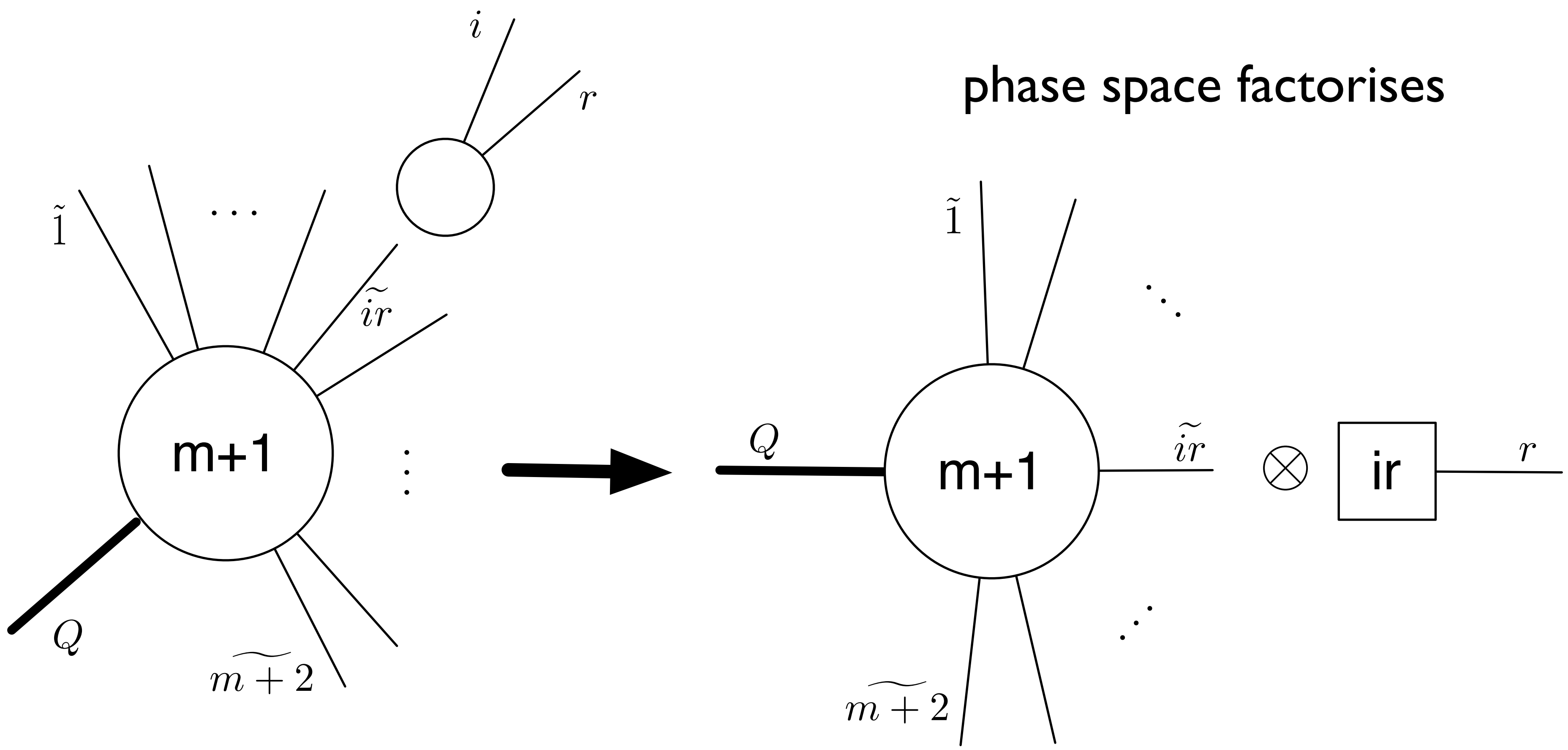
momentum is conserved $\tilde{p}_{ir}^{\mu} + \sum_n \tilde{p}_n^{\mu} = p_i^{\mu} + p_r^{\mu} + \sum_n p_n^{\mu}$

phase space factorises



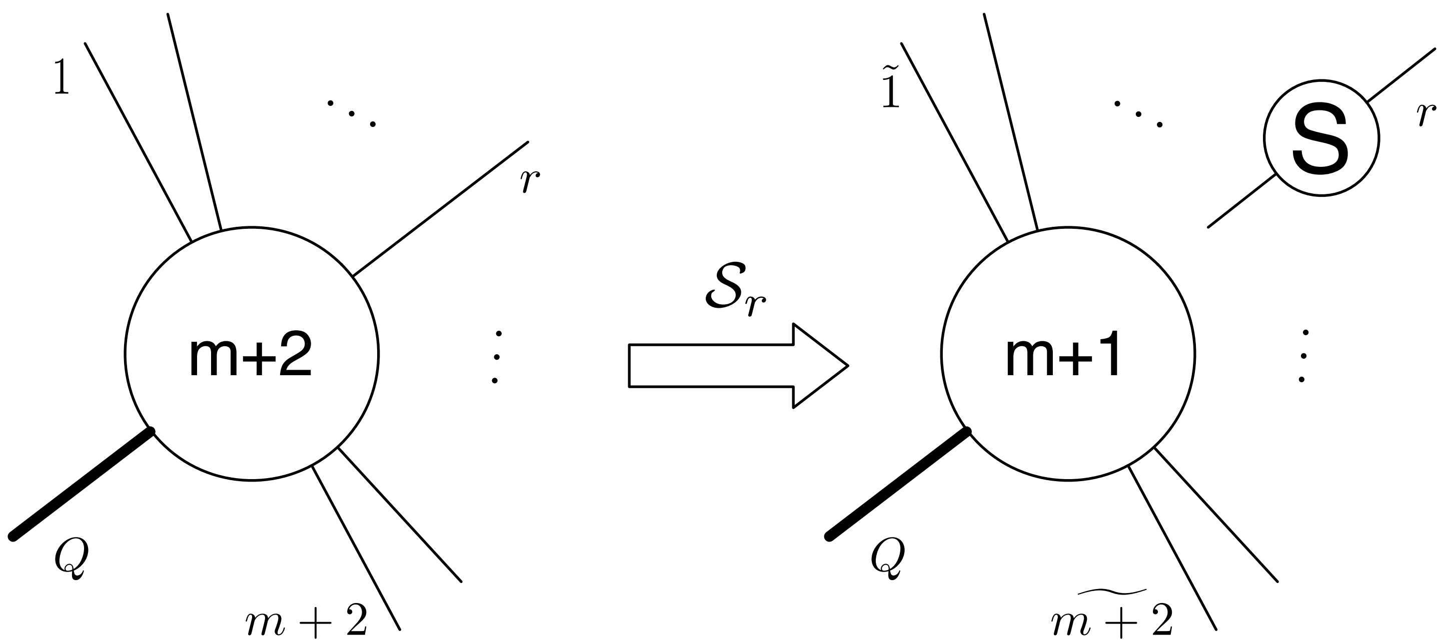
Soft mapping

$$\tilde{p}_n^\mu = \Lambda^\mu_\nu[Q,(Q-p_r)/\lambda_r](p_n^\nu/\lambda_r)\,, \qquad n \neq r$$

$$\lambda_r = \sqrt{1-y_{rQ}}$$

$$\Lambda^\mu_\nu[K,\widetilde{K}\,] = g^\mu_\nu - \frac{2(K+\widetilde{K}\,)^\mu(K+\widetilde{K}\,)_\nu}{(K+\widetilde{K}\,)^2} + \frac{2K^\mu\widetilde{K}_\nu}{K^2}$$

Lorentz transformation that preserves total momentum



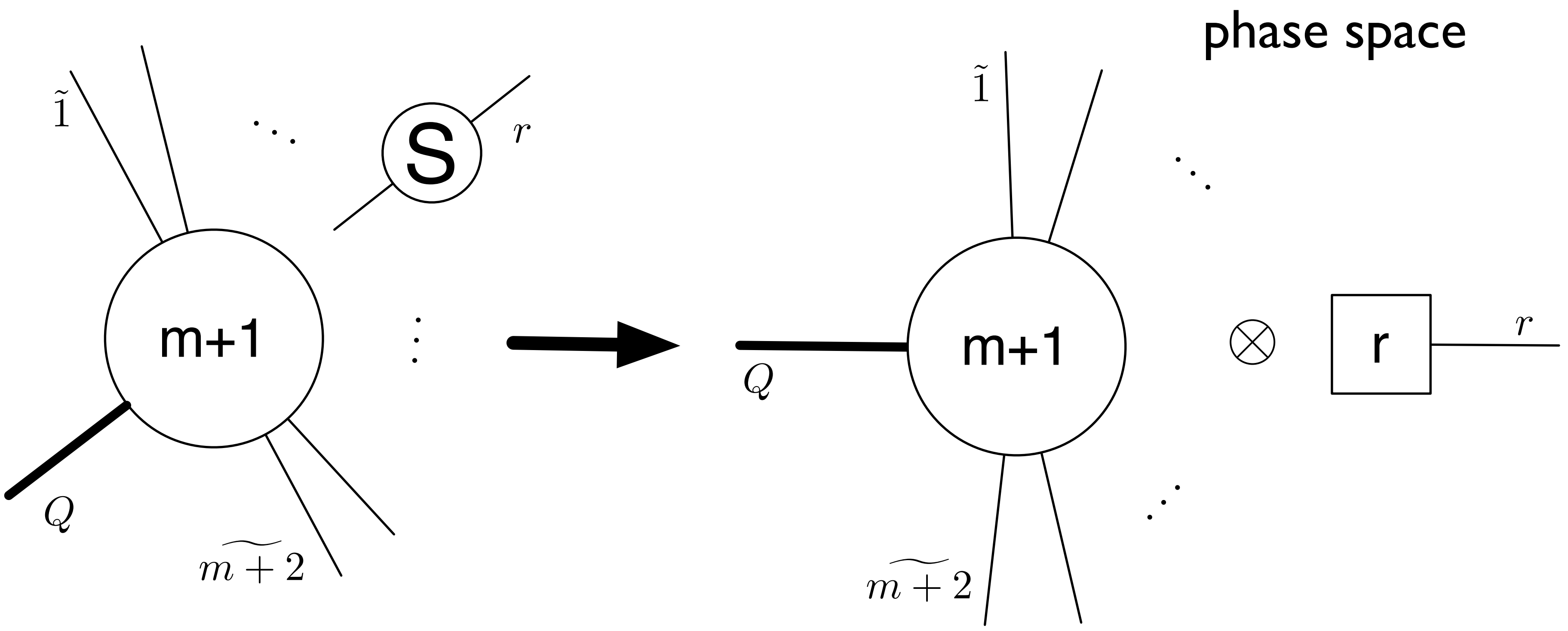
Soft mapping

$$\tilde{p}_n^\mu = \Lambda^\mu_\nu[Q, (Q - p_r)/\lambda_r](p_n^\nu/\lambda_r)\,, \qquad n \neq r$$

$$\lambda_r = \sqrt{1 - y_{rQ}}$$

$$\Lambda^\mu_\nu[K, \widetilde{K}\,] = g^\mu_\nu - \frac{2(K + \widetilde{K}\,)^\mu (K + \widetilde{K}\,)_\nu}{(K + \widetilde{K}\,)^2} + \frac{2K^\mu \widetilde{K}_\nu}{K^2}$$

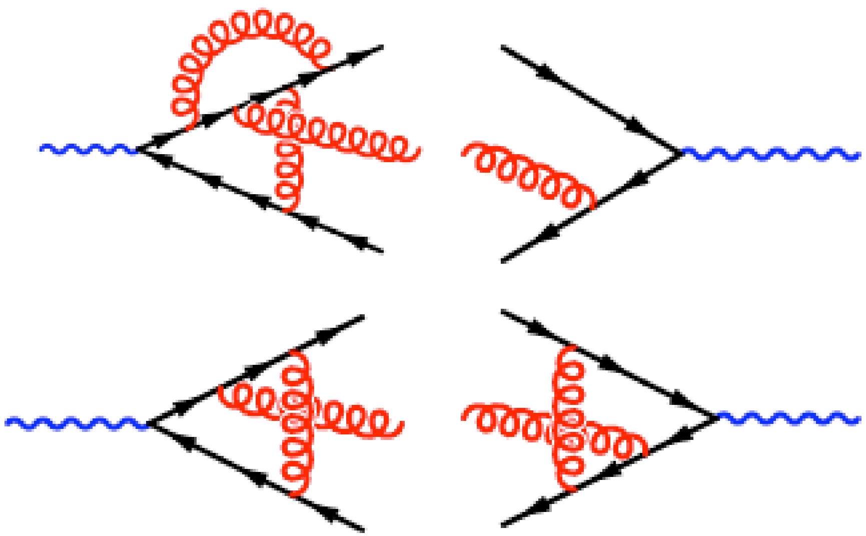
Lorentz transformation that preserves total momentum



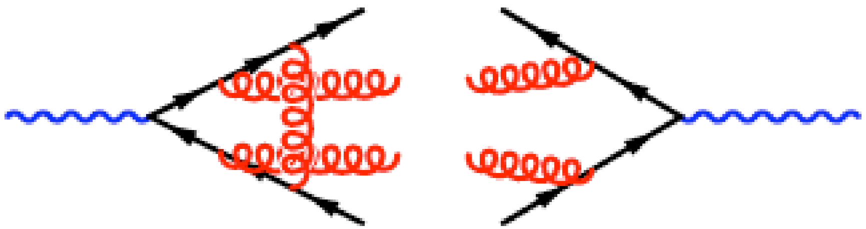
NNLO assembly kit

$e^+e^- \to 3$ jets

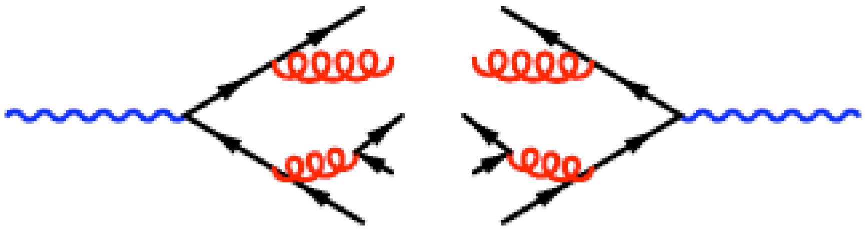
double virtual



real-virtual



double real



Two-loop matrix elements

- two-jet production $qq' \to qq', \; q\bar{q} \to q\bar{q}, \; q\bar{q} \to gg, \; gg \to gg$

  C. Anastasiou N. Glover C. Oleari M. Tejeda-Yeomans 2000-01

  Z. Bern A. De Freitas L. Dixon 2002

- photon-pair production $q\bar{q} \to \gamma\gamma, \; gg \to \gamma\gamma$

  C. Anastasiou N. Glover M. Tejeda-Yeomans 2002
  Z. Bern A. De Freitas L. Dixon 2002

- $e^+e^- \to 3$ jets $\gamma^* \to q\bar{q}g$

  L. Garland T. Gehrmann N. Glover A. Koukoutsakis E. Remiddi 2002

- $V + 1$ jet production $q\bar{q} \to Vg$

  T. Gehrmann E. Remiddi 2002

- Drell-Yan V production $q\bar{q} \to V$

  R. Hamberg W. van Neerven T. Matsuura 1991

- Higgs production $gg \to H$ (in the $m_t \to \infty$ limit)

  R. Harlander W. Kilgore; C. Anastasiou K. Melnikov 2002

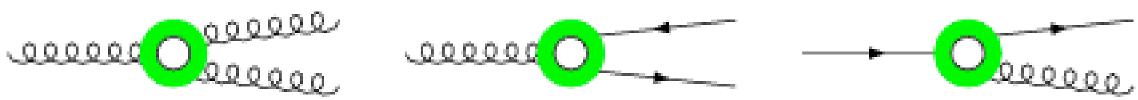
Collinear and soft currents

- universal IR structure ➡ process-independent procedure

- universal collinear and soft currents
  - 3-parton tree splitting functions

J. Campbell N. Glover 1997; S. Catani M. Grazzini 1998; A. Frizzo F. Maltoni VDD 1999; D. Kosower 2002

  - 2-parton one-loop splitting functions



Z. Bern L. Dixon D. Dunbar D. Kosower 1994; Z. Bern W. Kilgore C. Schmidt VDD 1998-99; D. Kosower P. Uwer 1999; S. Catani M. Grazzini 1999; D. Kosower 2003

- universal subtraction counterterms
  - several ideas and works in progress

D. Kosower; S. Weinzierl; A. De Ridder, T. Gehrmann, G. Heinrich 2003
S. Frixione M. Grazzini 2004; G. Somogyi Z. Trocsanyi VDD 2005

  - but devised only for $e^+e^- \to 3$ jets

A. De Ridder, T. Gehrmann, N. Glover 2005; G. Somogyi Z. Trocsanyi VDD 2006

NNLO subtraction

$$\sigma^{\rm NNLO} = \int_{m+2} d\sigma^{\rm RR}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\rm RV}_{m+1} J_{m+1} + \int_{m} d\sigma^{\rm VV}_{m} J_{m}$$

the 3 terms on the rhs are divergent in d=4

use universal IR structure to subtract divergences

NNLO subtraction

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} d\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} d\sigma^{\mathrm{VV}}_{m} J_{m}$$

the 3 terms on the rhs are divergent in d=4
use universal IR structure to subtract divergences

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \left[d\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - d\sigma^{\mathrm{RR,A_2}}_{m+2} J_{m} \right]$$

takes care of doubly-unresolved regions,
but still divergent in singly-unresolved ones

$$+ \int_{m+1} \left[d\sigma^{\mathrm{RV}}_{m+1} J_{m+1} - d\sigma^{\mathrm{RV,A_1}}_{m+1} J_{m} \right]$$

still contains $1/\epsilon$ poles in regions away from 1-parton IR regions

$$+ \int_{m} \left[d\sigma^{\mathrm{VV}}_{m} + \int_{2} d\sigma^{\mathrm{RR,A_2}}_{m+2} + \int_{1} d\sigma^{\mathrm{RV,A_1}}_{m+1} \right] J_{m}$$

2-step procedure

- construct subtraction terms that regularise the singularities of the SME in all unresolved parts of the phase space, avoiding multiple subtractions

G. Somogyi Z. Trocsanyi VDD 2005

- perform momentum mappings, such that the phase space factorises exactly over the unresolved momenta and such that it respects the structure of the cancellations among the subtraction terms

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→ Gabor's talk

NNLO counterterm

- construct the 2-unresolved-parton counterterm using the IR currents

$$\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2 = \sum_r \sum_{s\neq r} \Bigg\{ \sum_{i\neq r,s} \Bigg[\frac{1}{6}\mathbf{C}_{irs} + \sum_{j\neq i,r,s} \frac{1}{8}\mathbf{C}_{ir;js} + \frac{1}{2}\mathbf{S}_{rs}$$

$$+\frac{1}{2}\left(\mathbf{CS}_{ir;s} - \mathbf{C}_{irs}\mathbf{CS}_{ir;s} - \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\mathbf{CS}_{ir;s} \right) \Bigg]$$

$$- \sum_{i\neq r,s} \left[\mathbf{CS}_{ir;s}\mathbf{S}_{rs} + \mathbf{C}_{irs}\left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \right) \right.$$

$$\left. + \sum_{j\neq i,r,s} \mathbf{C}_{ir;js}\left(\frac{1}{2}\mathbf{S}_{rs} - \mathbf{CS}_{ir;s}\mathbf{S}_{rs} \right) \right] \Bigg\} |\mathcal{M}^{(0)}_{m+2}|^2$$

G. Somogyi Z. Trocsanyi VDD 2005

performing double and triple subtractions in overlapping regions

$$\mathrm{C}_{irs}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \mathrm{S}_{rs}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

$$\mathrm{C}_{ir;js}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0 \qquad \mathrm{CS}_{ir;s}\,(1-\mathrm{A}_2)\,|\mathcal{M}^{(0)}_{m+2}|^2 = 0$$

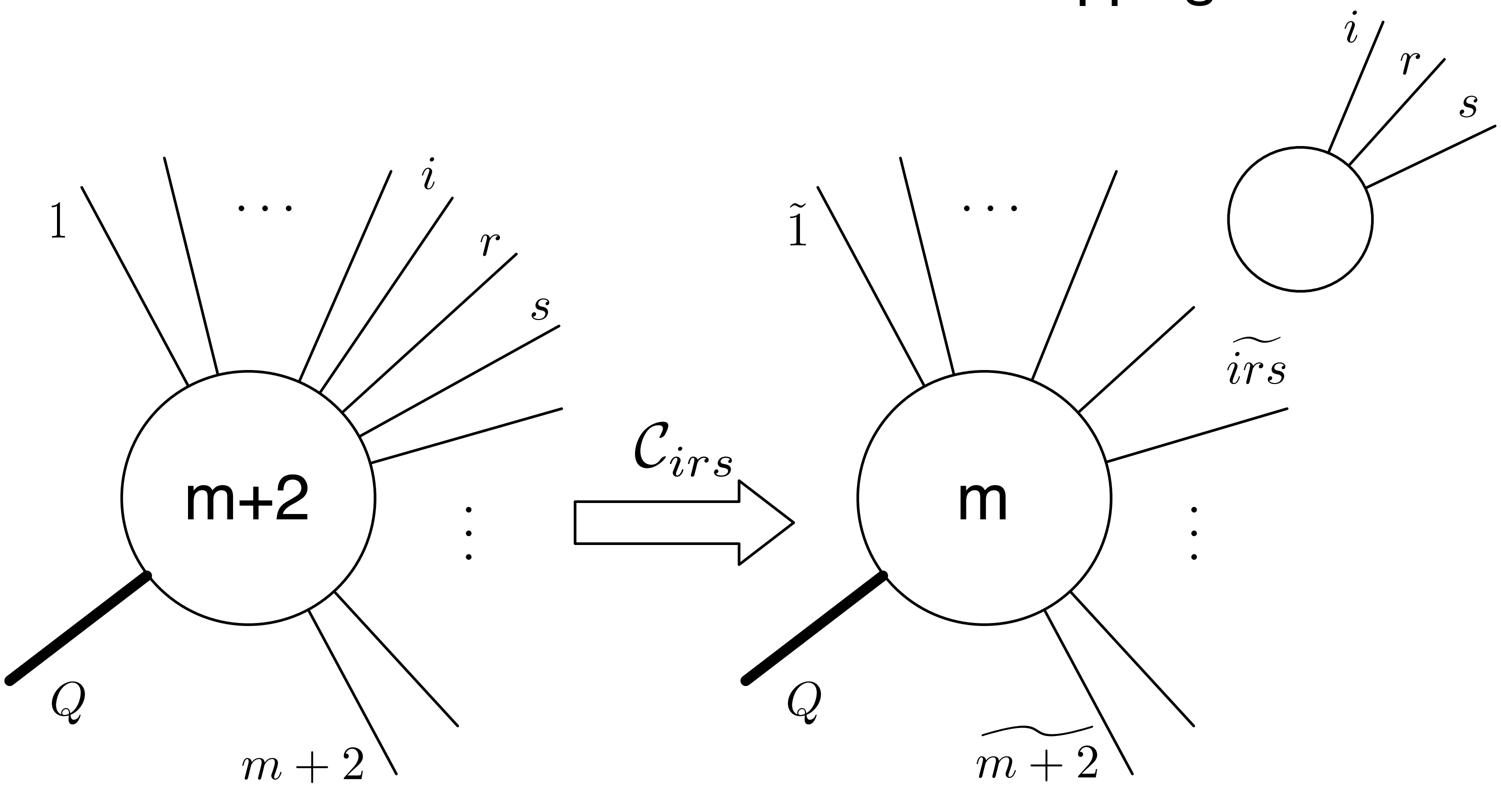
Triple-collinear mapping

$$\tilde{p}^\mu_{irs} = \frac{1}{1-\alpha_{irs}}(p^\mu_i + p^\mu_r + p^\mu_s - \alpha_{irs} Q^\mu)\,, \qquad \tilde{p}^\mu_n = \frac{1}{1-\alpha_{irs}} p^\mu_n\,, \qquad n \neq i, r, s$$

$$\alpha_{irs} = \frac{1}{2}\left[y_{(irs)Q} - \sqrt{y^2_{(irs)Q} - 4 y_{irs}}\,\right]$$

momentum conservation $\quad \tilde{p}^\mu_{irs} + \sum_n \tilde{p}^\mu_n = p^\mu_i + p^\mu_r + p^\mu_s + \sum_n p^\mu_n$

straightforward extension of NLO collinear mapping



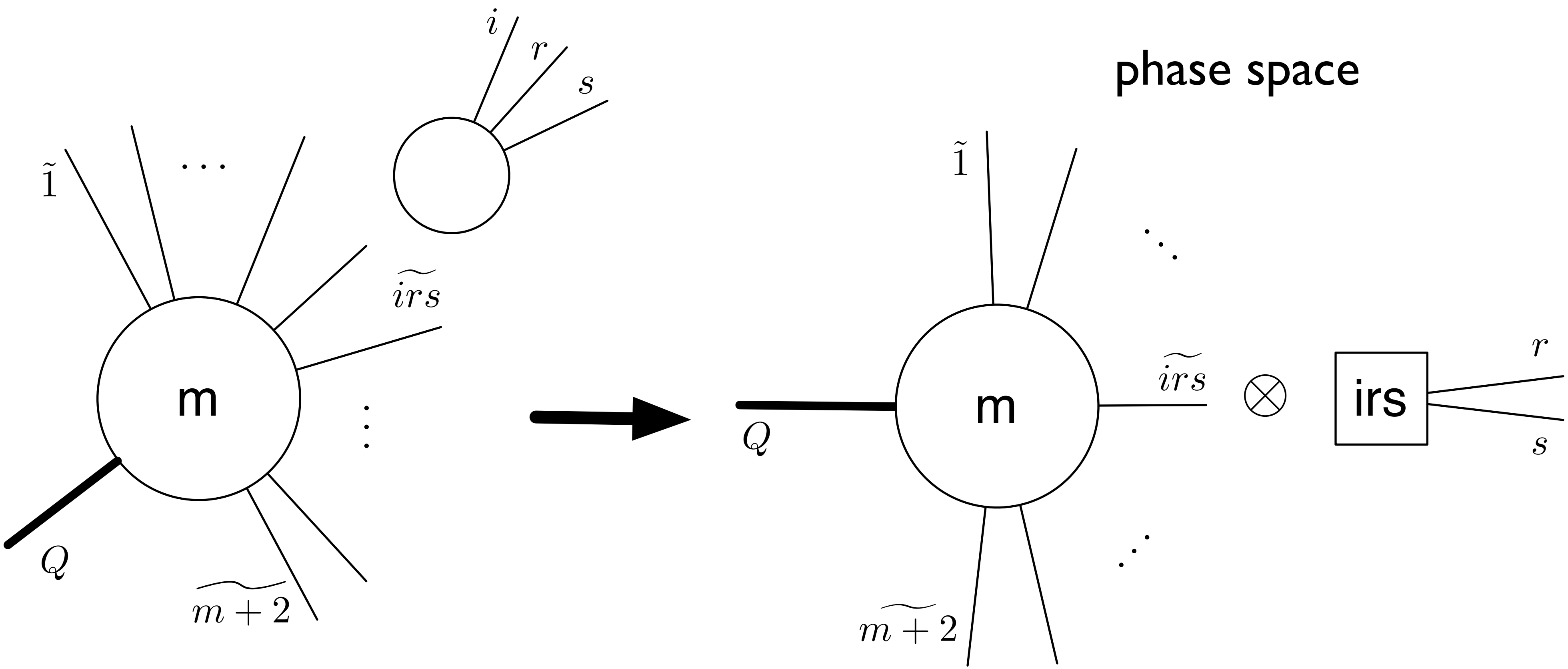
Triple-collinear mapping

$$\tilde{p}^{\mu}_{irs} = \frac{1}{1-\alpha_{irs}}\left(p^{\mu}_{i} + p^{\mu}_{r} + p^{\mu}_{s} - \alpha_{irs} Q^{\mu}\right) , \qquad \tilde{p}^{\mu}_{n} = \frac{1}{1-\alpha_{irs}} p^{\mu}_{n} , \qquad n \neq i, r, s$$

$$\alpha_{irs} = \frac{1}{2}\left[y_{(irs)Q} - \sqrt{y^{2}_{(irs)Q} - 4 y_{irs}}\,\right]$$

momentum conservation $\quad \tilde{p}^{\mu}_{irs} + \sum_{n} \tilde{p}^{\mu}_{n} = p^{\mu}_{i} + p^{\mu}_{r} + p^{\mu}_{s} + \sum_{n} p^{\mu}_{n}$

phase space

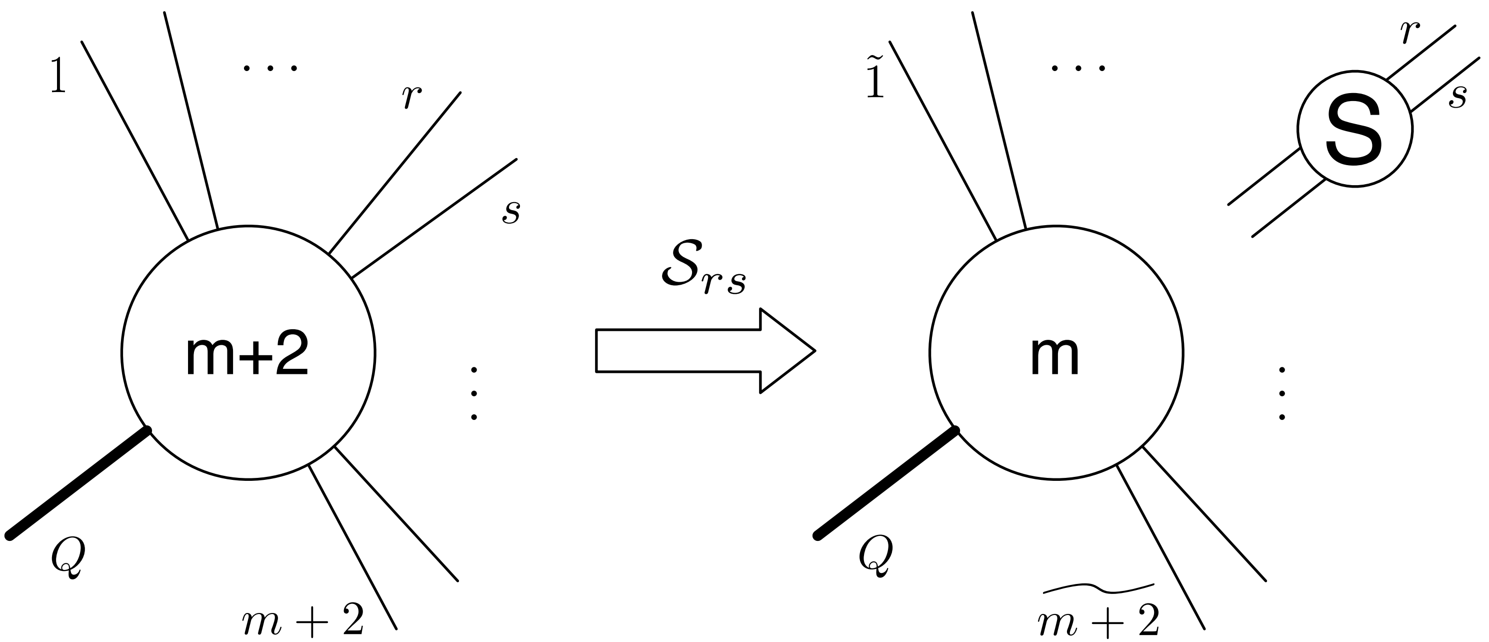


Double-soft mapping

$$\tilde{p}_n^\mu = \Lambda^\mu_\nu[Q,(Q-p_r-p_s)/\lambda_{rs}](p_n^\nu/\lambda_{rs})\,, \qquad n \neq r,s$$

$$\lambda_{rs} = \sqrt{1-\left(y_{(rs)Q}-y_{rs}\right)}$$

straightforward extension of NLO soft mapping



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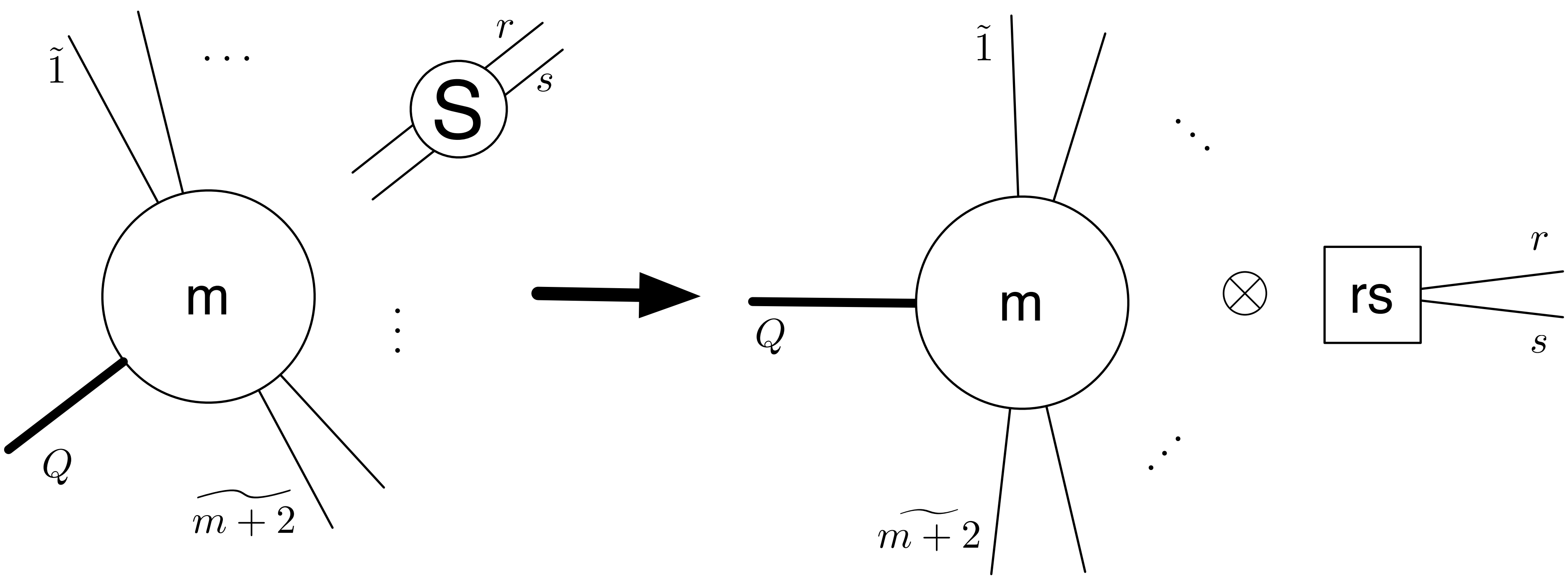
straightforward extension of NLO soft mapping

phase space



Double-collinear mapping

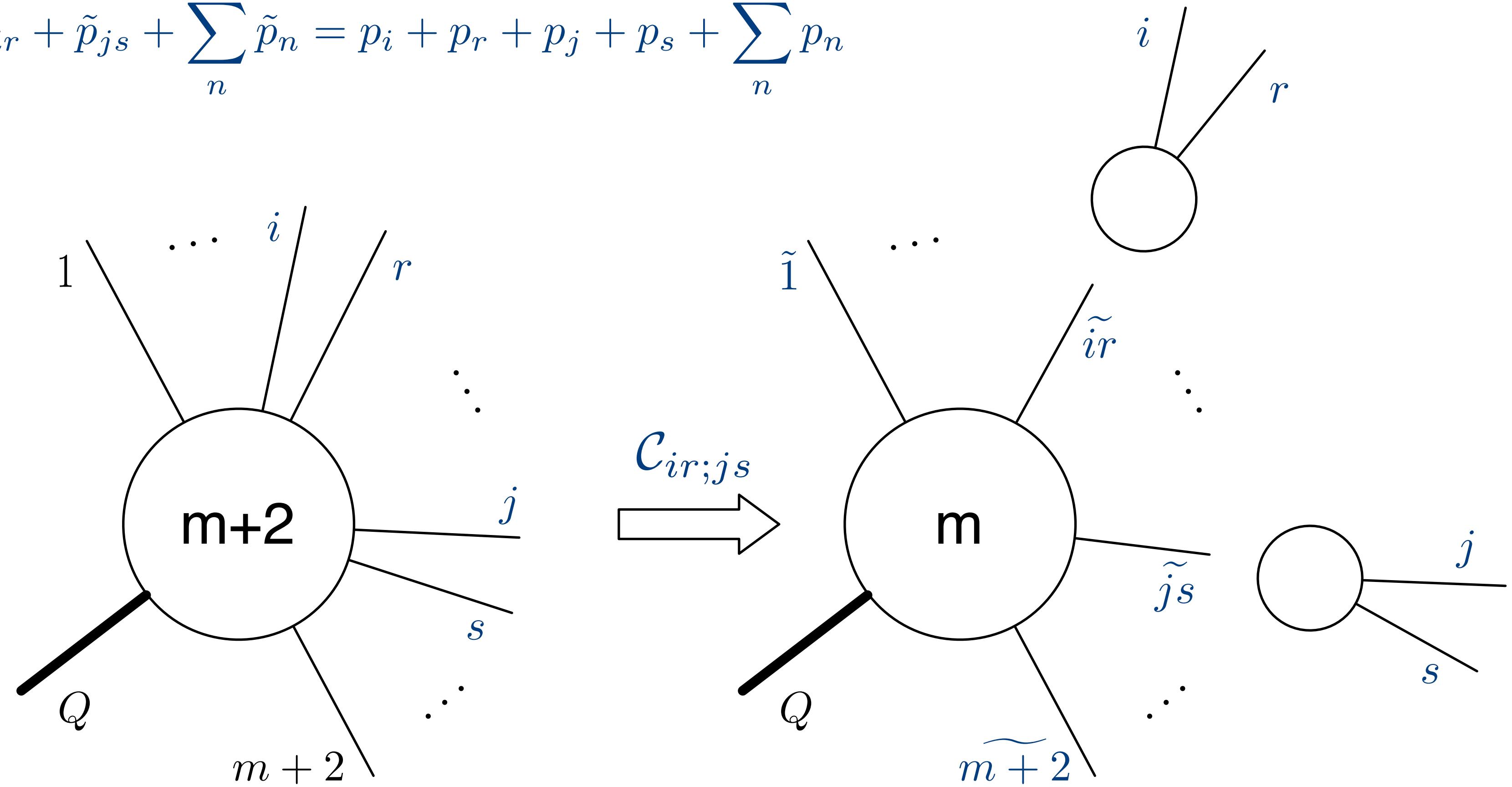
$$\tilde{p}^\mu_{ir} = \frac{1}{1-\alpha_{ir}-\alpha_{js}}(p^\mu_i + p^\mu_r - \alpha_{ir}Q^\mu)\,, \qquad \tilde{p}^\mu_{js} = \frac{1}{1-\alpha_{ir}-\alpha_{js}}(p^\mu_j + p^\mu_s - \alpha_{js}Q^\mu)$$

$$\tilde{p}^\mu_n = \frac{1}{1-\alpha_{ir}-\alpha_{js}} p^\mu_n\,, \qquad n \neq i, r, j, s$$

$$\alpha_{js} = \frac{1}{2}\left[y_{(js)Q} - \sqrt{y^2_{(js)Q} - 4y_{js}}\,\right]$$

momentum conservation

$$\tilde{p}_{ir} + \tilde{p}_{js} + \sum_n \tilde{p}_n = p_i + p_r + p_j + p_s + \sum_n p_n$$



Double-collinear mapping

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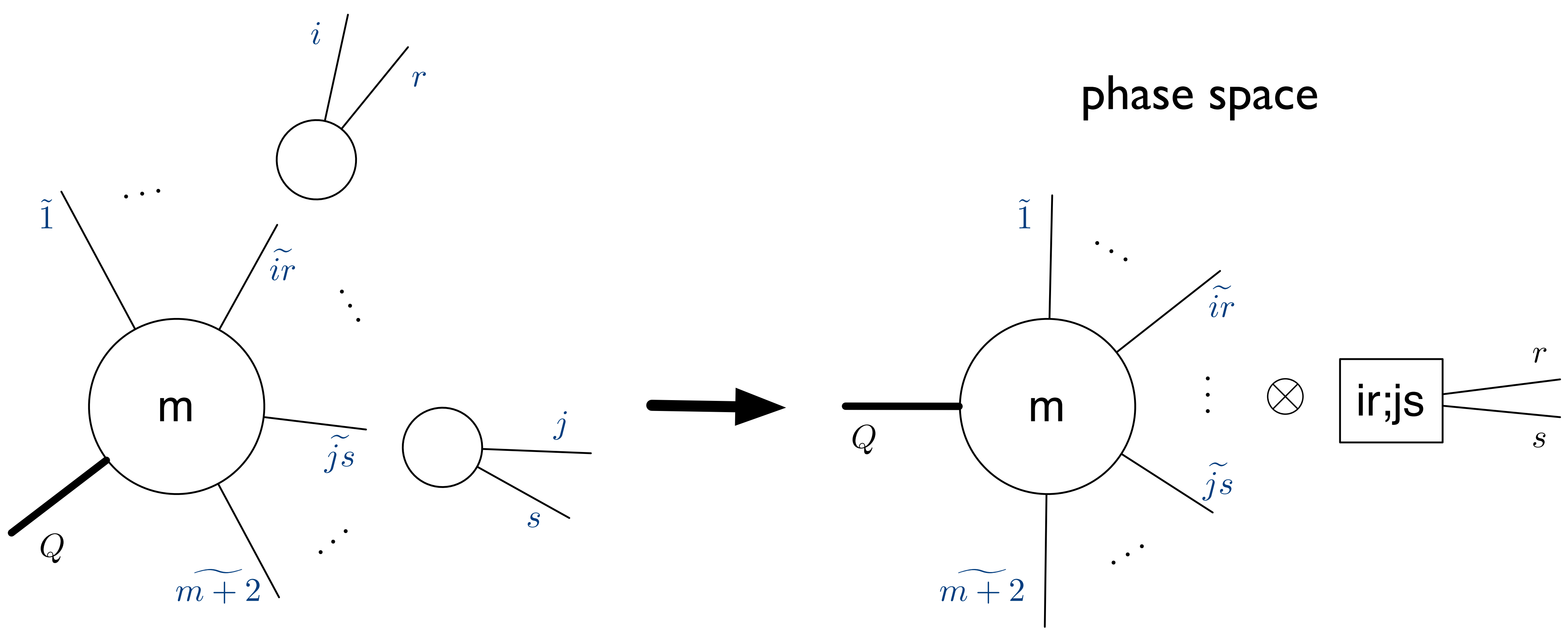
$$\tilde{p}^{\mu}_{n} = \frac{1}{1-\alpha_{ir}-\alpha_{js}}p^{\mu}_n\,, \qquad n \neq i, r, j, s$$

$$\alpha_{js} = \frac{1}{2}\left[y_{(js)Q} - \sqrt{y^2_{(js)Q} - 4y_{js}}\,\right]$$



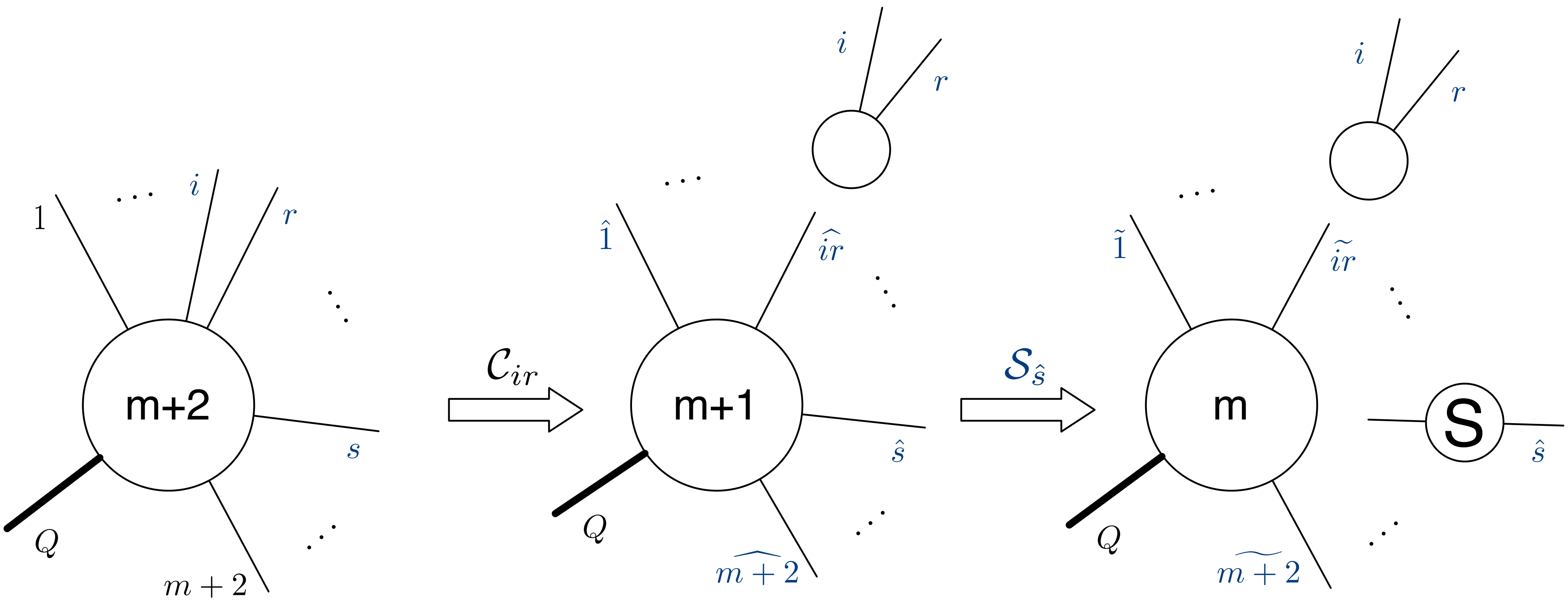
Soft-collinear mapping

composition of a collinear and a soft mapping

$$\hat{p}^\mu_{ir} = \frac{1}{1-\alpha_{ir}}(p^\mu_i + p^\mu_r - \alpha_{ir} Q^\mu)\,, \qquad \hat{p}^\mu_n = \frac{1}{1-\alpha_{ir}} p^\mu_n\,, \qquad n \neq i, r$$

$$\tilde{p}^\mu_n = \Lambda^\mu_\nu[Q,(Q-\hat{p}_s)/\lambda_{\hat{s}}](\hat{p}^\nu_n/\lambda_{\hat{s}})\,, \qquad n \neq \hat{s}$$

in this case, the order of the mappings is irrelevant



Soft-collinear mapping

composition of a collinear and a soft mapping

$$\hat{p}_{ir}^{\mu} = \frac{1}{1-\alpha_{ir}}(p_i^{\mu} + p_r^{\mu} - \alpha_{ir} Q^{\mu})\,, \qquad \hat{p}_n^{\mu} = \frac{1}{1-\alpha_{ir}} p_n^{\mu}\,, \qquad n \neq i, r$$

$$\tilde{p}_n^{\mu} = \Lambda^{\mu}_{\nu}[Q, (Q - \hat{p}_s)/\lambda_{\hat{s}}](\hat{p}_n^{\nu}/\lambda_{\hat{s}})\,, \qquad n \neq \hat{s}$$

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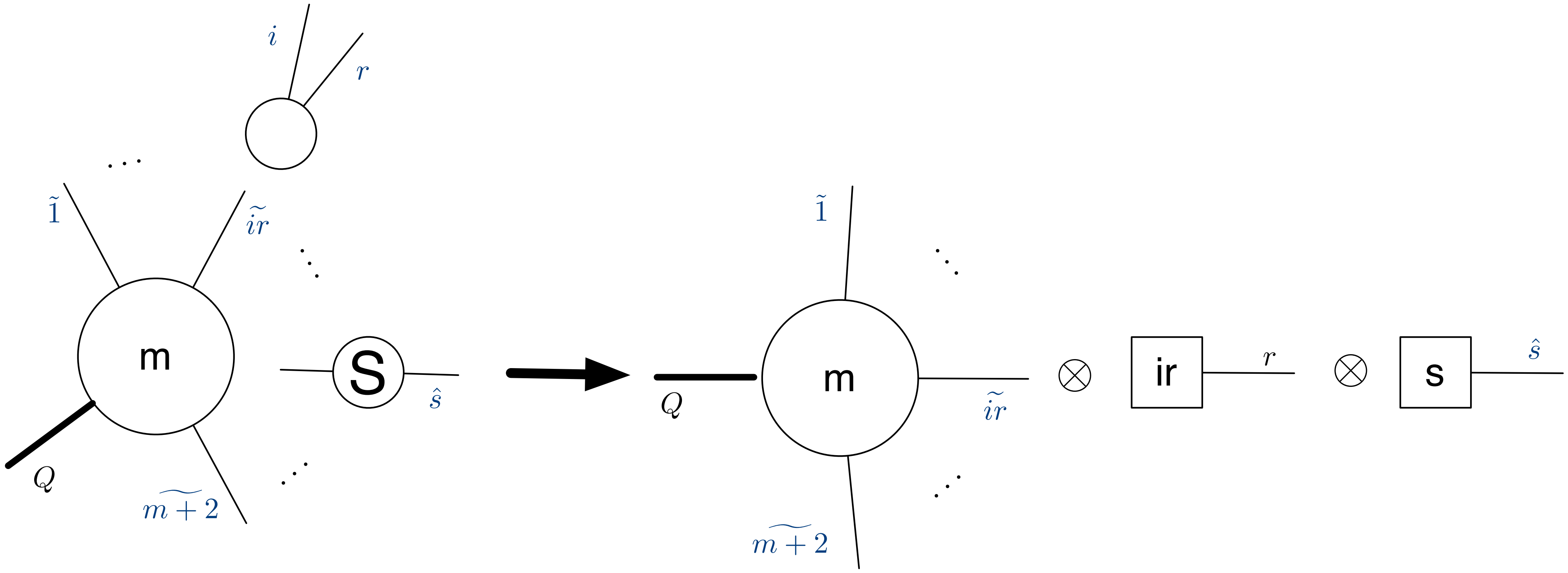
Soft-collinear mapping

composition of a collinear and a soft mapping

$$\hat{p}^\mu_{ir} = \frac{1}{1-\alpha_{ir}} (p^\mu_i + p^\mu_r - \alpha_{ir} Q^\mu) \,, \qquad \hat{p}^\mu_n = \frac{1}{1-\alpha_{ir}} p^\mu_n \,, \qquad n \neq i, r$$

$$\tilde{p}^\mu_n = \Lambda^\mu_\nu [Q, (Q - \hat{p}_s)/\lambda_{\hat{s}}] (\hat{p}^\nu_n / \lambda_{\hat{s}}) \,, \qquad n \neq \hat{s}$$

in this case, the order of the mappings is irrelevant



needs a NLO-type subtraction
between the m+2- and the m+1-parton contributions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{NNLO}}_{\{m+2\}} + \sigma^{\mathrm{NNLO}}_{\{m+1\}} + \sigma^{\mathrm{NNLO}}_{\{m\}}$$

$$\sigma^{\mathrm{NNLO}}_{\{m+2\}} = \int_{m+2} \Big[\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} + \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \Big]_{d=4}$$

must be finite in
the doubly-unresolved regions

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A_1 takes care of the singly-unresolved regions and A_{12} of the over-subtracting need to construct A_{12} such that all overlapping regions in 1-parton and 2-parton IR phase space regions are counted only once

$$\mathbf{C}_{ir}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{r}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{r}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{irs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{irs}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{C}_{ir;js}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{C}_{ir;js}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{CS}_{ir;s}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{CS}_{ir;s}|\mathcal{M}^{(0)}_{m+2}|^2$$

$$\mathbf{S}_{rs}(\mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_{12})|\mathcal{M}^{(0)}_{m+2}|^2 = \mathbf{S}_{rs}|\mathcal{M}^{(0)}_{m+2}|^2$$

the definition of A_{12} is rather simple

$$\mathbf{A}_{12}|\mathcal{M}^{(0)}_{m+2}|^2 \equiv \mathbf{A}_1\mathbf{A}_2|\mathcal{M}^{(0)}_{m+2}|^2$$

but showing that it has the right properties is non trivial, and requires considering iterated singly-unresolved limits and strongly-ordered doubly-unresolved limits

Iterated counterterms

$$\boldsymbol{\mathcal{A}}_{12}|\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 = \sum_t \left[\sum_{k\neq t} \frac{1}{2} \mathcal{C}_{kt} \boldsymbol{\mathcal{A}}_2 |\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 + \left(\mathcal{S}_t \boldsymbol{\mathcal{A}}_2 |\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 - \sum_{k\neq t} \mathcal{C}_{kt} \mathcal{S}_t \boldsymbol{\mathcal{A}}_2 |\mathcal{M}^{(0)}_{m+2}(\{p\})|^2 \right) \right]$$

where

$$\mathcal{C}_{kt}\boldsymbol{\mathcal{A}}_2 = \sum_{r\neq k,t} \left[\mathcal{C}_{kt}\mathcal{C}_{ktr} + \mathcal{C}_{kt}\mathcal{CS}_{kt;r} - \mathcal{C}_{kt}\mathcal{C}_{ktr}\mathcal{CS}_{kt;r} - \mathcal{C}_{kt}\mathcal{C}_{rkt}\mathcal{S}_{kt} + \sum_{i\neq r,k,t} \left(\frac{1}{2} \mathcal{C}_{kt}\mathcal{C}_{ir;kt} - \mathcal{C}_{kt}\mathcal{C}_{ir;kt}\mathcal{CS}_{kt;r} \right) \right] + \mathcal{C}_{kt}\mathcal{S}_{kt}$$

and likewise for $\mathcal{S}_t\boldsymbol{\mathcal{A}}_2\,,\ \mathcal{C}_{kt}\mathcal{S}_t\boldsymbol{\mathcal{A}}_2$

Iterated counterterms

- the momentum mapping for each of the iterated counterterms is built out of a composition of either the NLO collinear or the NLO soft mappings, or of both

- the treatment of colour in iterated singly-unresolved limits differs for spin-correlated SME from that of colour-correlated SME

  ⇒ no soft factorization formulae for simultaneously colour-correlated and spin-correlated SME.
  This was a no-go in the direction of generalised dipole-type counterterms

needs a NLO-type subtraction
between the m+2- and the m+1-parton contributions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{NNLO}}_{\{m+2\}} + \sigma^{\mathrm{NNLO}}_{\{m+1\}} + \sigma^{\mathrm{NNLO}}_{\{m\}}$$

$$\sigma^{\mathrm{NNLO}}_{\{m+2\}} = \int_{m+2} \Big[\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} + \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m \Big]_{d=4}$$

must be finite in
the doubly-unresolved regions →

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A_1 takes care of the singly-unresolved regions and A_{12} of the over-subtracting

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$$\sigma^{\rm NNLO}_{\{m+2\}} = \int_{m+2} \Big[{\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m$$

must be finite in
the doubly-unresolved regions

$$- {\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} + {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \Big]_{d=4}$$

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A_1 takes care of the singly-unresolved regions and A_{12} of the over-subtracting

$$\sigma^{\rm NNLO}_{\{m+1\}} = \int_{m+1} \Big\{ \Big[{\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big] J_{m+1}$$

$$- \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] J_m \Big\}_{\varepsilon=0}$$

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$$\sigma^{\rm NNLO}_{\{m+1\}} = \int_{m+1} \bigg\{ \Big[{\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big] J_{m+1}$$

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$$- \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] J_m \bigg\}_{\varepsilon=0}$$

remainder is finite by KLN theorem

$$\sigma^{\rm NNLO}_{\{m\}} = \int_m \Big\{ {\rm d}\sigma^{\rm VV}_m + \int_2 \Big[{\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \Big] + \int_1 \Big[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \Big(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \Big)^{\rm A_1} \Big] \Big\}_{\varepsilon=0} J_m$$

Conclusions

- we devised a NNLO subtraction scheme for $e^+e^- \to n$ jets
- the calculation is organised into 3 contributions, RR, RV, VV, each of which supposed to be finite in *d*=4 dimensions
- For $e^+e^- \to 3$ jets the RR and RV pieces are shown to be finite (see Gabor's talk)
- The VV piece still needs be done (but must be finite in *d*=4 dimensions, because of the KLN theorem)