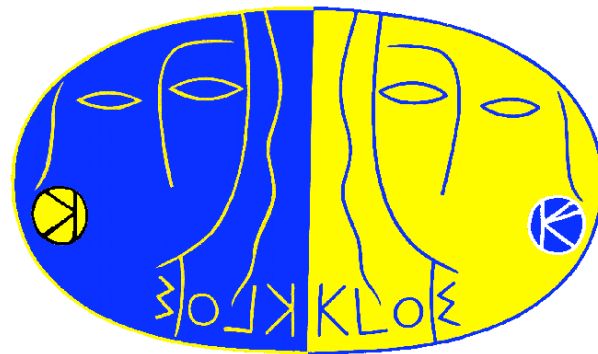


Measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma(\gamma))$ and
the dipion contribution to the muon anomaly
with the KLOE detector

Phys. Lett. B 670, 285 (2009)

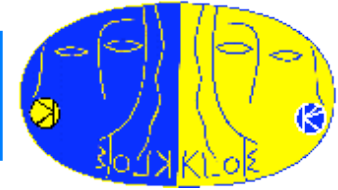
Graziano Venanzoni
(for the KLOE collaboration)
Laboratori Nazionali di Frascati



CIPANP 2009

San Diego, 30 May 2009

Outlook

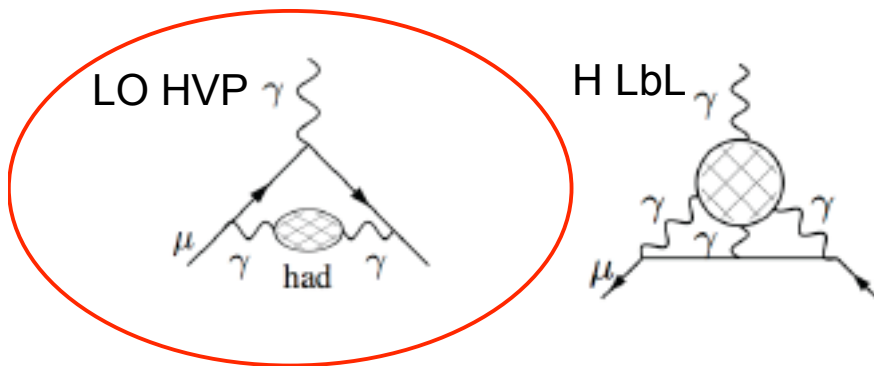


- Hadronic contribution to a_μ and ISR measurement (“Radiative Return”)
- New KLOE measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ via ISR (KLOE08)
- Evaluation of $a_\mu^{\pi\pi}$ and comparison with CMD-2/SND
- Conclusion & Outlook

Muon anomaly

$$a_\mu = \frac{(g_\mu - 2)}{2}$$

- Long established discrepancy ($>3\sigma$) between SM prediction and BNL E821 experiment
- Theoretical error δa_μ^{SM} ($\sim 6 \times 10^{-10}$) dominated by LO HVP ($4 \div 5 \cdot 10^{-10}$) and HLbL ($2.5 \div 4$)
- Experimental error $\delta a_\mu^{\text{EXP}} \sim 6 \times 10^{-10}$ (E821). Plan to reduce it to $1.5 \cdot 10^{-10}$ (new g-2 experiment @FNAL)



$$a_\mu^{\text{had,LO}} = (690.9 \pm 4.4) \cdot 10^{-10}$$

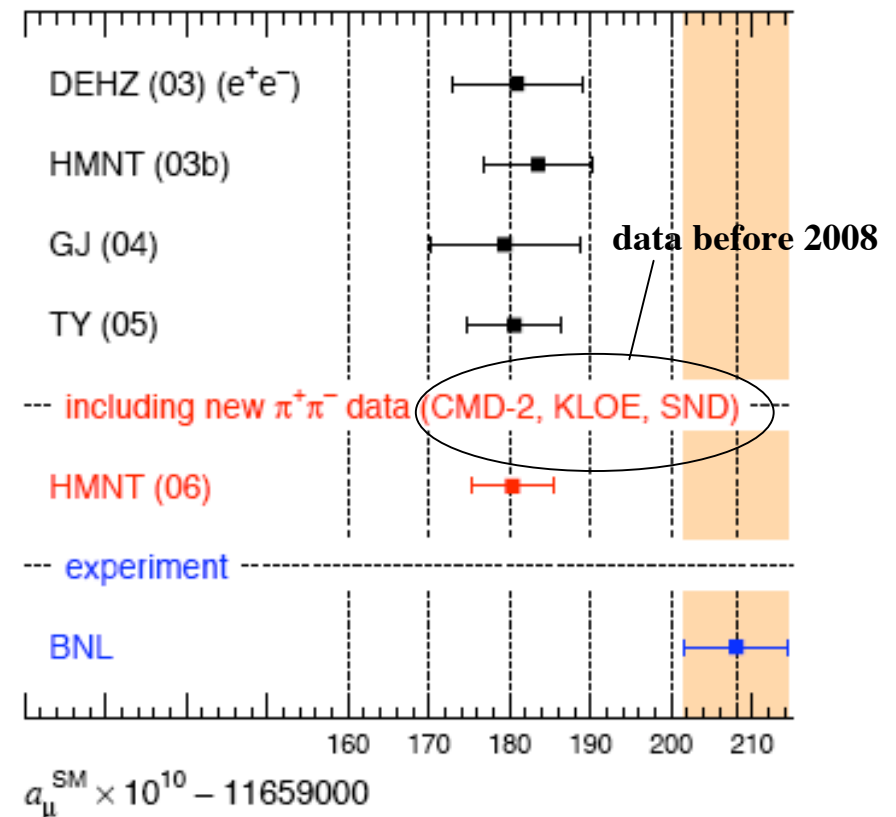
[Eidelman, TAU08]

$$a_\mu^{\text{had,LbL}} \sim (11 \pm 4) \cdot 10^{-10}$$

$$= 10.5 \pm 2.6$$

[Prades, de Rafael & A. Vainshtein 08]

a_μ^{SM} compared to BNL world av.

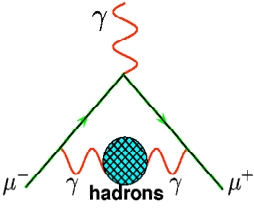


$$a_\mu^{\text{EXP}} - a_\mu^{\text{TH}} = (27.6 \pm 8.1) \cdot 10^{-10}, \sim 3.4\sigma$$

T.Teubner, PHIPSI08

$a_{\mu}^{\text{had,LO}}$

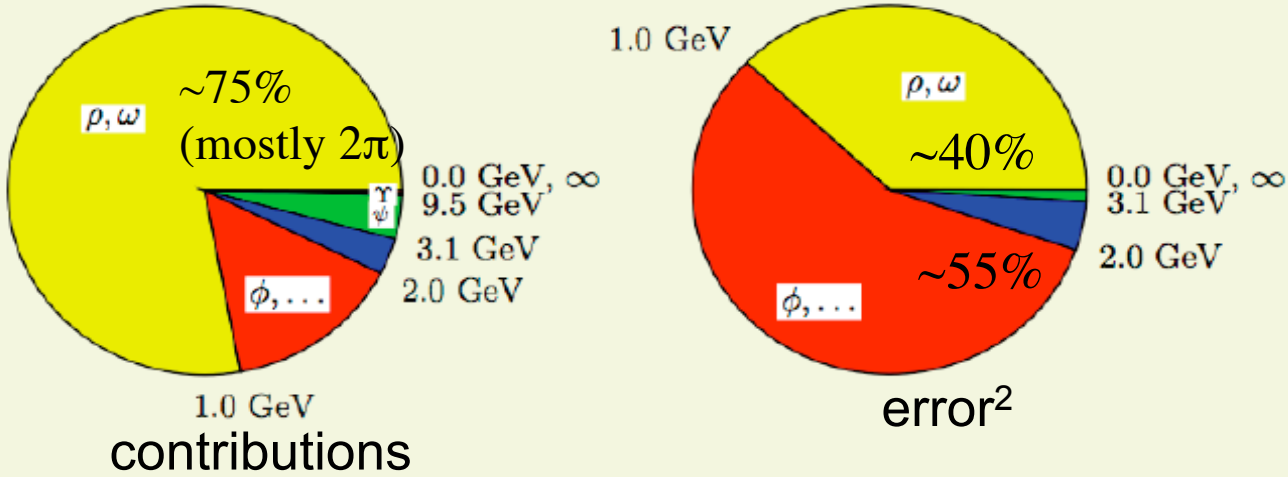
Hadronic contribution to a_{μ} can be estimated by means of a dispersion integral on e^+e^- data:



$$a_{\mu}^{\text{had,lo}} = \frac{1}{4\pi^3} \int_{4m_{\pi}^2}^{\infty} \sigma_{e^+e^- \rightarrow \text{hadr}}(s) K(s) ds$$

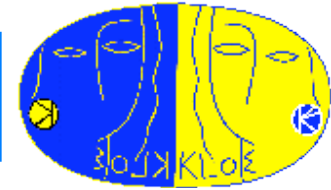
$K(S) \sim 1/s$; it emphasizes the role of low energies; particular important is the reaction $e^+e^- \rightarrow \pi^+\pi^-$ (below 1 GeV, contributes to $\sim 70\%$ to a_{μ}^{had})

F. Jegerlehner, Talk at PHIPSI08



σ_{HAD} traditionally measured by an energy scan (CMD-2/SND). At KLOE we use ISR to scan the energy below the ϕ peak

$\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ with ISR:

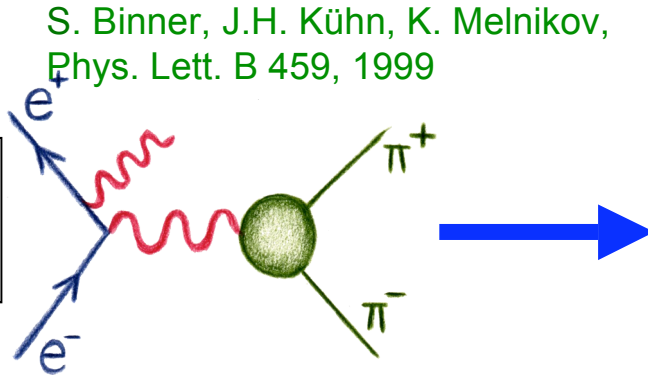


Since DAΦNE runs at fixed energy (1020 MeV), we measure the cross section $\sigma(e^+e^- \rightarrow \text{hadrons})$ as a function of the hadronic c.m. energy M_{hadr} by using

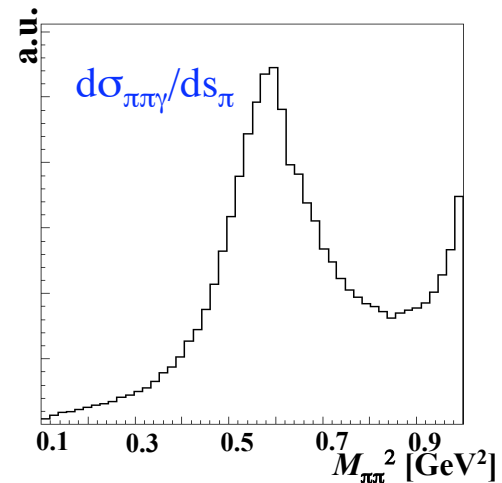
ISR ("radiative return"):

$$s \frac{d\sigma_{\pi\pi\gamma}}{dM_{\pi\pi}^2} = \sigma_{\pi\pi}(s) \times H(s)$$

Neglecting FSR effects



S. Binner, J.H. Kühn, K. Melnikov, Phys. Lett. B 459, 1999

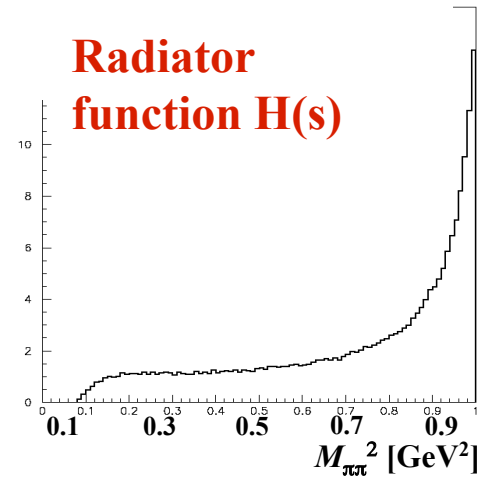


$$4m_{\pi}^2 \leq s_{\pi} = M_{\pi\pi}^2 \leq m_{\phi}^2$$

This method is a complementary approach to the standard energy scan.

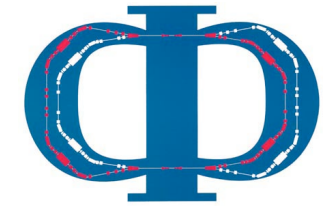
- Luminosity, acceptance (and other normalizations) enter only once !
- Requires precise calculations of the radiator **H**
 - PHOKHARA MC NLO Generator

(H. Czyż, A. Grzebińska, J.H. Kühn, G. Rodrigo, Eur. Phys. J. C 27, 2003)

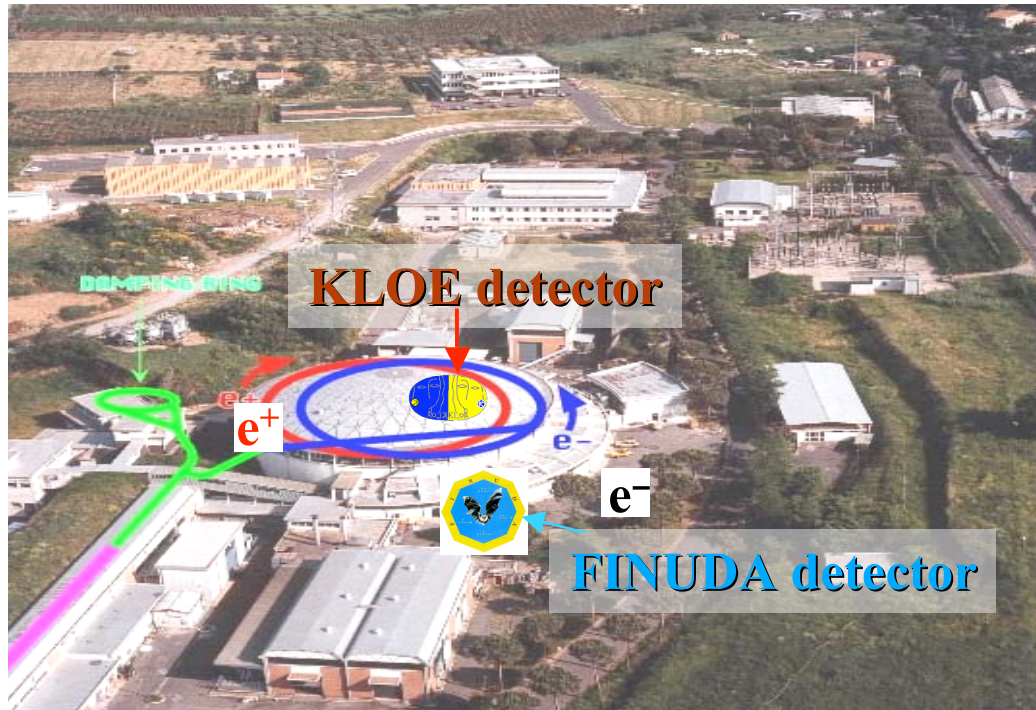


KLOE has published a first measurement of $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ with ISR, using 2001 data (140pb⁻¹) PLB606(2005)12
⇒ ~3σ discrepancy on a_μ between SM and BNL exp

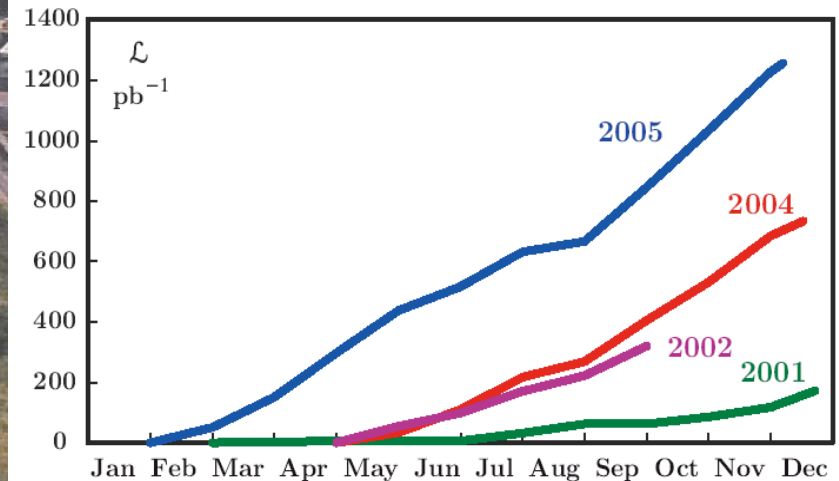
DAΦNE: A Φ-Factory



e^+e^- - collider with $\sqrt{s}=m_\Phi\approx 1.0195$ GeV



Integrated Luminosity



Luminosity $L_{\text{peak}} = 1.5 \cdot 10^{32} \text{cm}^{-2}\text{s}^{-1}$

Our published results (PLB606(2005)12) were based on 140 pb⁻¹ of 2001 data!

Our new results (2008) are based on 240 pb⁻¹ from 2002 data!

Total KLOE int. Luminosity:
 $\int \mathcal{L} dt \sim 2500 \text{ pb}^{-1}$ (2001 - 05)

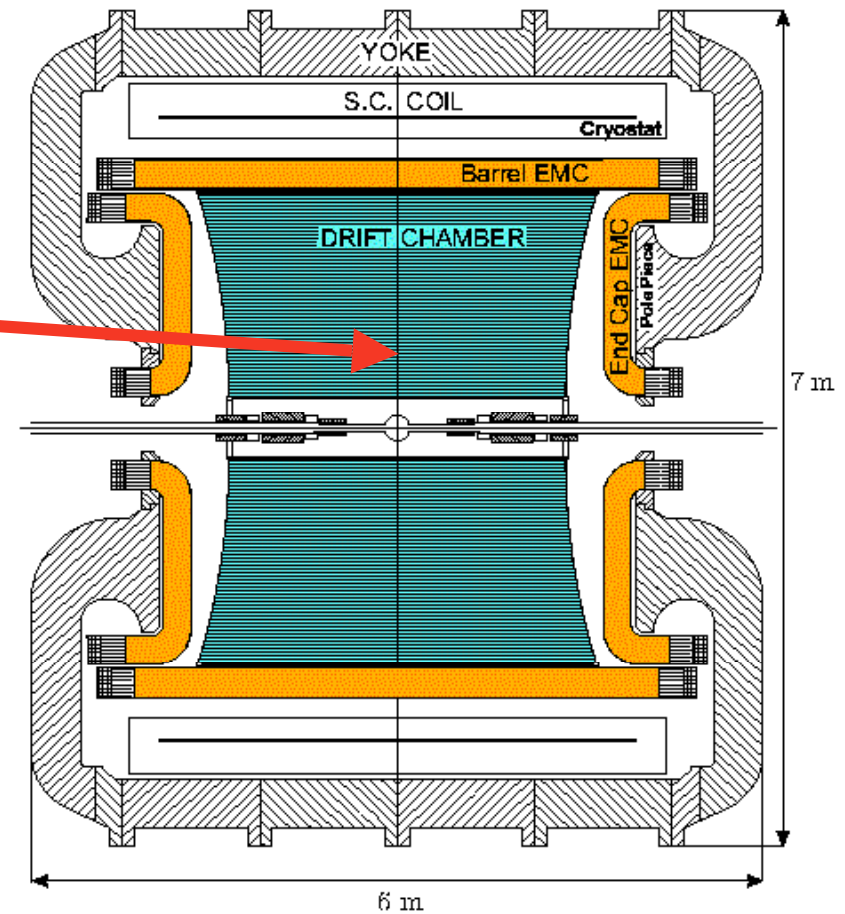
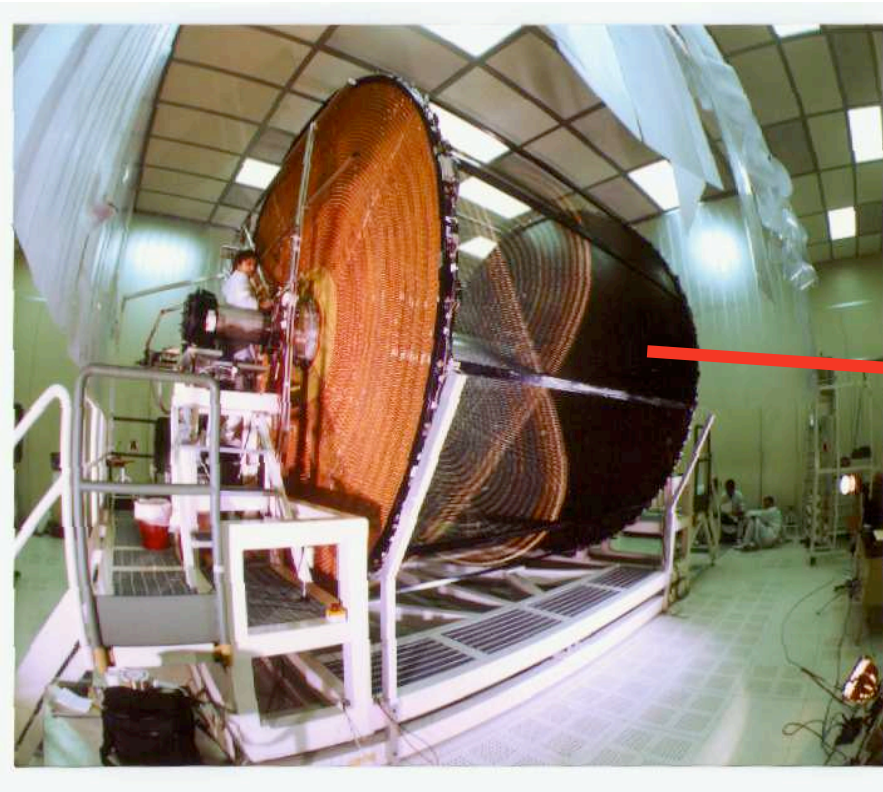
2006:

- Energy scan with 4 points around m_Φ -peak
- 220 pb⁻¹ at $\sqrt{s} = 1000$ MeV (off-peak data)

KLOE Detector



Drift chamber

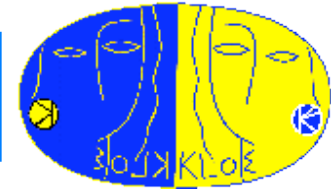


$$\sigma_p/p = 0.4\% \text{ (for } 90^\circ \text{ tracks)}$$

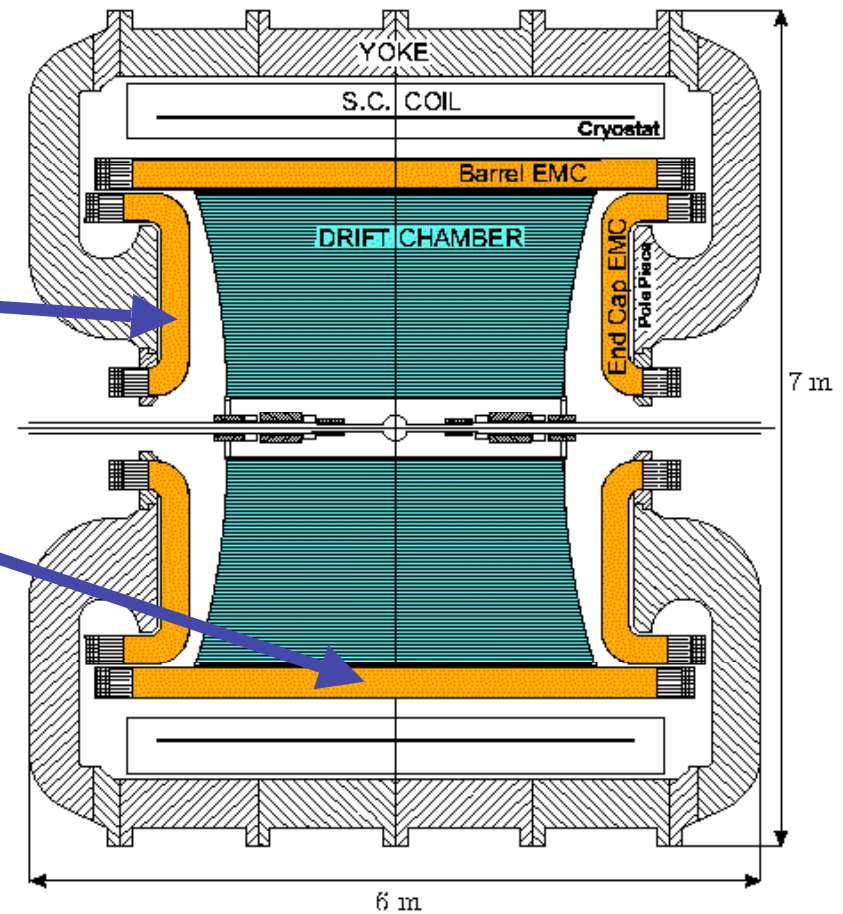
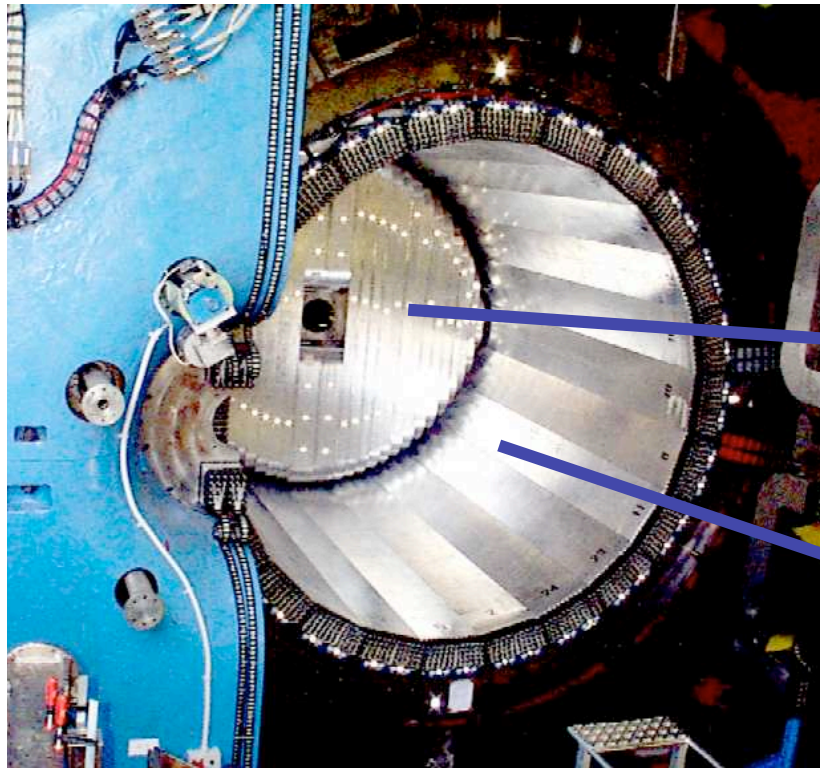
$$\sigma_{xy} \approx 150 \mu\text{m}, \sigma_z \approx 2 \text{ mm}$$

***Excellent momentum
resolution***

KLOE Detector



Electromagnetic Calorimeter



$$\sigma_E/E = 5.7\% / \sqrt{E(\text{GeV})}$$
$$\sigma_T = 54 \text{ ps} / \sqrt{E(\text{GeV})} \oplus 100 \text{ ps}$$

(Bunch length contribution subtracted from constant term)

Excellent timing resolution

Extracting $|F_\pi|^2$ from $\pi\pi\gamma$ events



a) Via absolute Normalisation to VLAB Luminosity (as in 2005 analysis):

1)
$$\frac{d\sigma_{\pi\pi\gamma(\gamma)}^{obs}}{dM_{\pi\pi}^2} = \frac{\Delta N_{Obs} - \Delta N_{Bkg}}{\Delta M_{\pi\pi}^2} \cdot \frac{1}{\epsilon_{Sel}} \cdot \frac{1}{\int L dt}$$

$d\sigma_{\pi\pi\gamma(\gamma)}/dM^2$ is obtained by subtracting background from observed event spectrum, divide by selection efficiencies, and *int. luminosity*.

2)
$$\sigma_{\pi\pi}(s) \approx s \frac{d\sigma_{\pi\pi\gamma(\gamma)}^{obs}}{dM_{\pi\pi}^2} \cdot \frac{1}{H(s)}$$

Obtain $\sigma_{\pi\pi}$ from (ISR) - radiative cross section $d\sigma_{\pi\pi\gamma(\gamma)}/dM^2$ via theoretical radiator function $H(s)$:

3)
$$|F_\pi|^2 = \frac{3s}{\pi\alpha^2\beta_\pi^3} \sigma_{\pi\pi}(s)$$

Relation between $|F_\pi|^2$ and the cross section $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$

b) Via bin-by-bin Normalisation to rad. Muon events (*analysis is in progress*)

Radiative Corrections



Radiator-Function $H(s, s_\pi)$ (ISR):

- ISR-Process calculated at NLO-level

PHOKHARA generator

(H.Czyż, A.Grzelińska, J.H.Kühn, G.Rodrigo, EPJC27,2003)

Precision: 0.5%

$$s \cdot \frac{d\sigma_{\pi\pi\gamma}}{ds_\pi} = \sigma_{\pi\pi}(s_\pi) \times H(s, s_\pi)$$

Radiative Corrections:

i) Bare Cross Section

divide by Vacuum Polarisation $\delta(s) = (\alpha(s)/\alpha(0))^2$

→ from F. Jegerlehner

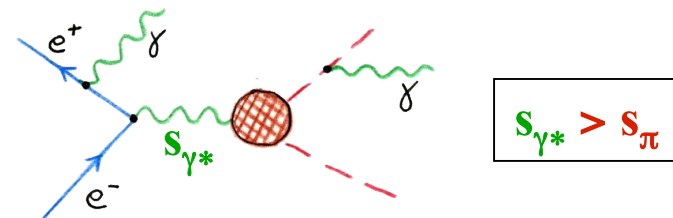
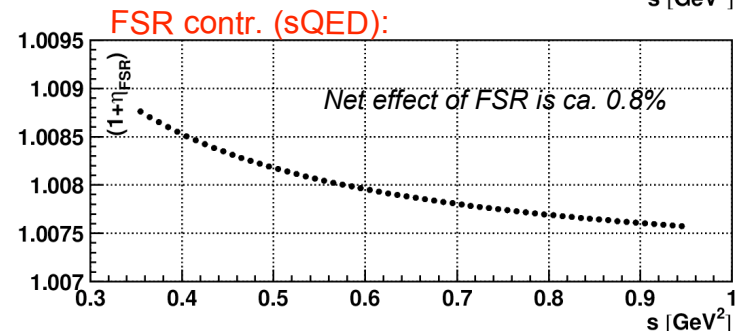
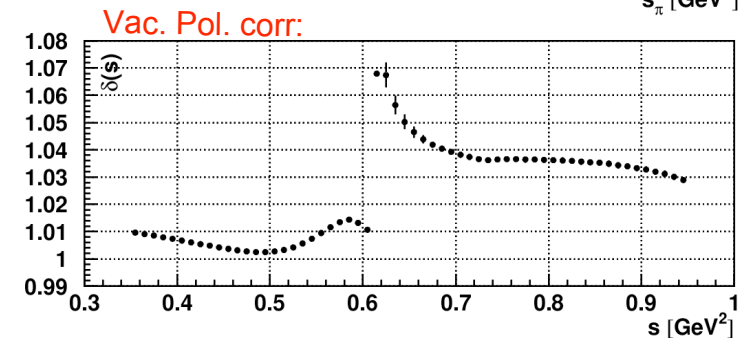
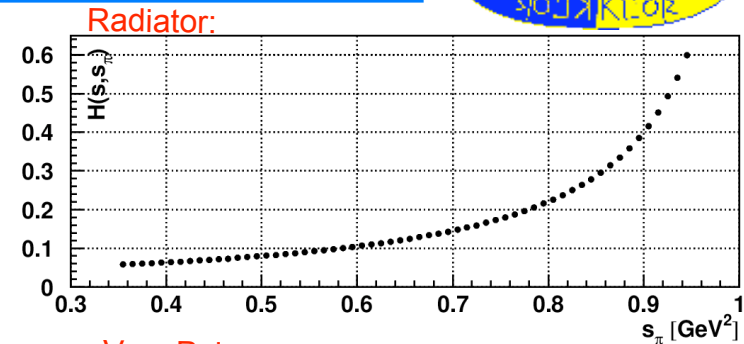
ii) FSR

Cross section $\sigma_{\pi\pi}$ must be incl. for FSR
for use in the dispersion integral of a_μ



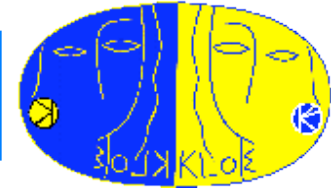
FSR corrections have to be taken into account
in the efficiency eval. (SA Acceptance, M_{Trk}) and
in the passage $s_\pi \rightarrow s_{\gamma^*}$

(H.Czyż, A.Grzelińska, J.H.Kühn, G.Rodrigo, EPJC33,2004)



$$s_{\gamma^*} > s_\pi$$

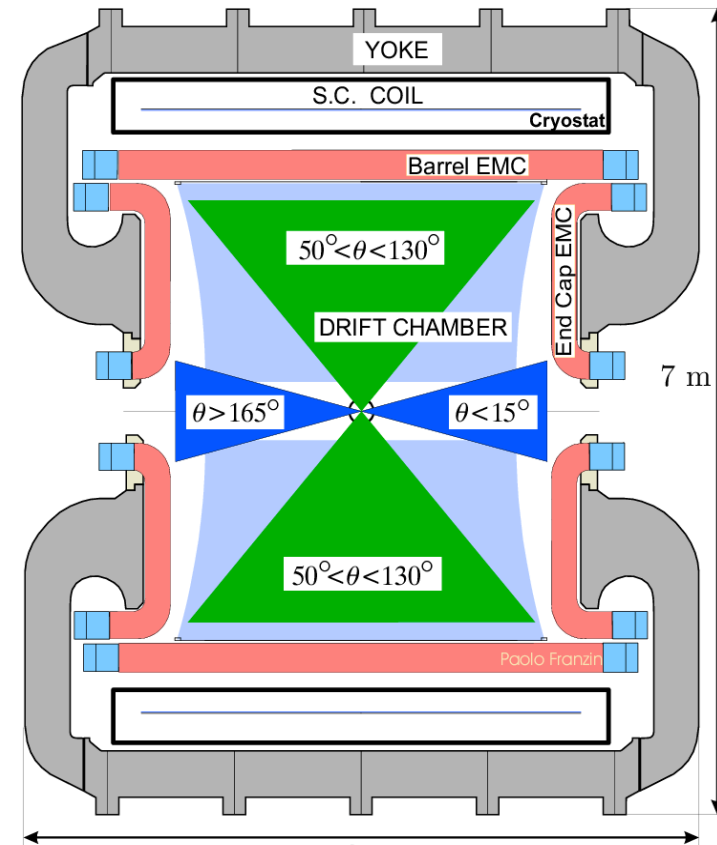
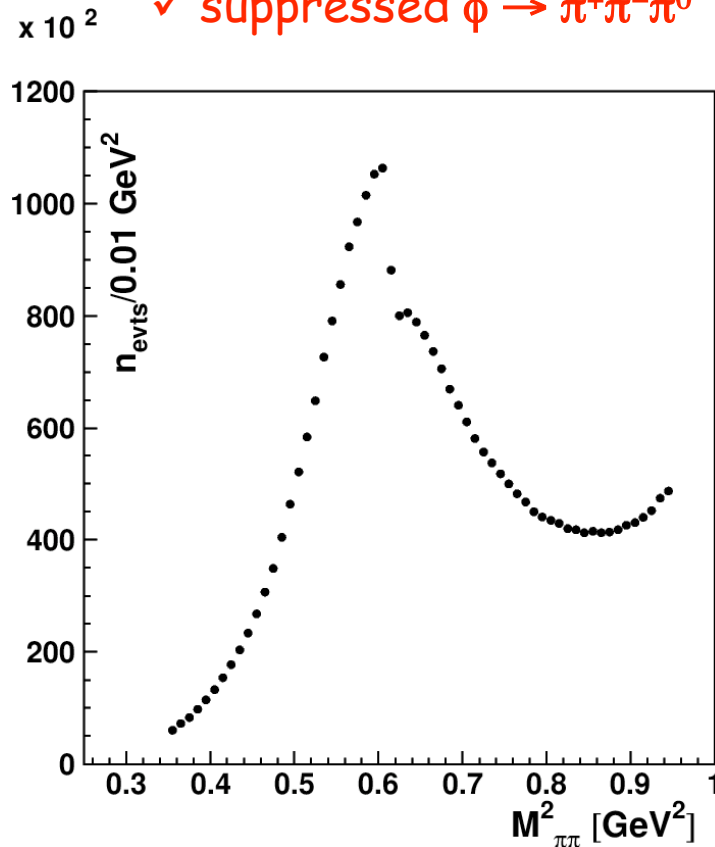
Event Selection



- a) 2 tracks with $50^\circ < \theta_{\text{track}} < 130^\circ$
- b) small angle γ ($\theta_{\pi\pi} < 15^\circ$ or $> 165^\circ$)

kinematics: $\vec{p}_\gamma = \vec{p}_{\text{miss}} = -(\vec{p}_+ + \vec{p}_-)$

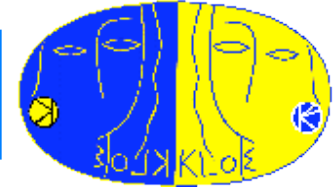
- ✓ high statistics for ISR
- ✓ low relative FSR contribution
- ✓ suppressed $\phi \rightarrow \pi^+\pi^-\pi^0$ wrt the signal



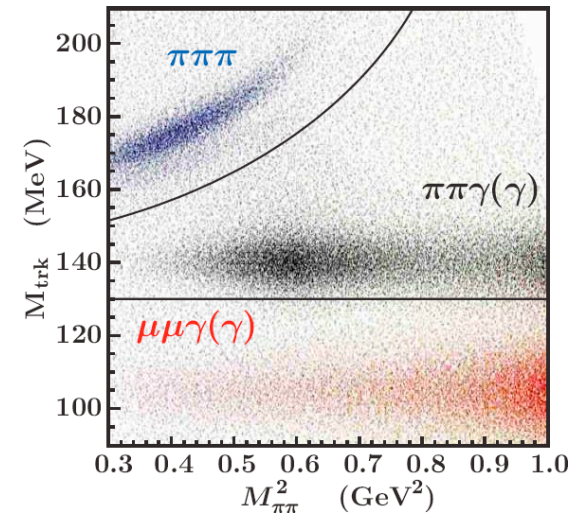
statistics: 242pb⁻¹ of 2002 data

3.1 Mill. Events between 0.35 and 0.95 GeV²

Event Selection



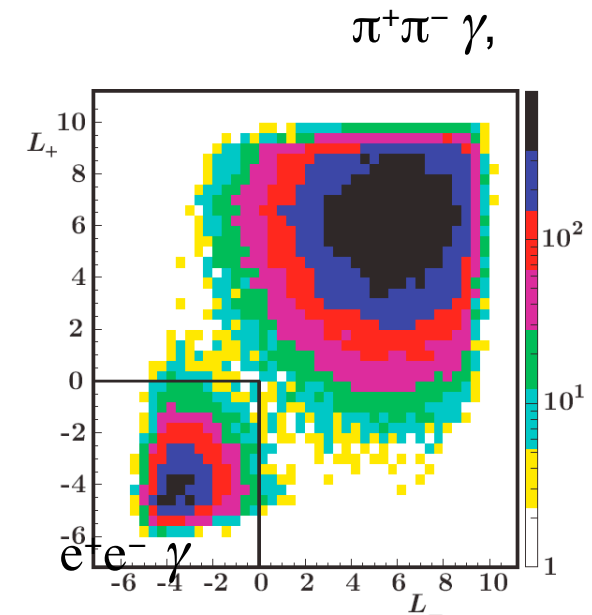
- Experimental challenge: control backgrounds from
 - $\phi \rightarrow \pi^+ \pi^- \pi^0$
 - $e^+ e^- \rightarrow e^+ e^- \gamma$
 - $e^+ e^- \rightarrow \mu^+ \mu^- \gamma$,
 removed using kinematical cuts in *trackmass* $M_{Trk} - M_{\pi\pi}^2$ plane



M_{Trk} :
 defined by 4-momentum conservation
 under the hypothesis of 2 tracks with
 equal mass and one γ

$$\left(\sqrt{s} - \sqrt{p_1^2 + M_{trk}^2} - \sqrt{p_2^2 + M_{trk}^2} \right)^2 - (p_1 + p_2)^2 = 0$$

To further clean the samples from radiative Bhabha events, we use a particle ID estimator for each charged track based on **Calorimeter** Information and Time-of-Flight.

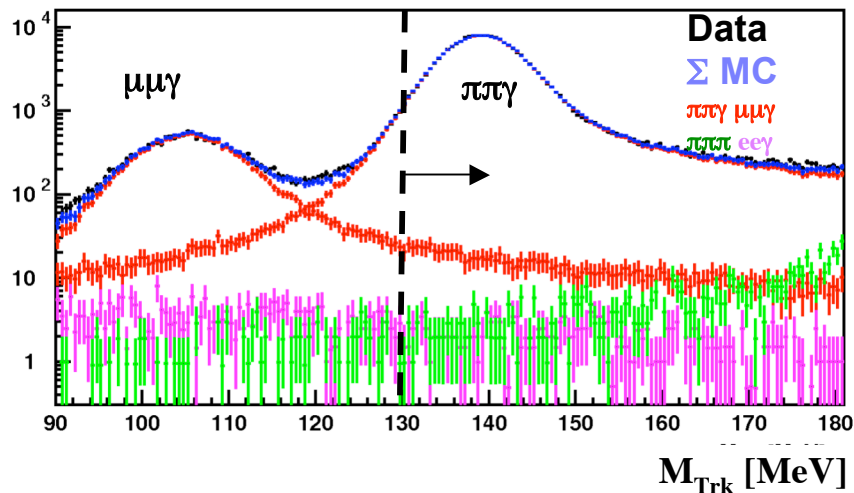


Background:

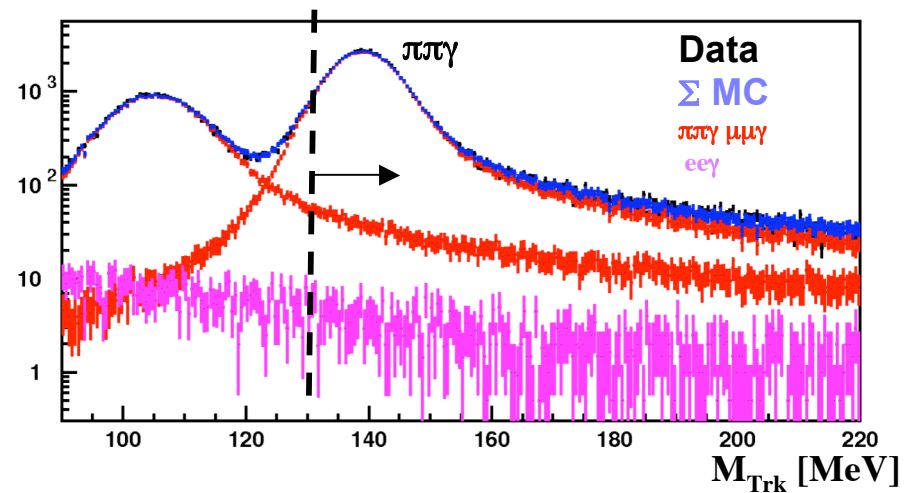


Main backgrounds estimated from MC shapes fitted to data distribution in M_{Trk}
 ($\pi\pi\gamma/\mu\mu\gamma$, $\pi\pi\pi$, $ee\gamma$)

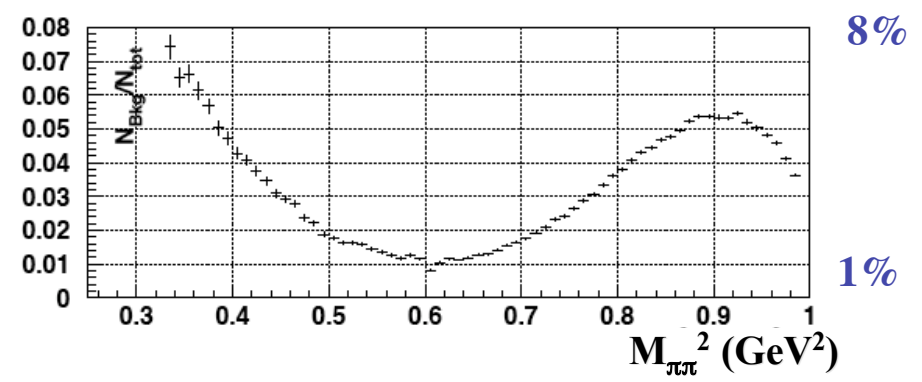
$0.60 < M_{\pi\pi}^2 < 0.62 \text{ GeV}^2, \chi^2/ndof = 158/180$



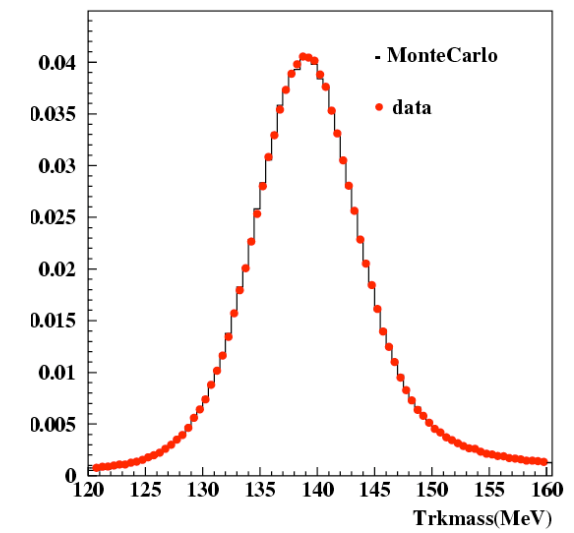
$0.84 < M_{\pi\pi}^2 < 0.86 \text{ GeV}^2, \chi^2/ndof = 179/258$



Tot bckg ($\mu\mu\gamma$, $\pi\pi\pi$ and $ee\gamma$) contribution



- Excellent agreement on M_{TRK} distribution between data and MC (2 different sets of „tuning“)



Tracking



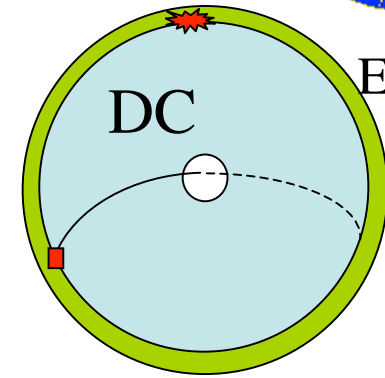
Two control samples

$\pi^+\pi^-\pi^0$

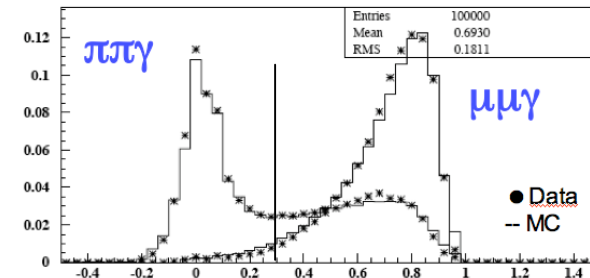
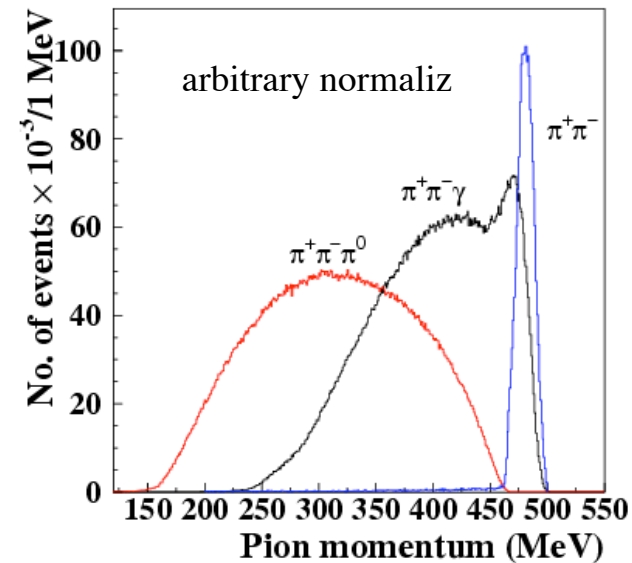
- 1) a tagging track recognized as a pion by PID, extrapolating back to the IP, which satisfies the trigger
- 2) 2 prompt clusters not associated to the tagging track with $E > 50$ MeV and distant each other 60 cm
- 3) A constraint on the photon energy and time to further clean the sample, and improve missing momentum and energy

$\pi^+\pi^-\gamma$

- 1) As for $\pi^+\pi^-\pi^0$ sample
- 2) 1 prompt clusters not associated to the tagging track with $E > 50$ MeV
- 3) The tagging track must have $p > 460$ MeV (to reject $\pi^+\pi^-\pi^0$ events), the *candidate* track must have mass (built from 4 momentum conservation) $M_{\text{miss}} > 120$ MeV and $NN < 0.3$, to suppress $\mu^+\mu^-\gamma$ events

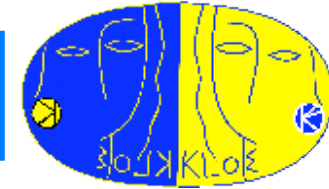


EMC



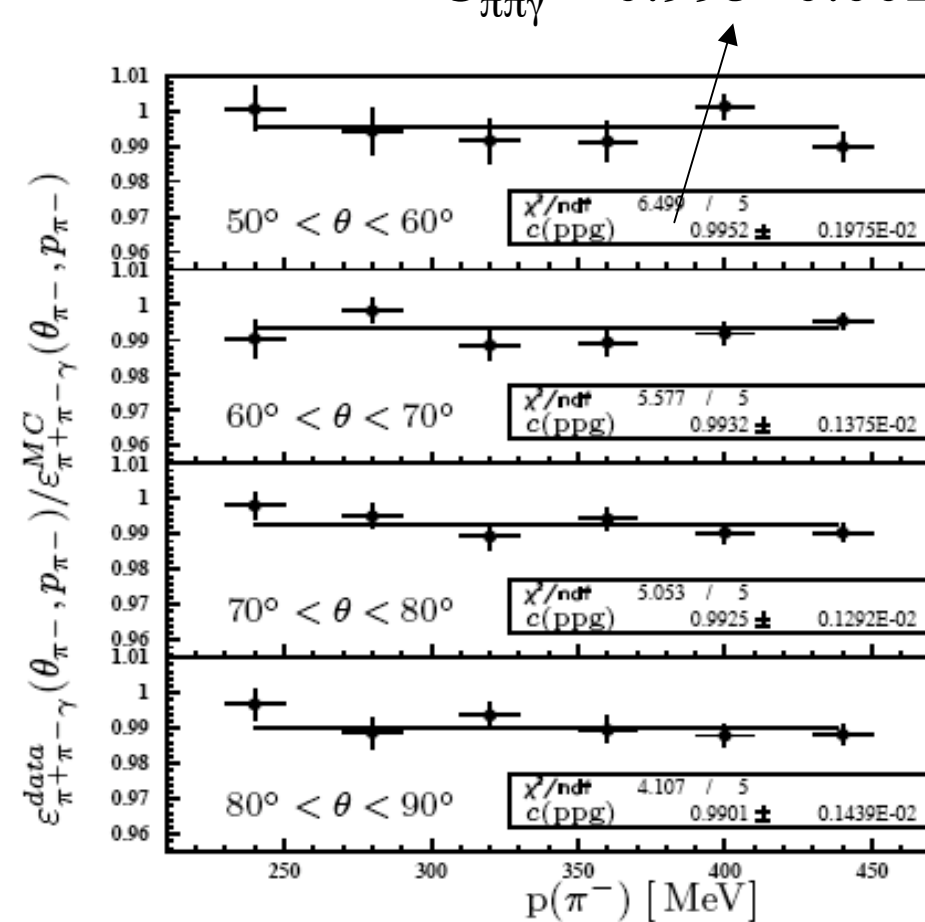
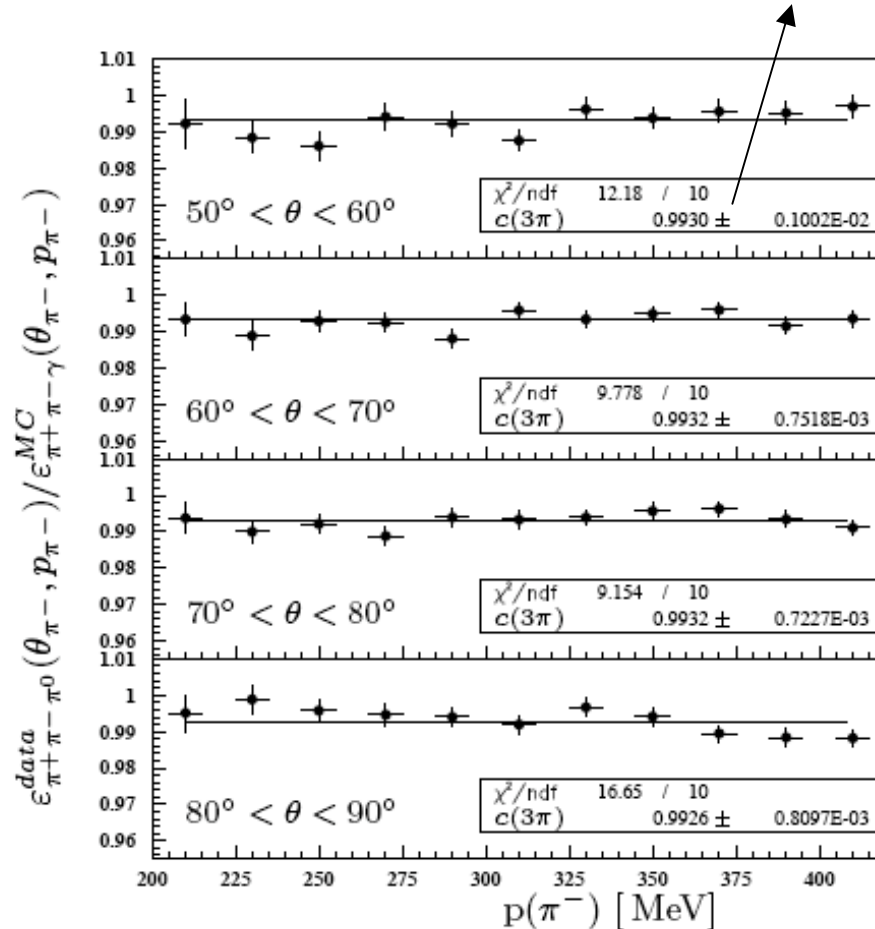
NN
output

Data/MC corrections from $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\gamma$



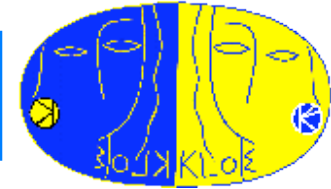
$$C_{3\pi} = 0.993 \pm 0.001$$

$$C_{\pi\pi\gamma} = 0.995 \pm 0.002$$



When “weighted” for the $\pi\pi\gamma$ event distribution the two methods gives 0.3% fractional difference in $M_{\pi\pi}^2$ which is the systematic error

π/e PID and TCA

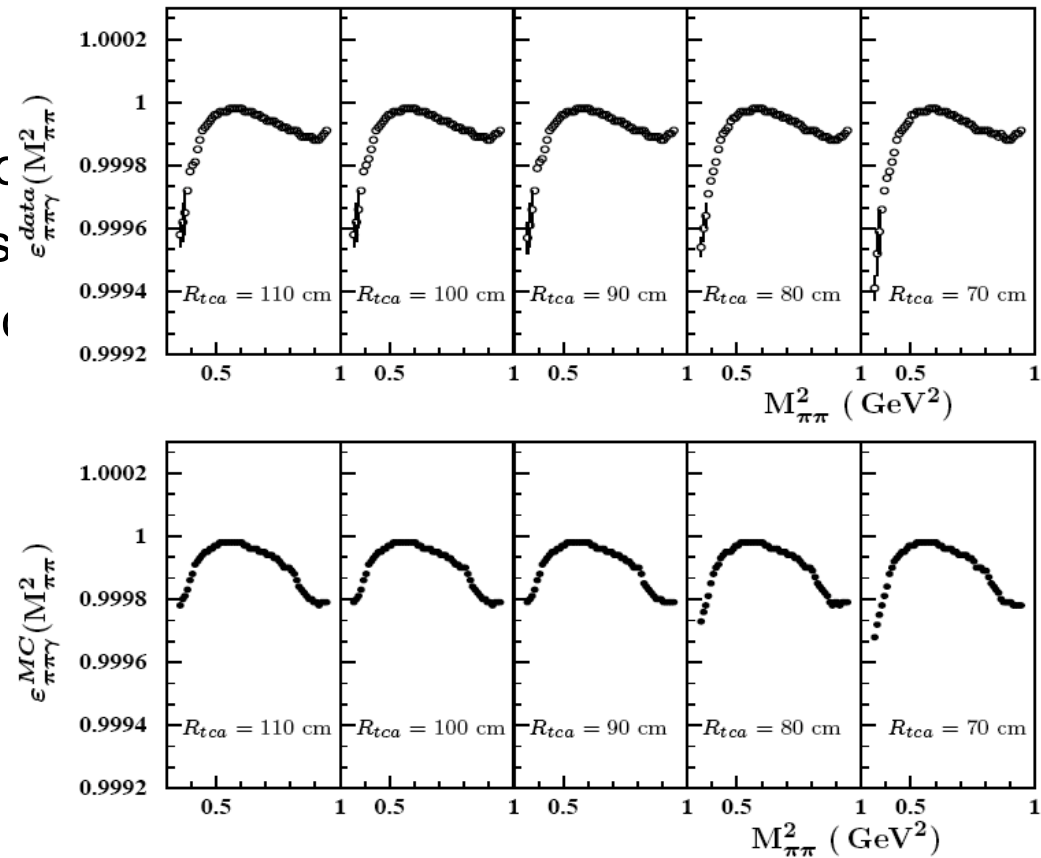


$\pi^+\pi^-\gamma$ sample

- 1) Two tracks satisfying $\pi\pi\gamma$ “tracking” acceptance selection
- 2) a tagging track recognized as pion by PID, extrapolating back to the IP, which satisfies the trigger
- 3) Look for a cluster with $PID > 0$ associated to the *candidate* track in slices of θ, p

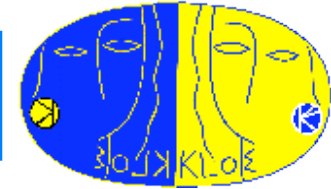
Efficiency ~ 1

data/MC correction = 1 at $R=90$ cm



the systematic error is given by varying the association radius, the effect on the correction data/MC is negligible

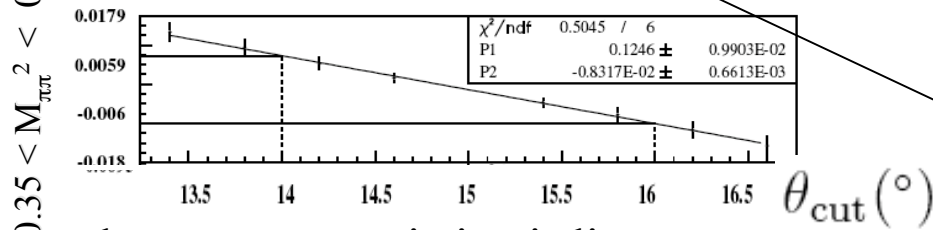
Acceptance



$\theta_{\pi\pi}$ is angle of the missing photon

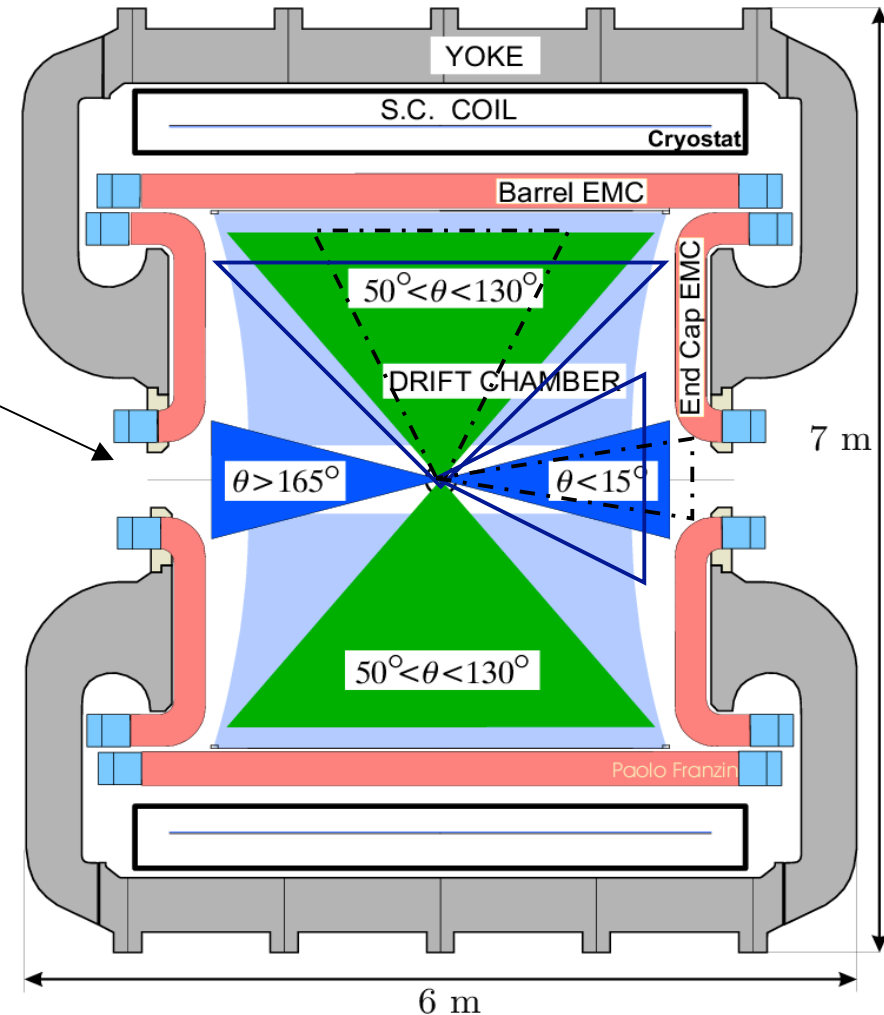
We study the impact of enlarging/reducing the fiducial volume on the geometrical acceptance in slices of $M_{\pi\pi}^2$

$$\frac{N_{MC}(\theta_{\pi\pi} < \theta_{cut})}{N_{MC}(\theta_{\pi\pi} < 15^\circ)} - \frac{N_{data}(\theta_{\pi\pi} < \theta_{cut})}{N_{data}(\theta_{\pi\pi} < 15^\circ)}$$

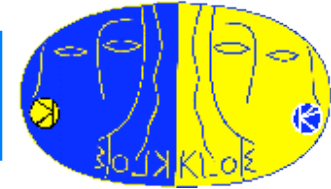


the spectrum variation is linear as a function of the cut, so the excursion at ± 1 degree is taken as systematic error

$M_{\pi\pi}^2$ range (GeV^2)	Systematic error (%)
$0.35 \leq M_{\pi\pi}^2 < 0.39$	0.6
$0.39 \leq M_{\pi\pi}^2 < 0.43$	0.5
$0.43 \leq M_{\pi\pi}^2 < 0.45$	0.4
$0.45 \leq M_{\pi\pi}^2 < 0.49$	0.3
$0.49 \leq M_{\pi\pi}^2 < 0.51$	0.2
$0.51 \leq M_{\pi\pi}^2 < 0.64$	0.1
$0.64 \leq M_{\pi\pi}^2 < 0.95$	-



Unfolding



Our bin width (0.01 GeV^2 is $\sim 5 \delta M_{\pi\pi}^2$) \Rightarrow Resolution
Matrix almost diagonal!

- We use Bayesian approach

G. D'Agostini, Nucl. Instrum. Meth. A 362 (1995) 487

- method based on Bayes' theorem
 - ◆ no matrix inversion needed
 - ◆ can be applied to multidimensional problems
 - ◆ **iterative** algorithm; can start with a uniform distribution

■ **Bayes formula:**
$$P(C_i|E_j) = \frac{P(E_j|C_i)P(C_i)}{\sum_{l=1}^{n_c} P(E_j|C_l)P(C_l)}$$

"true", normalized distribution

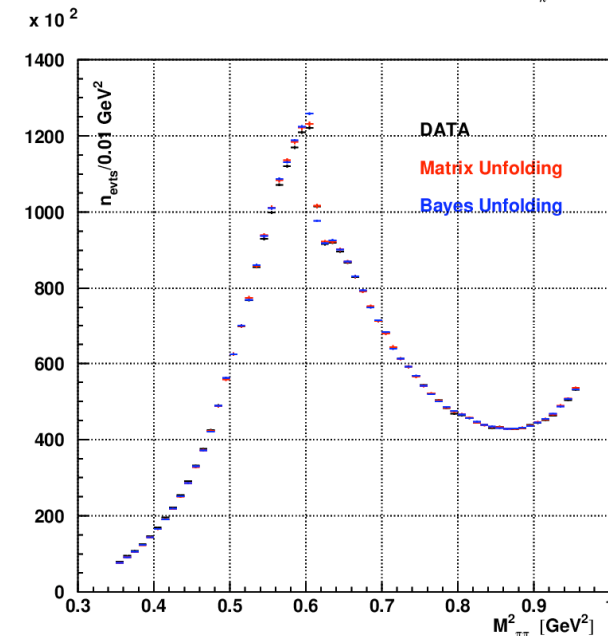
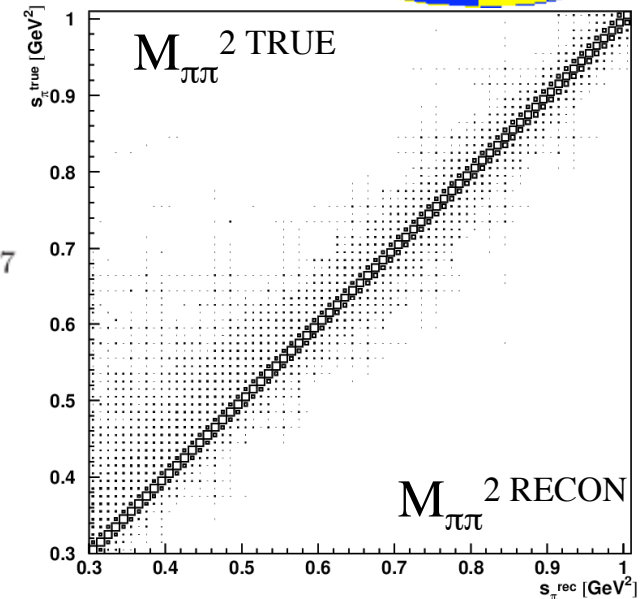
- ◆ "if we observe a single event "(effect E_j)", the probability that it has been due to the i -th cause "(C_i)," is proportional to the probability of the cause times probability of the cause to produce the effect"

D'Agostini

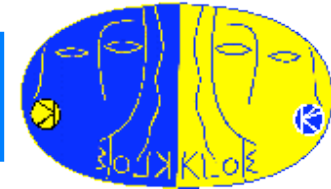
- We compare the result with the simple matrix procedure. They gives difference only around ρ - ω region

$M_{\pi\pi}^2 \text{ (GeV}^2\text{)}$	0.58	0.59	0.6	0.61	0.62
$\delta_{unf} \text{ (\%)} $	0.4	0.3	2.1	4.0	0.4

- Very small effect;
systematic error negligible on a_u !



Luminosity:



KLOE measures L with Bhabha scattering

F. Ambrosino et al. (KLOE Coll.)
Eur.Phys.J.C47:589-596,2006

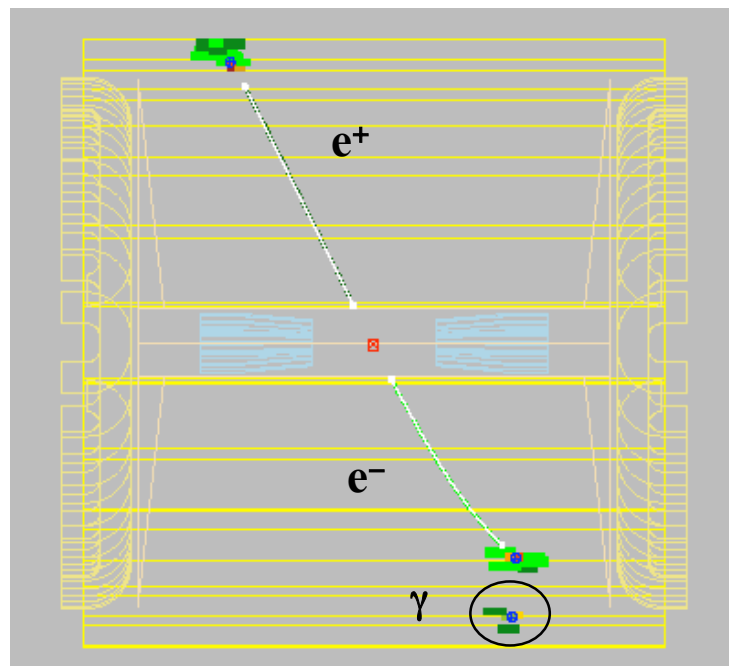
$55^\circ < \theta < 125^\circ$
 acollinearity $< 9^\circ$
 $p \geq 400$ MeV

$$\int \mathcal{L} dt = \frac{N_{obs} - N_{bkg}}{\sigma_{eff}}$$

generator used for σ_{eff}

BABAYAGA (Pavia group):

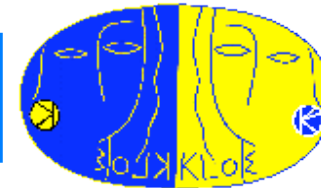
C. M.C. Calame et al., NPB758 (2006) 22



new version (**BABAYAGA@NLO**) gives
 0.7% decrease in cross section,
 and better accuracy: 0.1%

Systematics on Luminosity	
Theory	0.1 %
Experiment	0.3 %
TOTAL 0.1 % th \oplus 0.3% exp = 0.3%	

New KLOE result (KLOE08)



Systematic errors on $a_\mu^{\pi\pi}$:

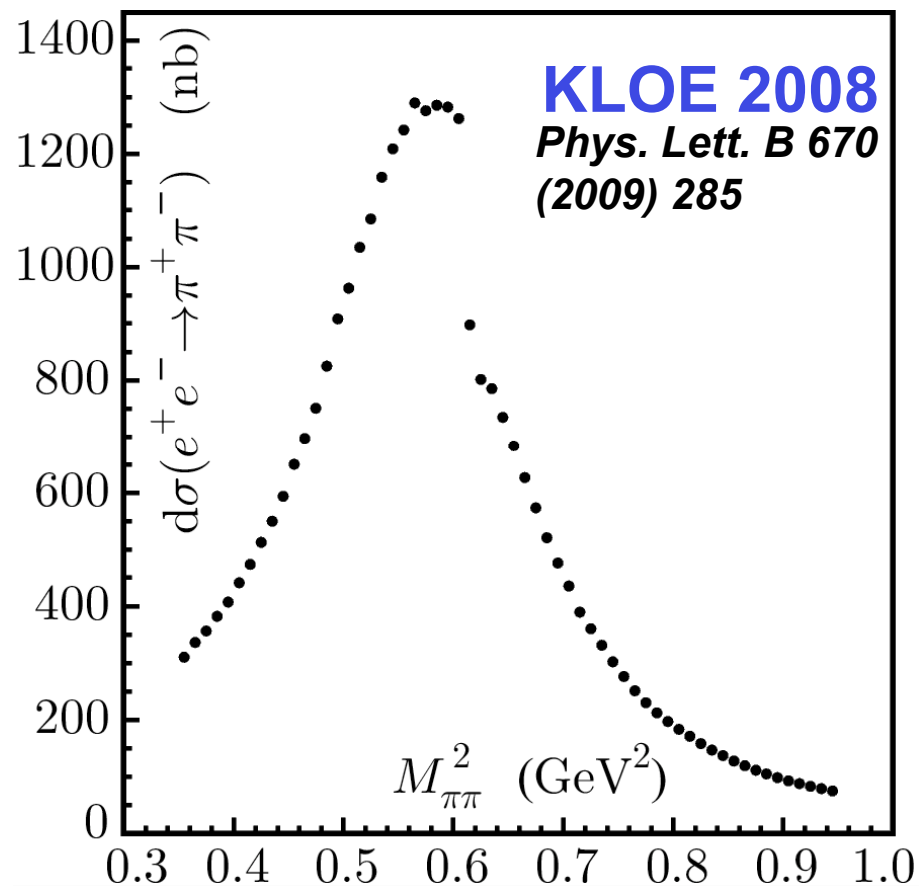
Reconstruction Filter	negligible
Background	0.3%
Trackmass/Miss. Mass	0.2%
π/e -ID and TCA	negligible
Tracking	0.3%
Trigger	0.1%
Acceptance ($\theta_{\pi\pi}$)	0.1%
Acceptance (θ_π)	negligible
Unfolding	negligible
Software Trigger	0.1%
\sqrt{s} dep. Of H	0.2%
Luminosity ($0.1_{\text{th}} \oplus 0.3_{\text{exp}}$)%	0.3%

experimental fractional error on $a_\mu = 0.6\%$

FSR resummation	0.3%
Radiator H	0.5%
Vacuum polarization	0.1%

theoretical fractional error on $a_\mu = 0.6\%$

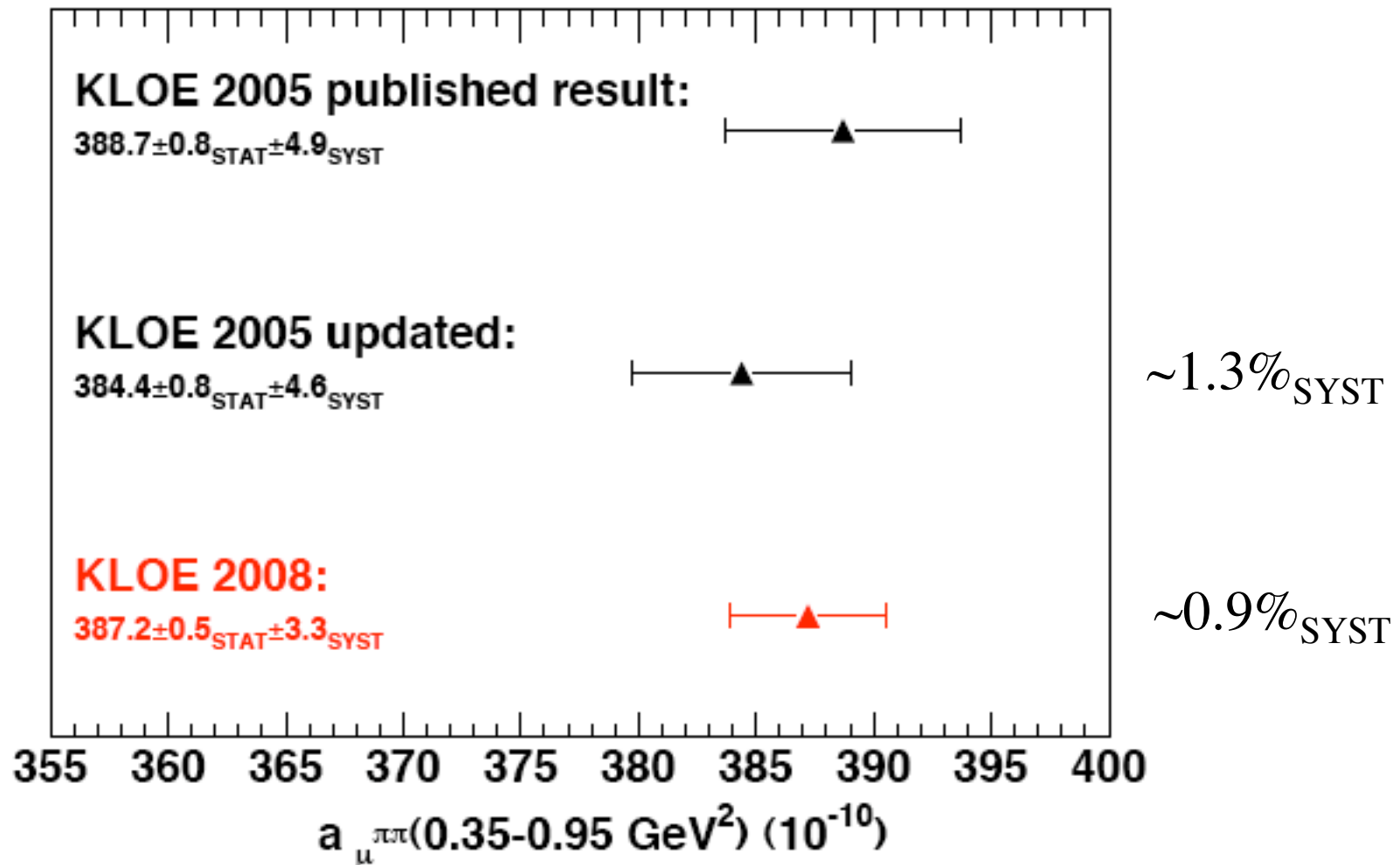
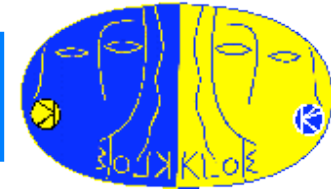
$\sigma_{\pi\pi}$, undressed from VP, inclusive for FSR as function of $(M_{\pi\pi}^0)^2$



stat. error only

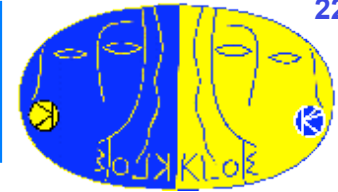
$$a_\mu^{\pi\pi}(0.35-0.95\text{GeV}^2) = (387.2 \pm 0.5_{\text{stat}} \pm 2.4_{\text{sys}} \pm 2.3_{\text{theo}}) \cdot 10^{-10}$$

$a_{\mu}^{\pi\pi}$ from KLOE:

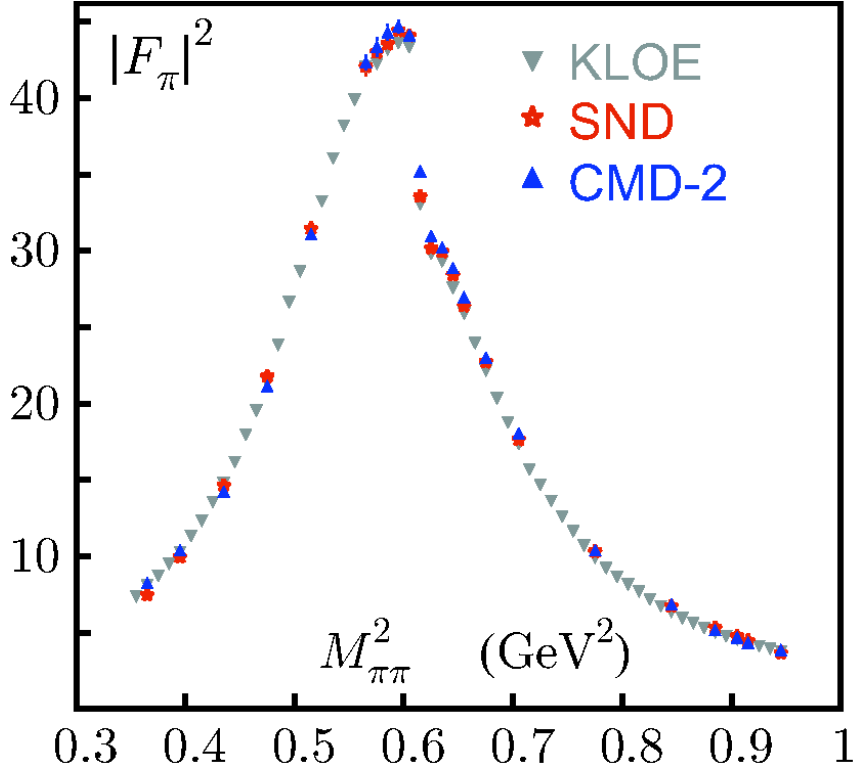


All results are in good agreement. New result has 30% better accuracy

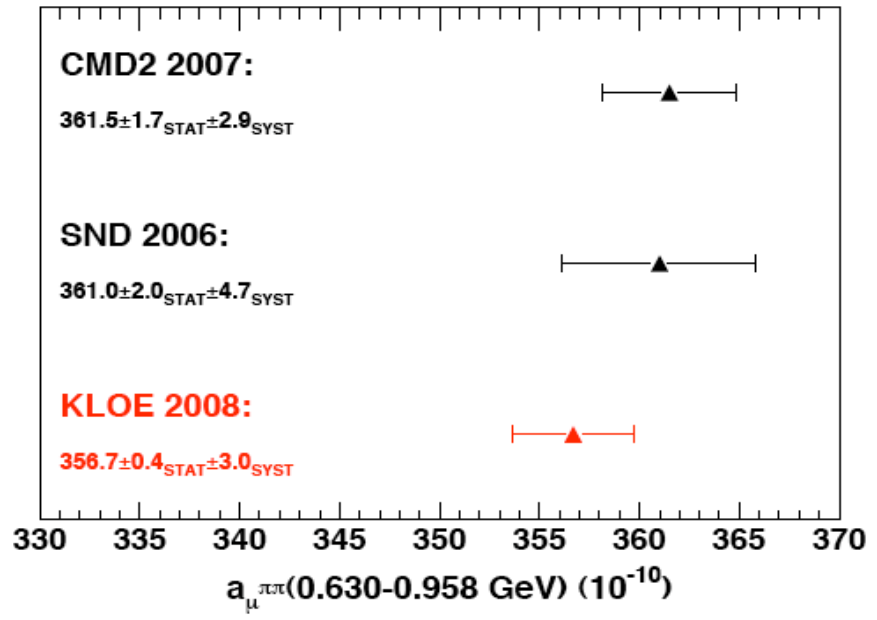
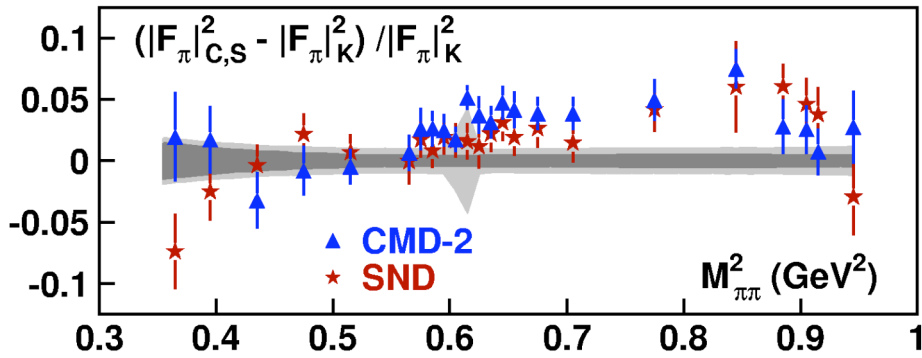
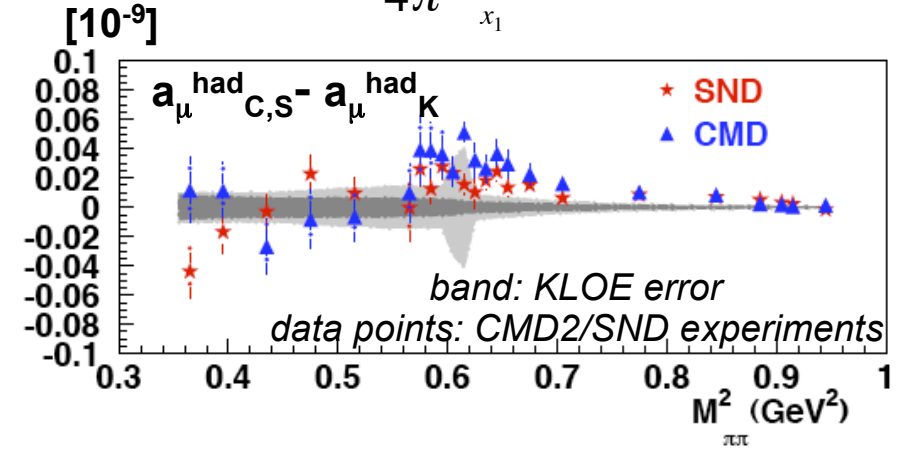
$|F_\pi|^2$ and $a_\mu^{2\pi}$ KLOE vs CMD2/SND



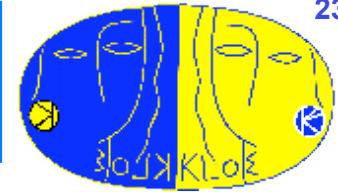
Phys. Lett. B 670 (2009) 285



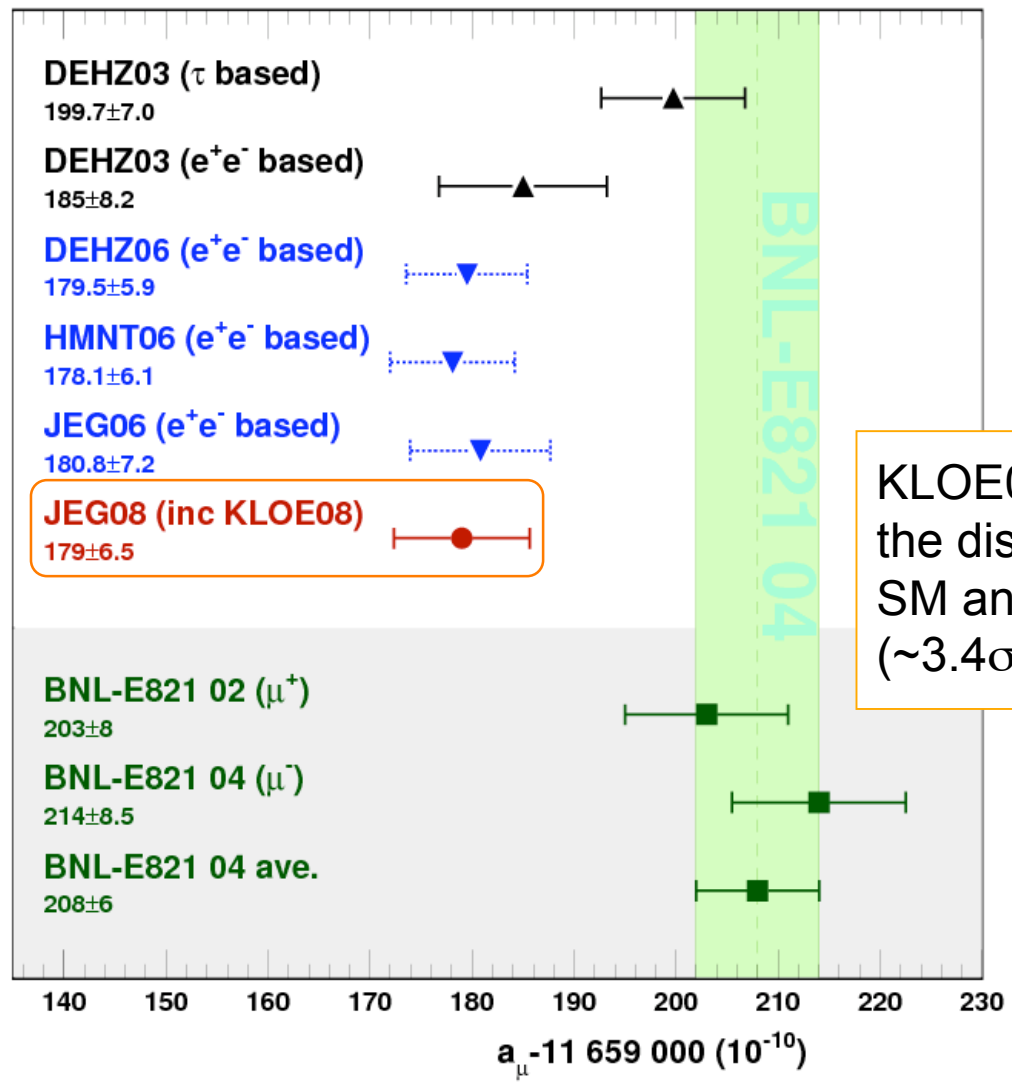
abs. contr. to $a_\mu^{\text{had}} = \frac{1}{4\pi^3} \int_{x_1}^{x_2} \sigma^{\text{had}}(s) K(s) ds$ per bin:



$a_\mu = (g_\mu - 2)/2:$

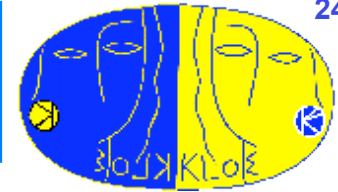


Theoretical predictions compared to the BNL result:

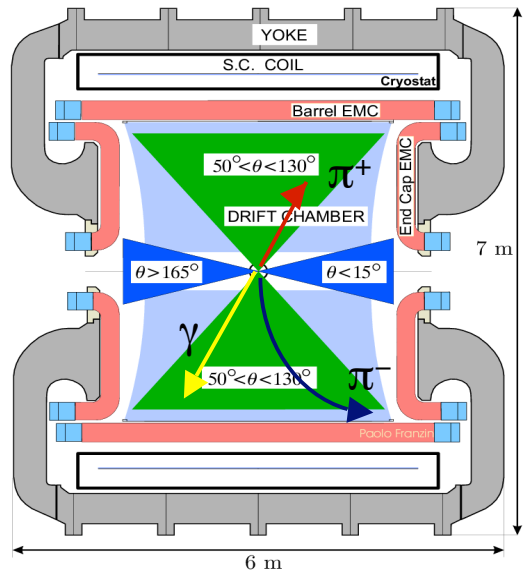


KLOE08 strengthens the discrepancy between SM and BNL experiment (~3.4σ)

New analysis in progress:



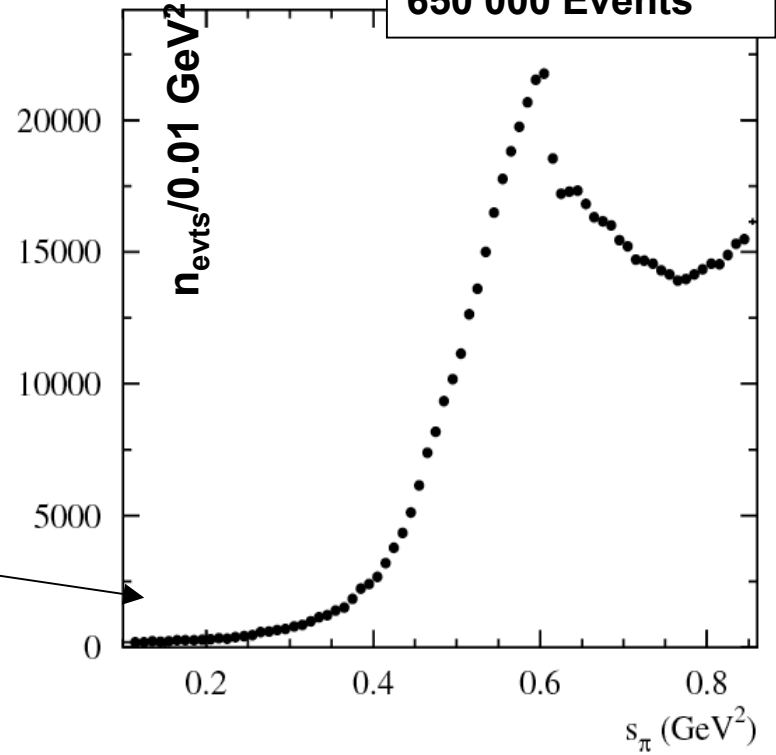
Use data sample taken at $\sqrt{s} \cong 1000$ MeV,
20 MeV below the ϕ -peak



2 pion tracks at large angles
 $50^\circ < \theta_p < 130^\circ$

Photons at large angles
 $50^\circ < \theta_\gamma < 130^\circ$

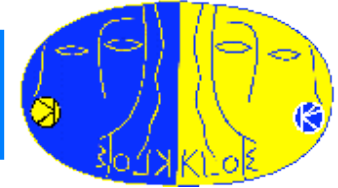
statistics: 233pb⁻¹
650 000 Events



- ✓ independent complementary analysis
- ✓ threshold region $(2m_\pi)^2$ accessible
- ✓ γ_{ISR} photon detected
(4-momentum constraints)
- ✓ lower background from ϕ decays
($\phi \rightarrow f_0 \gamma \rightarrow \pi\pi \gamma$, $\phi \rightarrow \pi^+\pi^-\pi^0$) off-peak

Analysis very close to be finalized

Conclusions



□ KLOE has performed the first precision measurement of 2π cross section using ISR (KLOE05, *Phys. Lett. B* 606, 12 (2005))

□ We have presented a new measurement of $\sigma_{\pi\pi}$ and to $a_{\mu}^{\pi\pi}$ in the range between 0.35 - 0.95 GeV² with 0.9% systematic error (KLOE08, *Phys. Lett. B* 670, 285 (2009))

- KLOE08 result on $a_{\mu}^{\pi\pi}$ agrees with KLOE05 and strengthens
- the discrepancy between SM and BNL experiment ($\sim 3.4\sigma$)
- KLOE08 $a_{\mu}^{\pi\pi}$ agrees with recent results from CMD2 and SND experiments at VEPP-2M in Novosibirsk

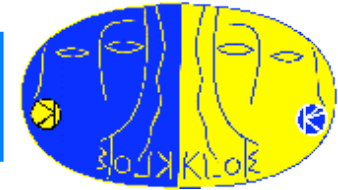
□ Results are also expected in near future from other KLOE analyses:

- F_{π} by photon detected at large angle off-resonance data (1 GeV)
- F_{π} by the π/μ ratio

Stay Tuned!

SPARE SLIDES

Correcting for γ_{FSR} energy:



Go from $M^2_{\pi\pi} \rightarrow S_{\gamma^*}$

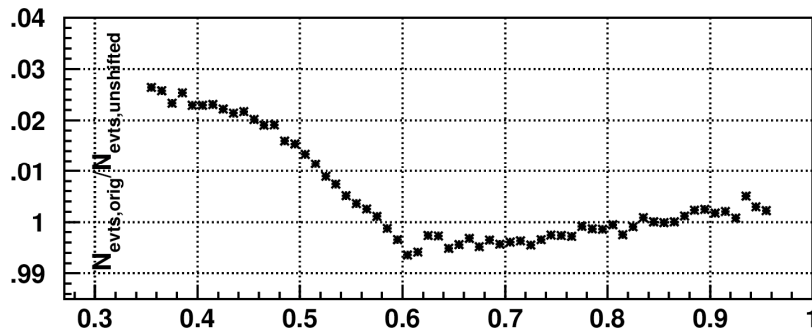
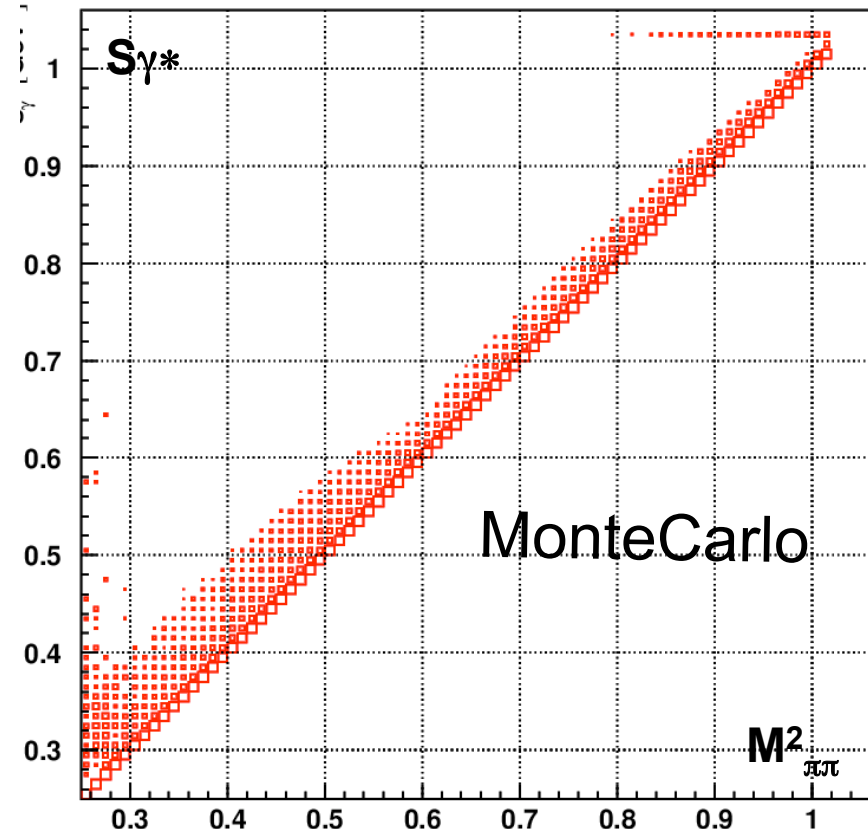
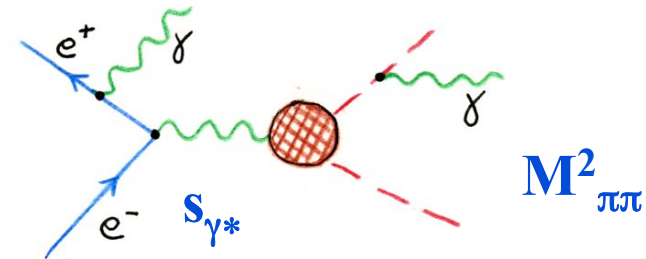
The presence of γ_{FSR} results in a shift of the measured quantity $M^2_{\pi\pi}$ towards lower values:

$$M^2_{\pi\pi} < S_{\gamma^*}$$

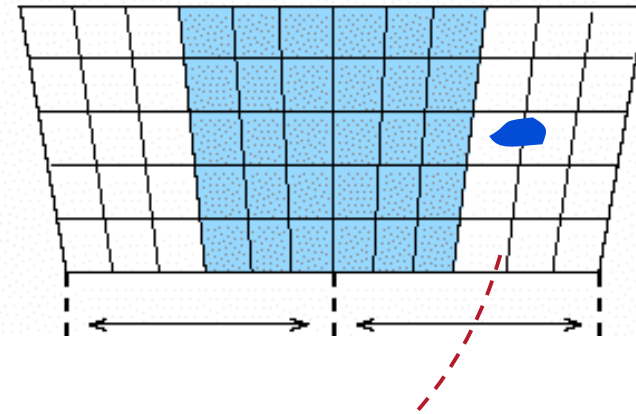
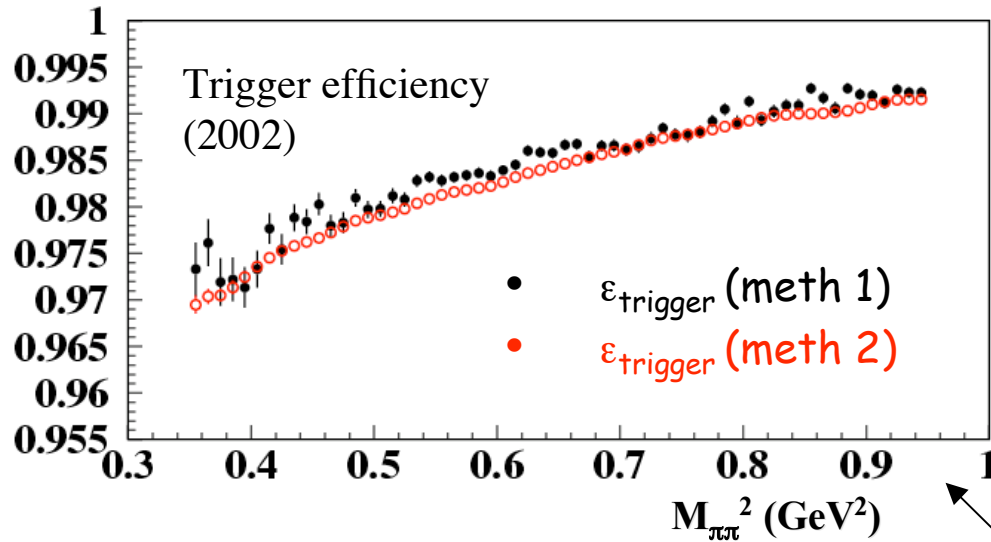
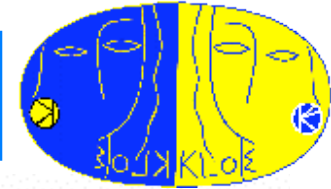
Use special version of PHOKHARA which allows to determine whether photon comes from initial or final state \rightarrow build matrix which relates $M^2_{\pi\pi}$ to $M^2_{\gamma^*}$.

ISR only: $S_{\gamma^*} = M^2_{\pi\pi}$

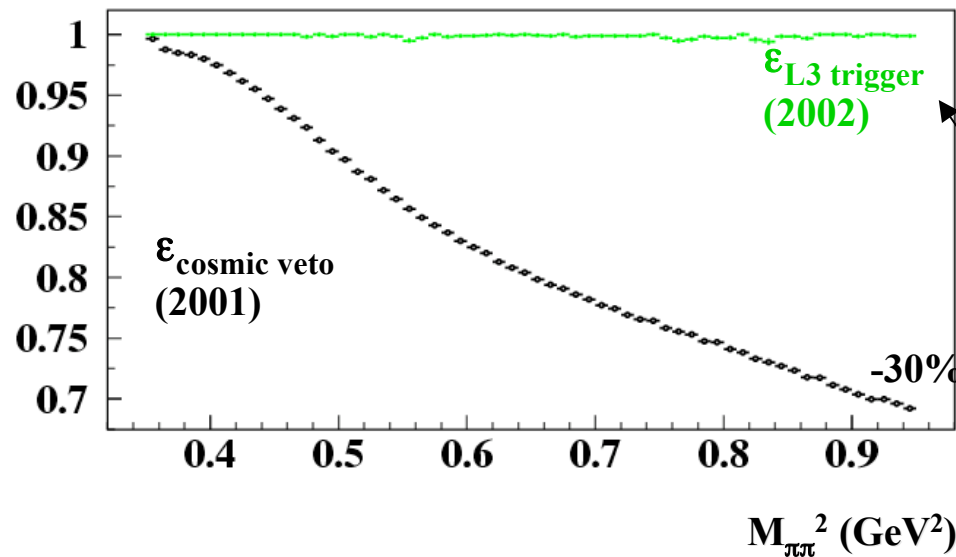
FSR photon present: $S_{\gamma^*} = M^2_{\pi\pi\gamma(\text{FSR})}$



Trigger



- The event is **triggered** by the (pion) tracks only which deposit $E > 50 \text{ MeV}$ in 2 sectors of the calorimeter



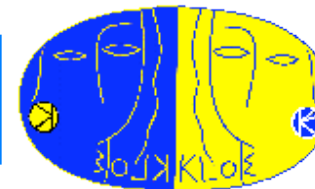
~99.9%

~70.%

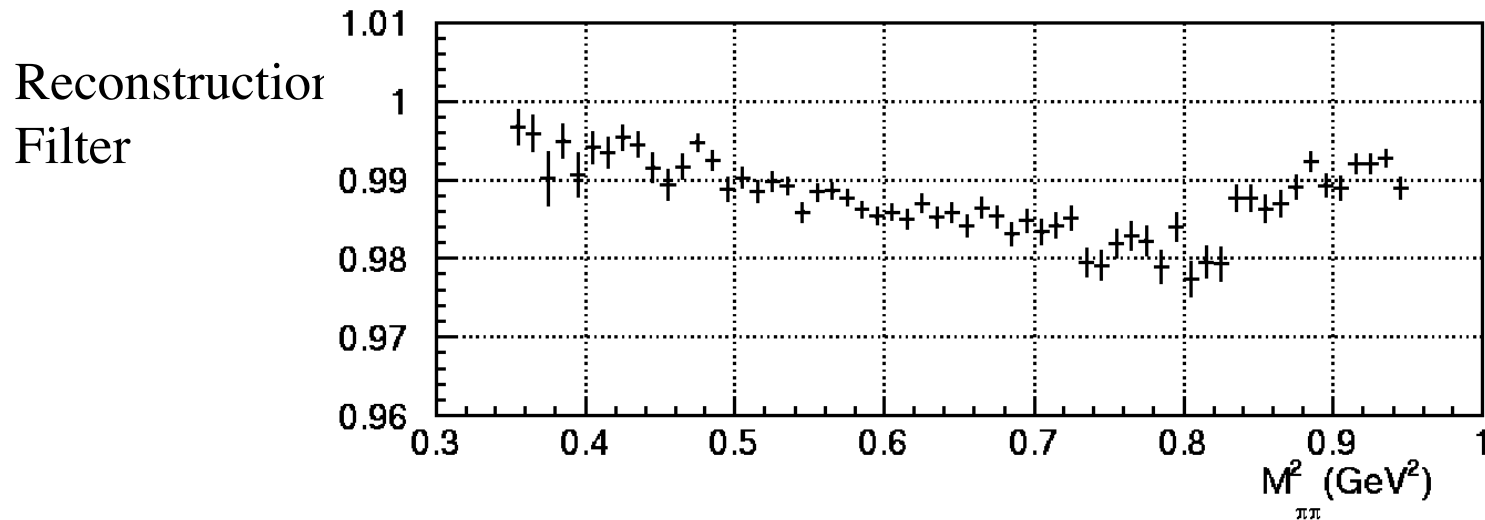
- trigger efficiency evaluated on data by 2 independent methods.
- Error is the fractional difference of the 2 methods: 0.1%

- The main source (**hardware veto of cosmic rays**) of inefficiency in the published result **has been replaced by an online filter (L3)**

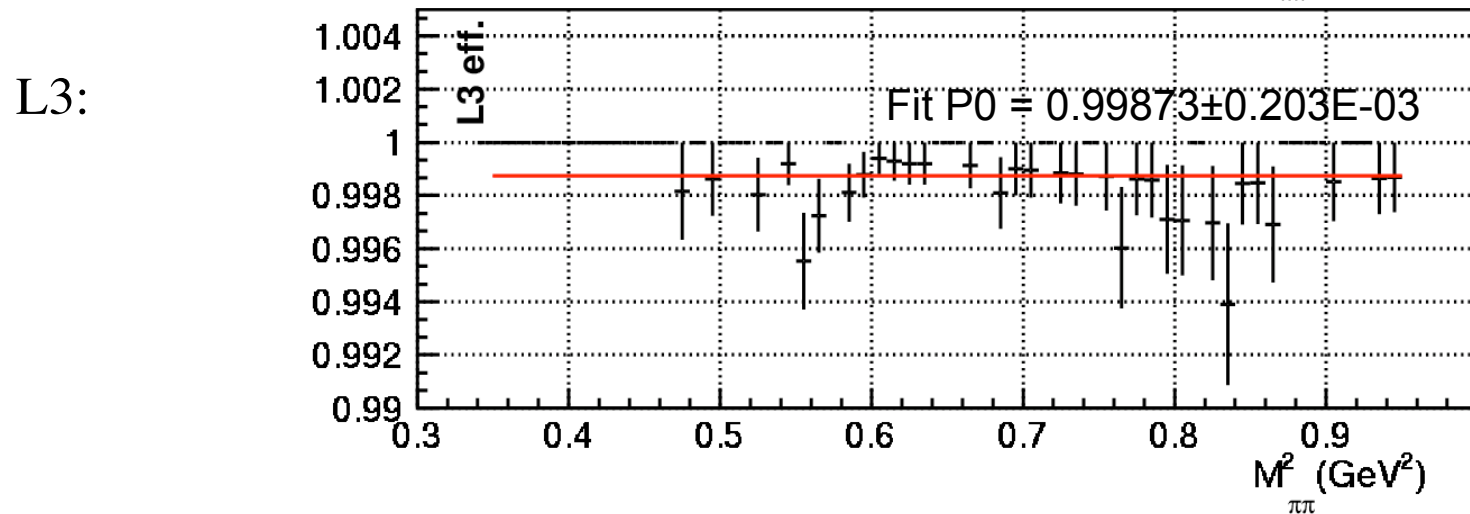
Reconstruction and L3 filters:



Both efficiencies estimated via downscaled control samples:

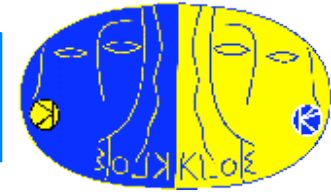


$\sim 2\%$

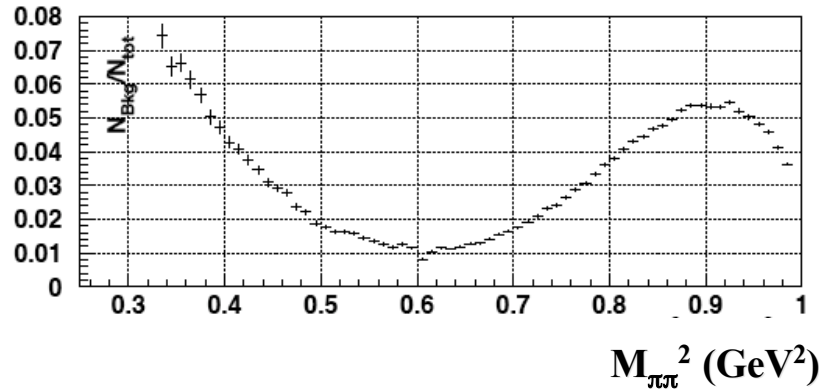


0.1% taken as uncertainty on the spectrum due to L3 trigger.

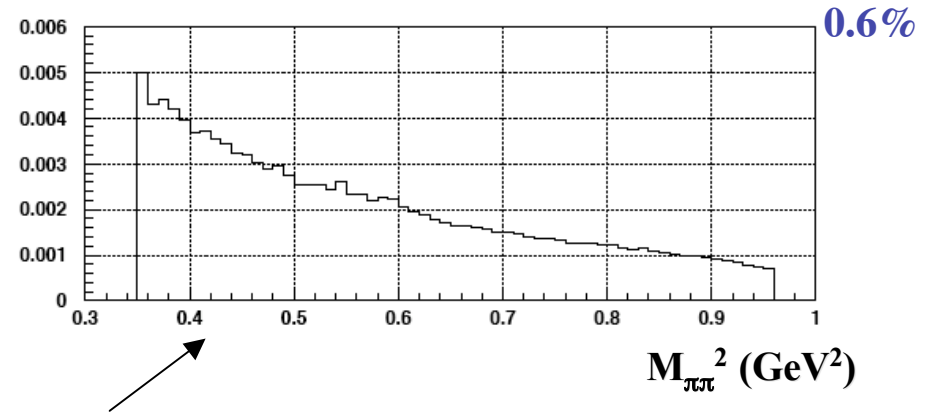
Background: total contribution and error



Tot bckg ($\mu\mu\gamma$, $\pi\pi\pi$ and $ee\gamma$) contribution **Error on bckg subtraction (in %)**



8%



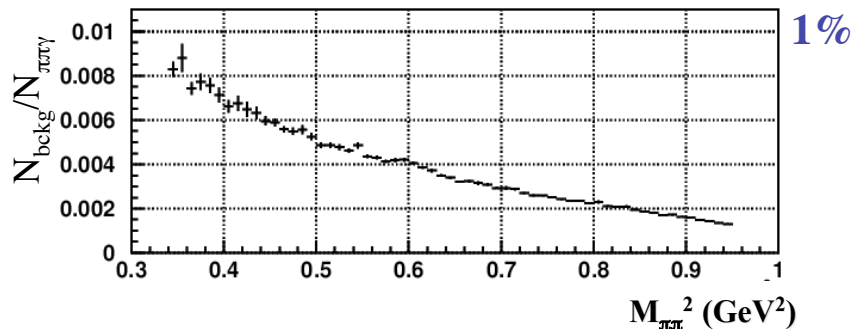
0.6%

Additional bckg channels:

- $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$ (Ekhara) $\sim 0.8\%$ at low $M_{\pi\pi}^2$
- $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ (Nextcalibur) negligible
- $\phi \rightarrow f_0\gamma \rightarrow \pi\pi\gamma$ (Phokhara, Fasterd) negligible
- $\phi \rightarrow \pi\rho \rightarrow \pi\pi\gamma$ (Phokhara, Fasterd) negligible
- $e^+e^- \rightarrow \omega\gamma_{ISR} \rightarrow \pi\pi\pi\gamma$ (Phokhara) negligible

Contribution to Bckg error :

- Uncertainty on $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$ contribution
- Error from normalization parameters obtained from the fit



1%

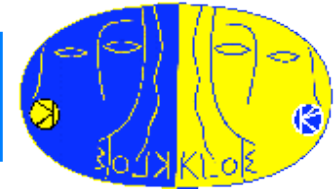
“Phokhara”: see talk of A. Grzielinska

“Ekhara”: C.zyz *et al*

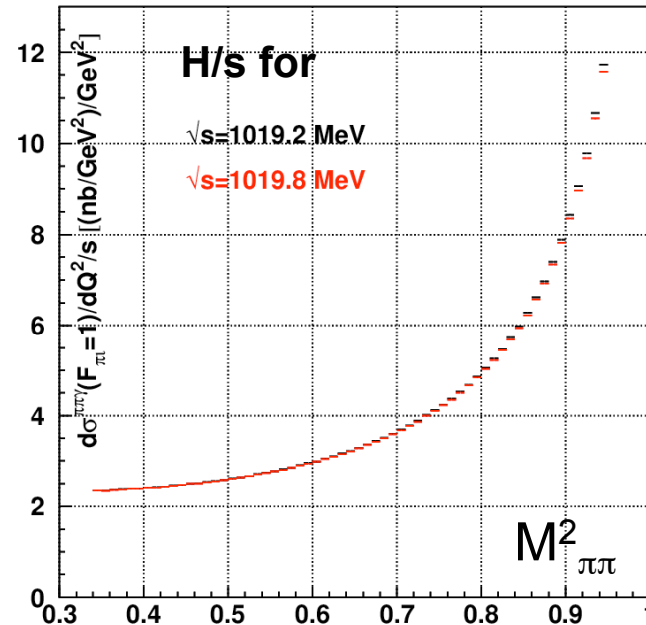
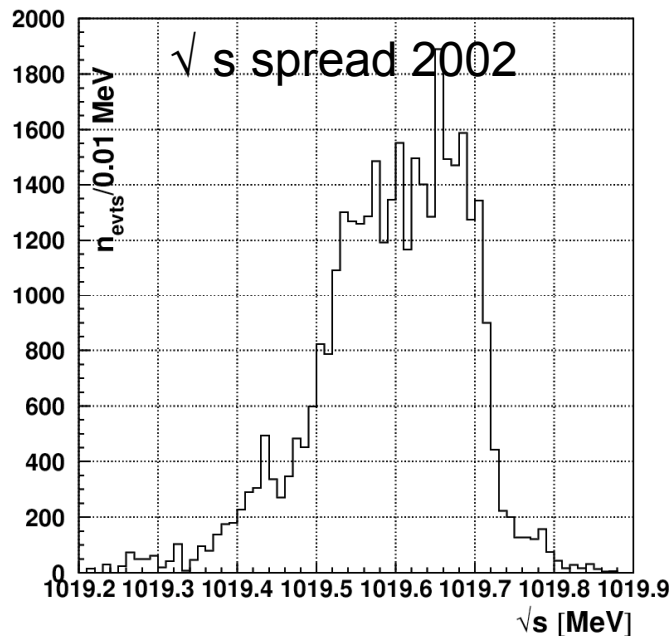
“Nextcalibur” : F.A. Berends *et al*

“Fasterd”: O. Shekhotvsova *et al*

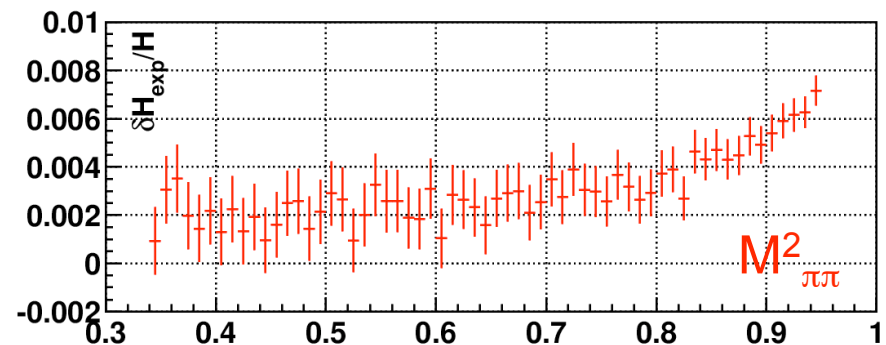
Radiator function (H)



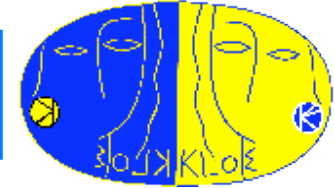
In addition to the 0.5% theoretical error we evaluate the experimental uncertainty due to the spread in \sqrt{s} during the data taking in 2002 (since we evaluated H at the fixed energy $\sqrt{s} = 1.019456$ GeV)



We take half the rel. difference between the radiator functions obtained at $\sqrt{s} = 1.0192$ GeV and $\sqrt{s} = 1.0198$ GeV as the experimental syst. uncertainty on the radiator function.

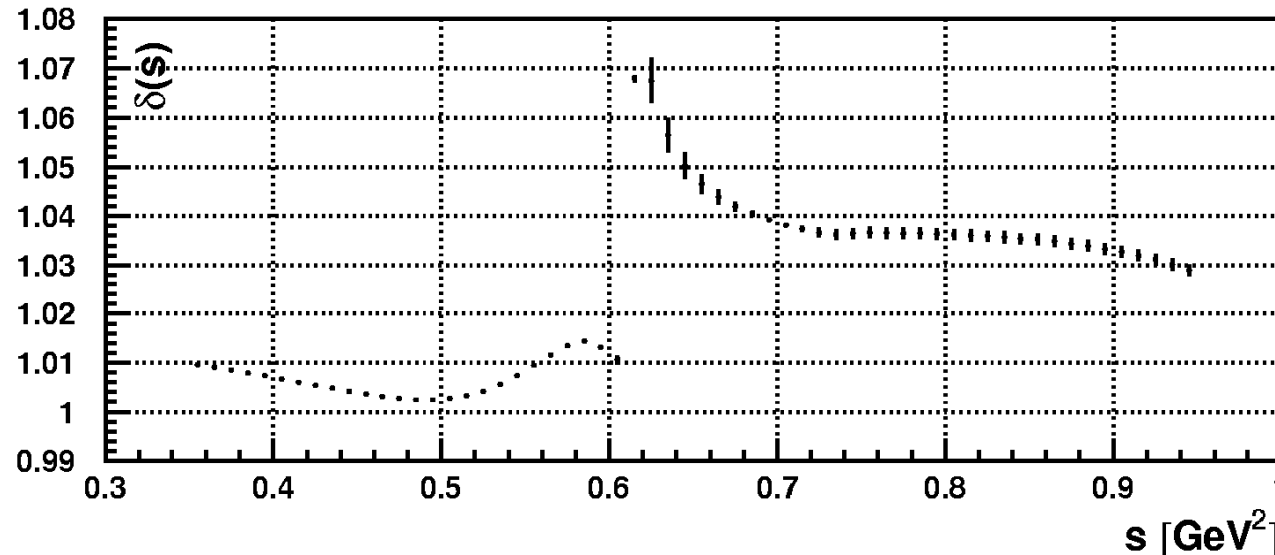


Vacuum Polarisation



For use in the dispersive integral for $\Delta^{\pi\pi}a_\mu$, one needs to subtract effects from vacuum polarization (VP) to obtain a *bare* cross section $\sigma_{\pi\pi}^0$:

$$\sigma_{\pi\pi}^0(s) = \sigma^{\text{dressed}}_{\pi\pi}(s) \left(\frac{\alpha(0)}{\alpha(s)} \right)^2 = \sigma_{\pi\pi}(s) / \delta(s)$$



Points obtained from F. Jegerlehner's webpage
(the only points which are publicly available!)

Correction is applied only to the cross section $\sigma_{\pi\pi}^0$ (not on $\sigma_{\pi\pi\gamma}$ and $|F_\pi|^2$).

Error on VP points introduces an relative error on the value of $\Delta^{\pi\pi}a_\mu$ of 0.1%.