Planck's constant

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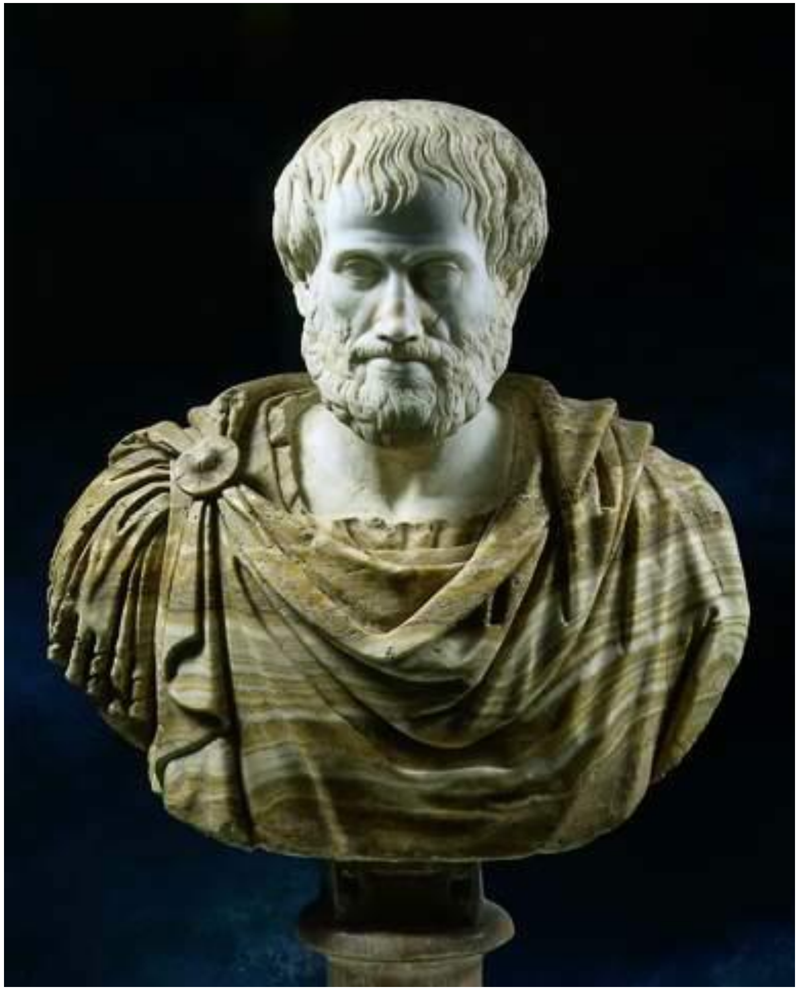
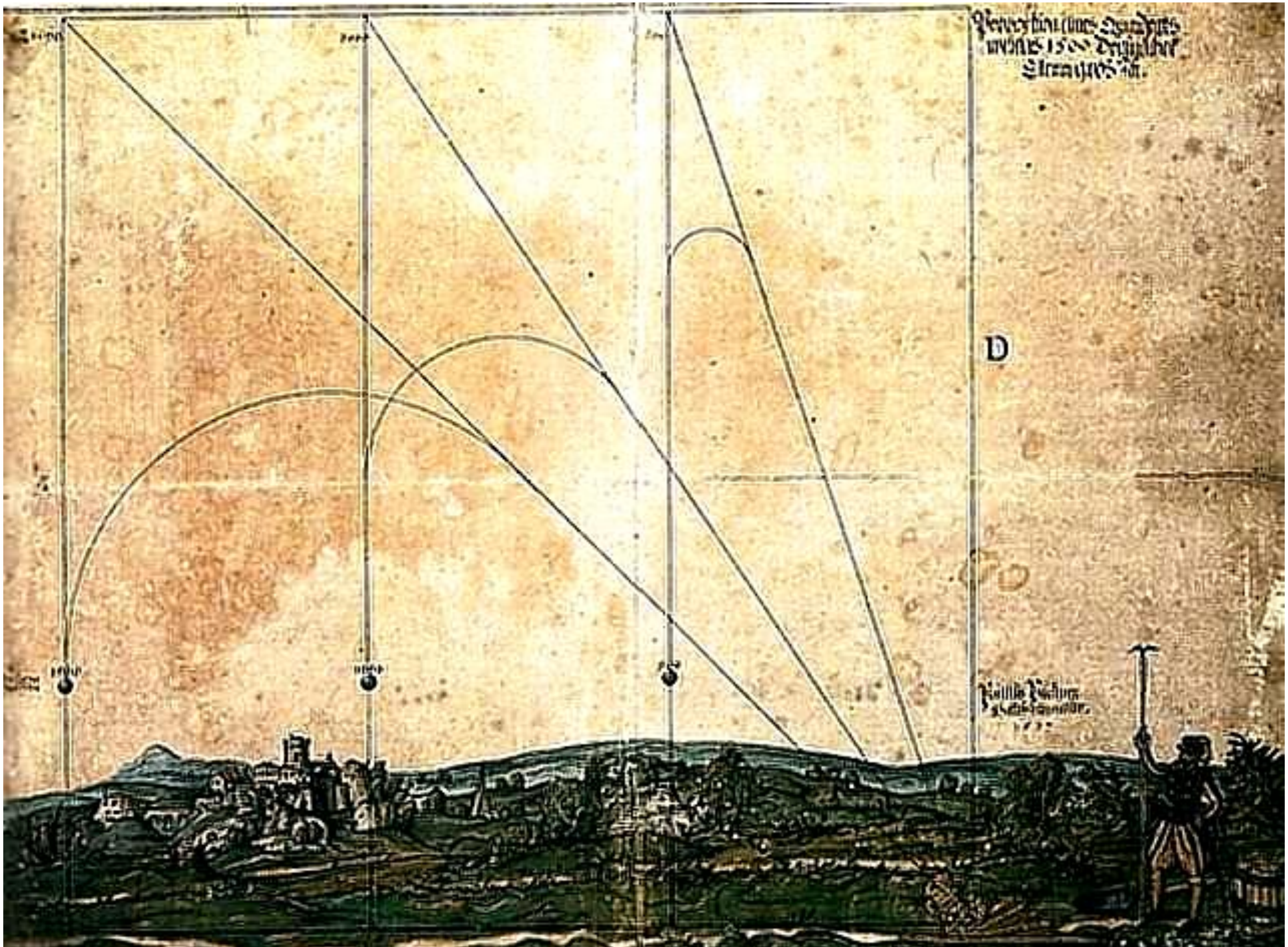
Ancient mechanics



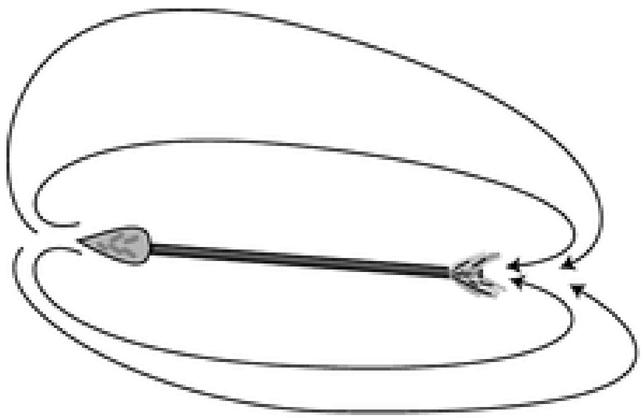
The speed of an object's motion is proportional to the force being applied.

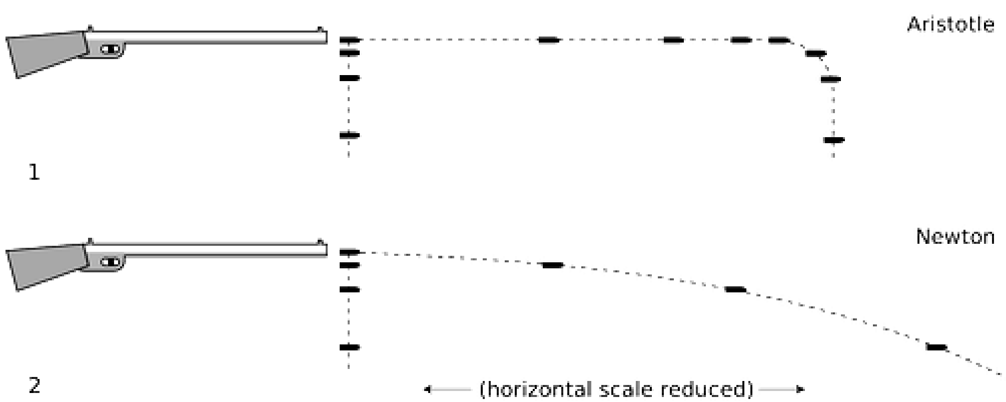
$$\boldsymbol{F} = m\boldsymbol{v}$$

Aristotle’s trajectories



Issues of Aristotelian mechanics



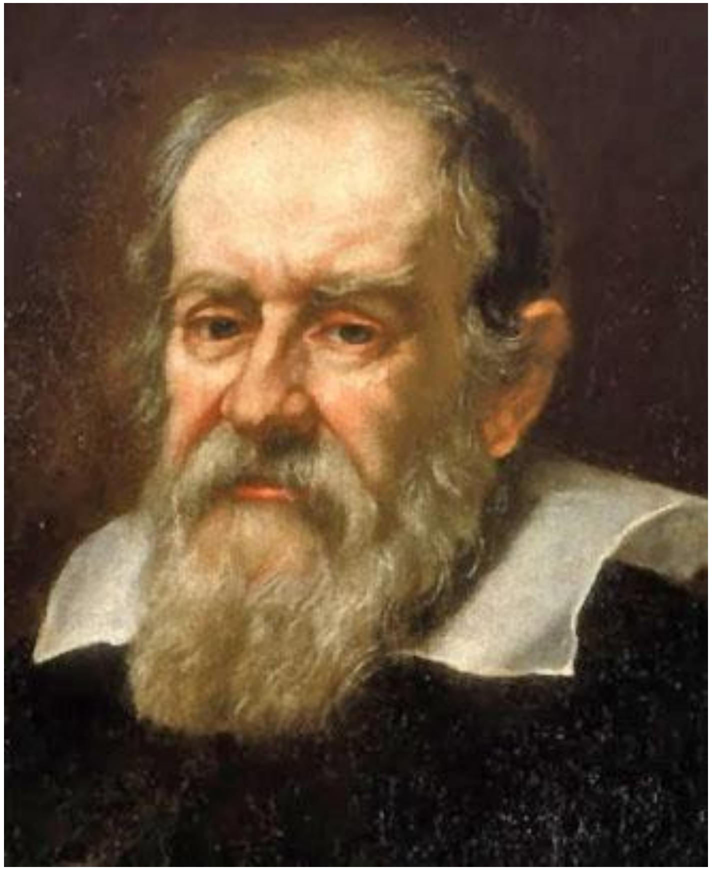


- It hardly explains how an arrow keeps on flying after being shot;
- Can be easily disproved through a simple experiment.

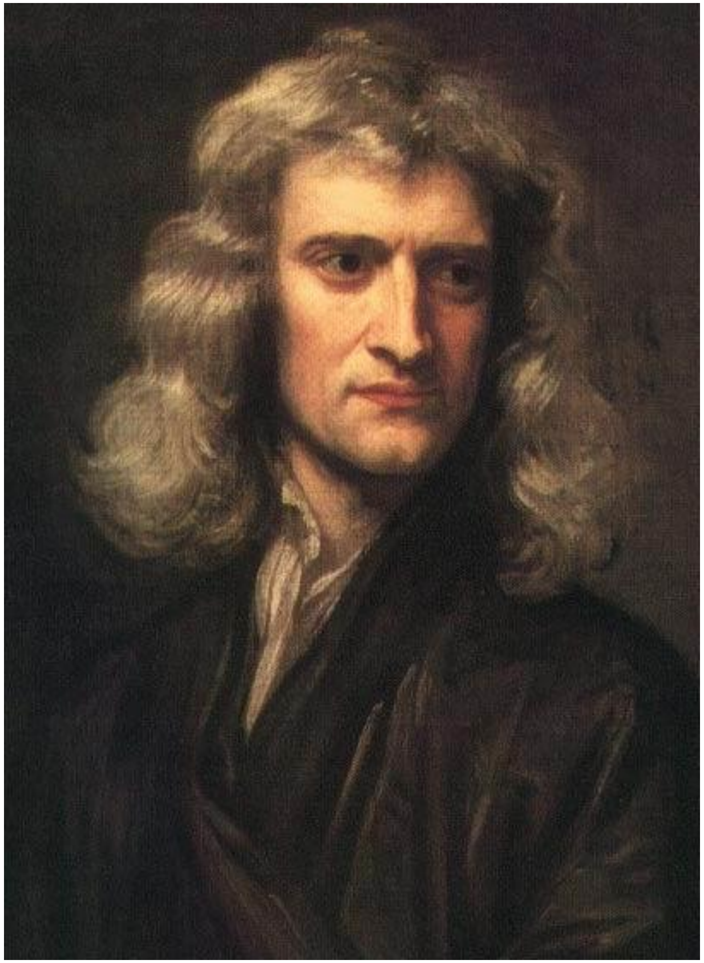
The scientific method

- Observation of phenomenon;
- Hypothesis and prediction;
- Experiment;
  - Reproducible;
- Write a law using the "language of mathematics".

Falsifiability of the law



1874: The task of physics is "nearly completed"



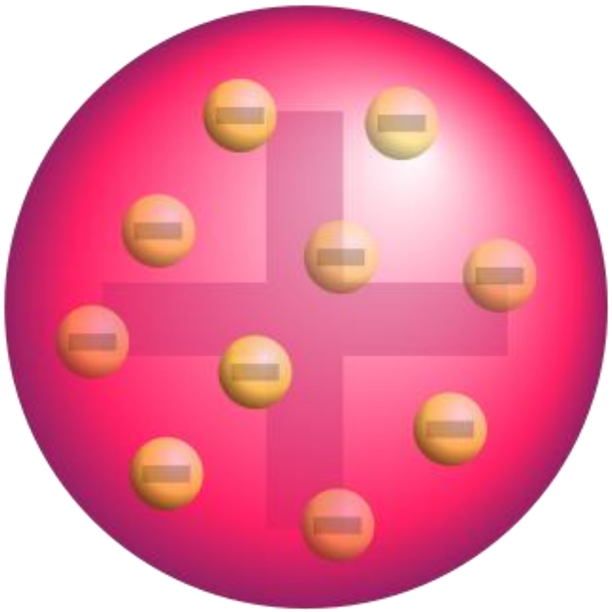
Quantum mechanics and relativity

- Is classical mechanics as wrong as Aristotelian mechanics?
- How can classical mechanics, relativity, and quantum mechanics to reconcile?

Every theory has its own scope of validity, one has to know its limits!

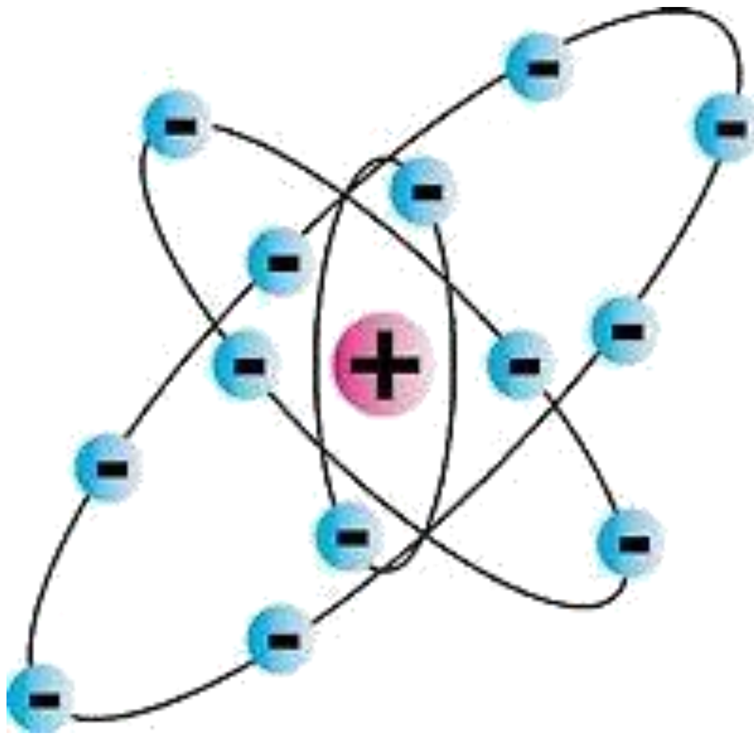
For example, how is the atom made?

The **plum pudding** model of the atom by J.J. Thomson



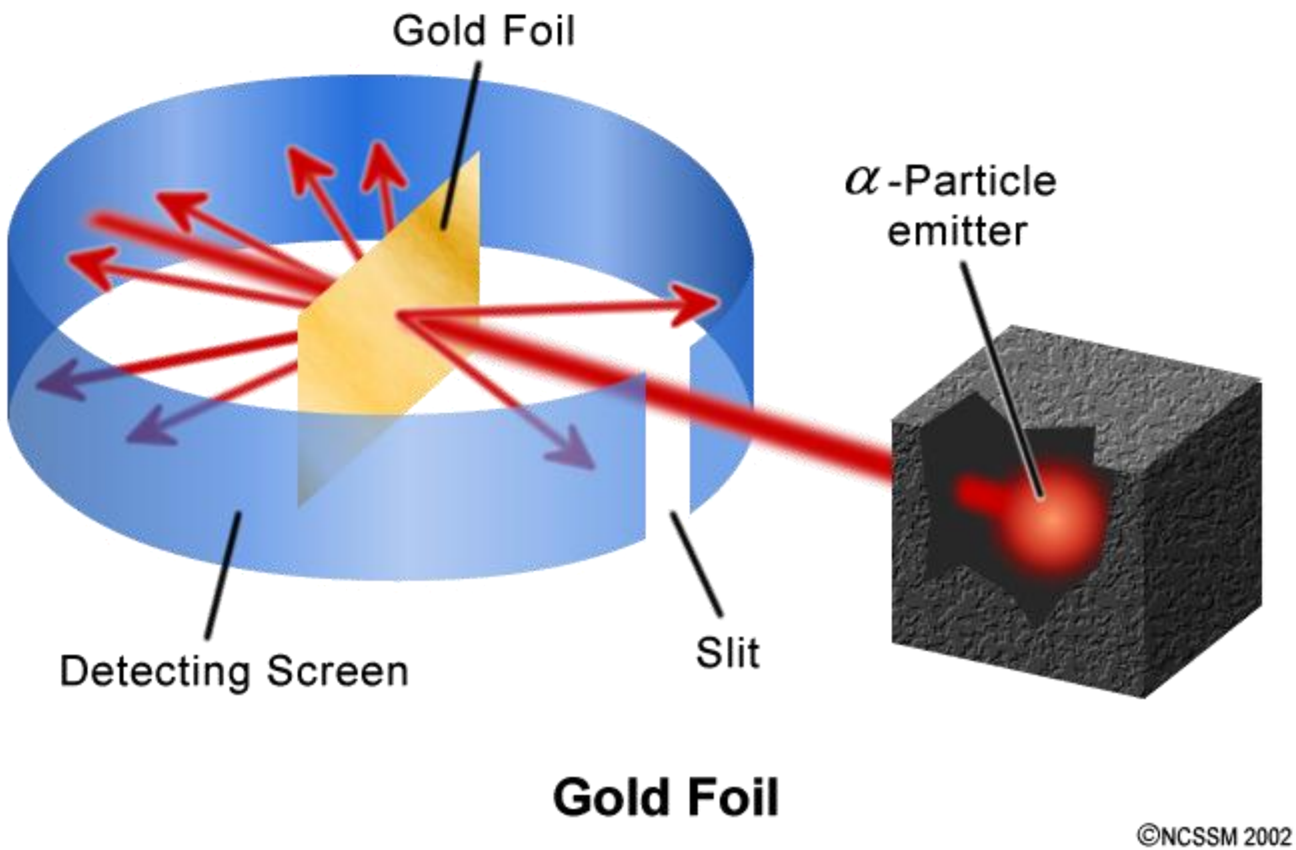
The atoms […] consist of a number of negatively electrified corpuscles enclosed in a sphere of uniform positive electrification.

Rutherford's model of the atom



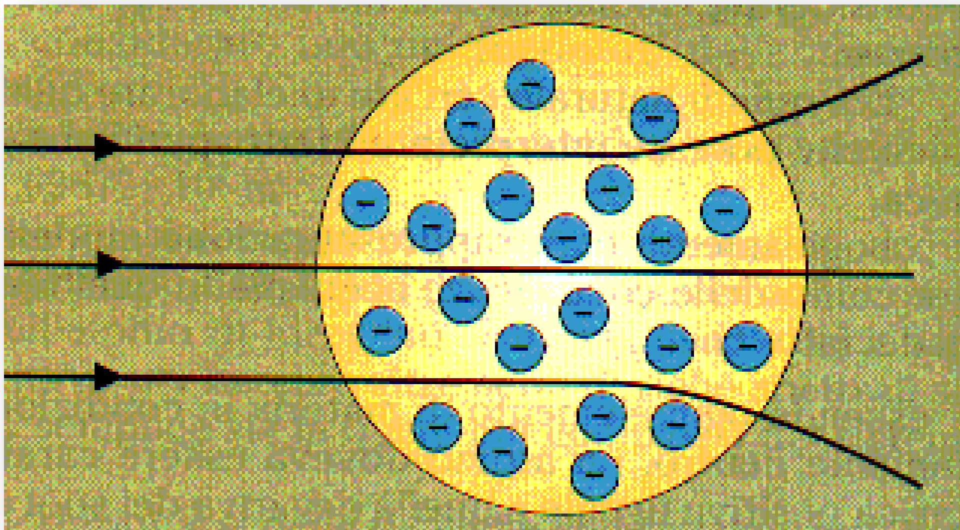
Negatively-charged electrons orbit around a positively charged nucleus.

Geiger–Marsden experiment 1911
(aka Rutherford's experiment)

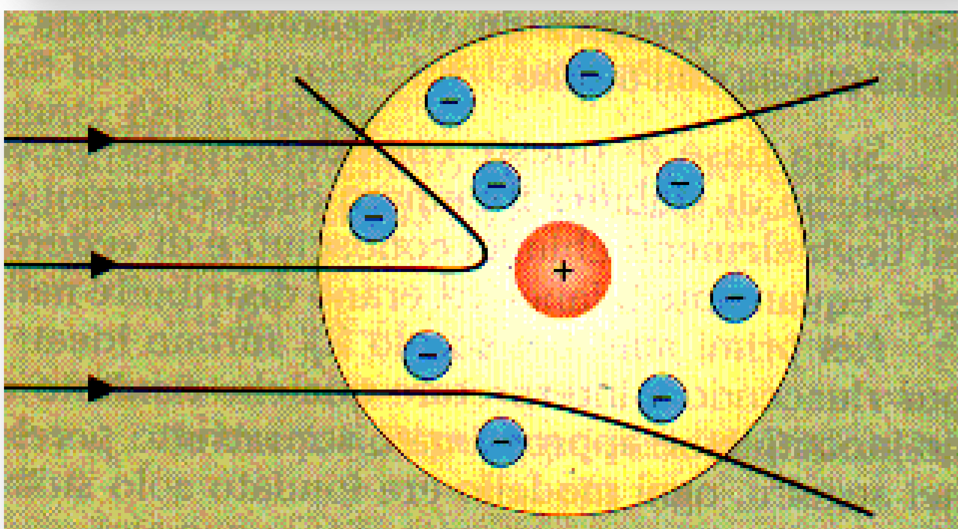


Scattering Helium nuclei (aka α-particles) off thin gold foil.

Thomson vs Rutherford



The more the particle travels close to the centre, the smaller is the deflection angle



The more the particle travels close to the centre, the larger is the deflection angle. Even backscattering occurs!

Some calculations

$$E = T + V = \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

The total energy is the kinetic energy of the a-particle ***T*** plus the Coulomb-energy ***V*** due to the repulsion between charges of the same sign. When the particle turns back ***T=V***.

Putting T=V we get: $T = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

If we solve for *r* we get: $r = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{T}$

Some more calculations

The elementary charge **e**, the dielectric constant of vacuum $\boldsymbol{\varepsilon_0}$ and the kinetic energy ***T*** appear in the previous formula:

$$e = 1.6 \cdot 10^{-19} C$$

$$\varepsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{m}^{-3}\mathrm{kg}^{-1}\mathrm{s}^{4}\mathrm{A}^{2}$$

$$T \approx 5MeV = 5 \cdot 10^{6} \cdot 1.6 \cdot 10^{-19} J$$

It is useful to know that $1/(4\pi\varepsilon_0)$ is about $9*10^9$ in the International System of Units.

Last calculations

Protons in a-particle

Protons in gold

Elementary charge

$$r = \frac{1}{4\pi\varepsilon_0}\frac{z_\alpha \cdot z_{Au} \cdot e^2}{T} = 9\cdot 10^9 \, \frac{2\cdot 79\cdot 1.6\cdot 10^{-19}\cdot 1.6\cdot 10^{-19}}{5\cdot 10^6\cdot 1.6\cdot 10^{-19}} =$$

Dielectric constant of vacuum

Kinetic energy of a-particle

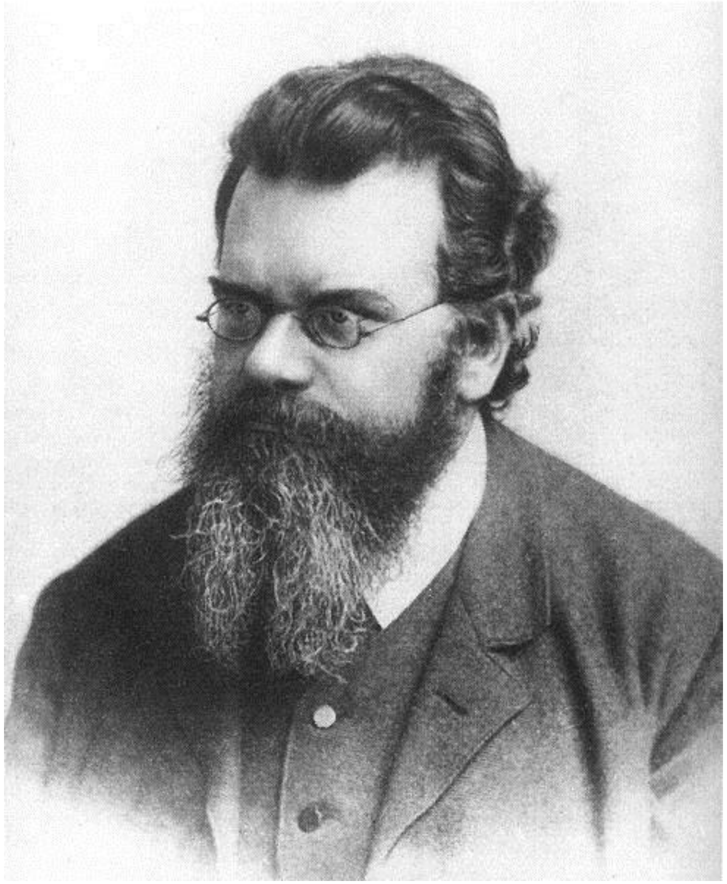
$$r = 4.55\cdot 10^{-14}\,\mathrm{m} \approx 45\,\mathrm{fm}$$

At that time, the radius of the atom of gold was well known: 150’000 fm.
It is 3000 times larger!

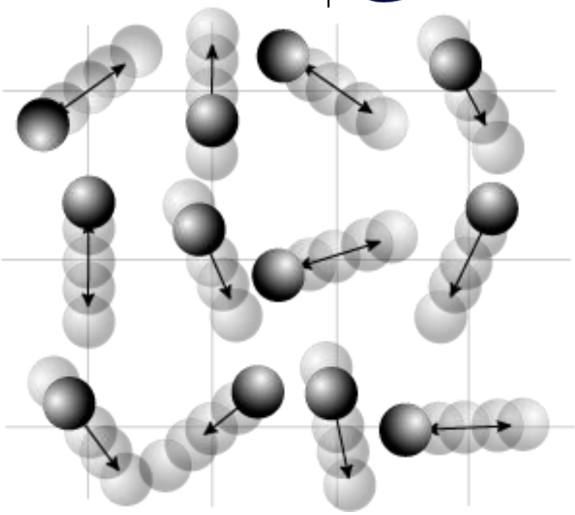
“It was quite the most incredible event that has ever happened to me in my life. It was almost as if you fired **a 15-inch shell into a piece of tissue paper and it came back and hit you**.”

On consideration, I realised that this scattering backward must be the result of a single collision, and when I made calculations I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greater part of the mass of the atom was concentrated in a minute nucleus. It was then that I had the idea of an **atom with a minute massive center, carrying a charge.**

Boltzmann proposes a statistical interpretation of thermodynamics



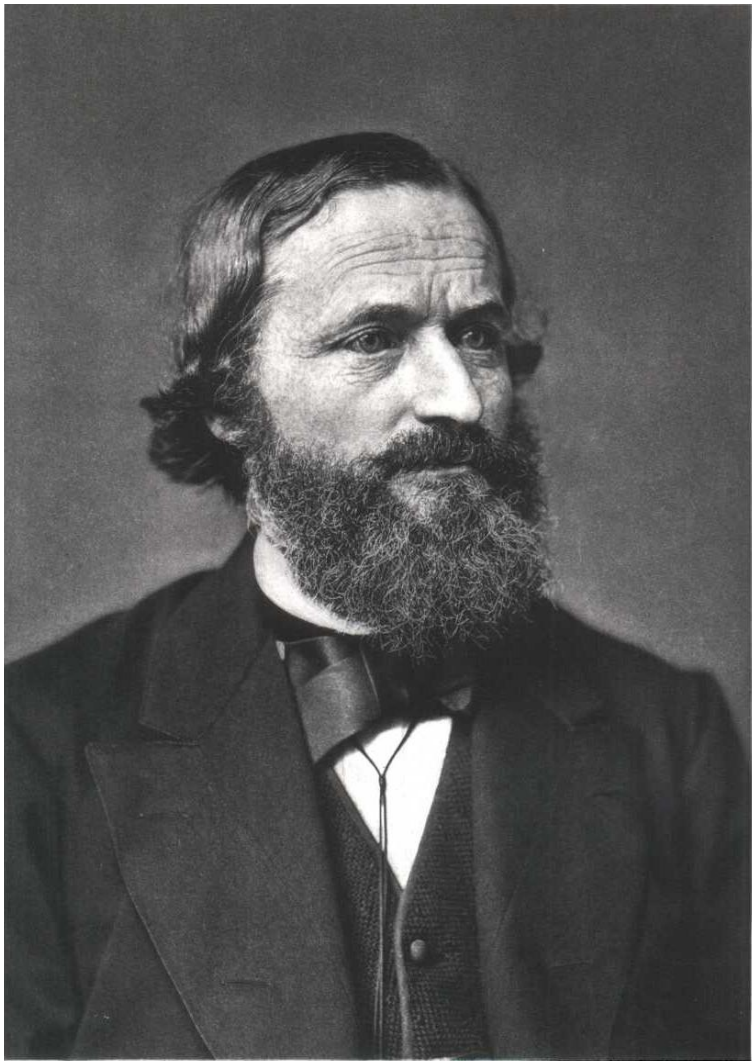
At the beginning of 1800, Boltzmann proposed a weird theory to explain laws of thermodynamics.



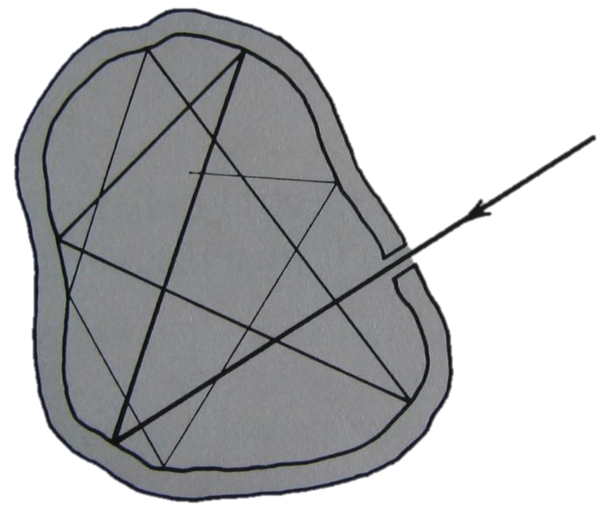
He said that gases were made of many tiny particles moving randomly and hitting one another. The well-known laws of thermodynamics could be interpreted as statistical laws.

People did not like this theory…

Kirchhoff: the black body radiation



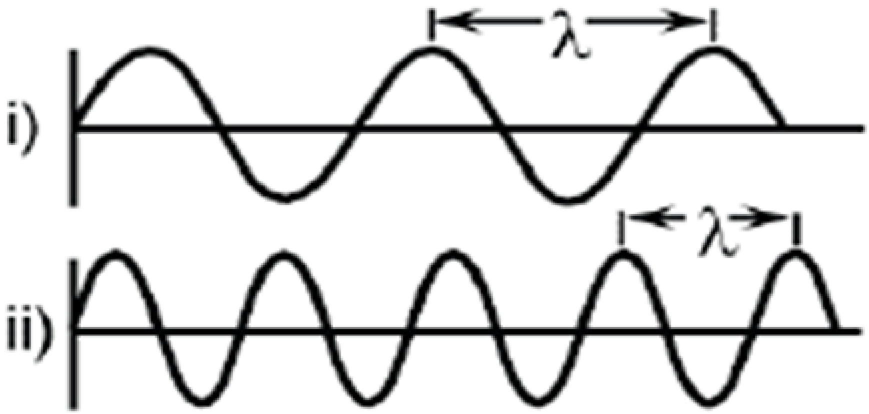
A **black body** is a cavity (e.g. an oven) with a tiny hole, kept at a constant temperature.



The radiation entering the black body is reflected by the inner walls a huge number of times before getting out.

What is an electromagnetic radiation?

Maxwell undestood that an electromagntic radiation isn't but a wave: a vibration of the electric and magnetic field travelling thorugh space at the speed of light.

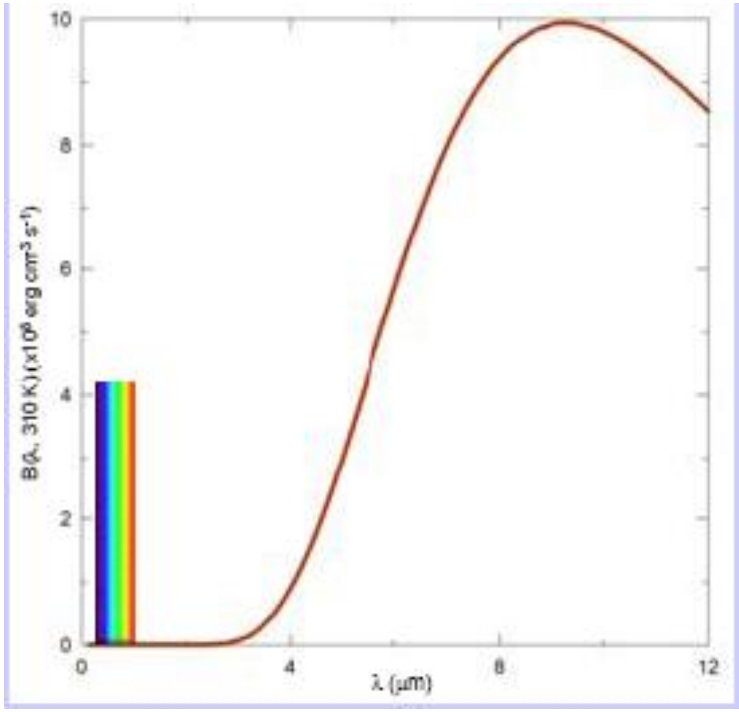


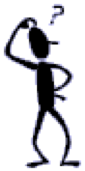
velocity $v = \frac{\lambda}{T} = c \simeq 3 \cdot 10^8 \, \frac{\mathrm{m}}{\mathrm{s}}$

frequency $\nu = f = \frac{1}{T} = \frac{\lambda}{c}$

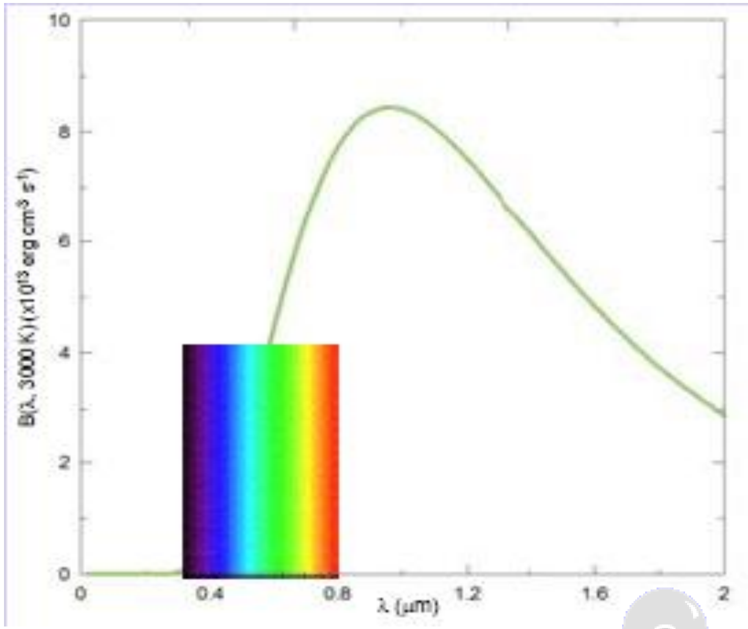
- The **wavelength** λ is the **space** between two peaks of the wave.
- The **period** T is the **time** between two peaks.
- The frequency ν or f is the amount of peaks per second.

Emission spectra of common objects





Human body

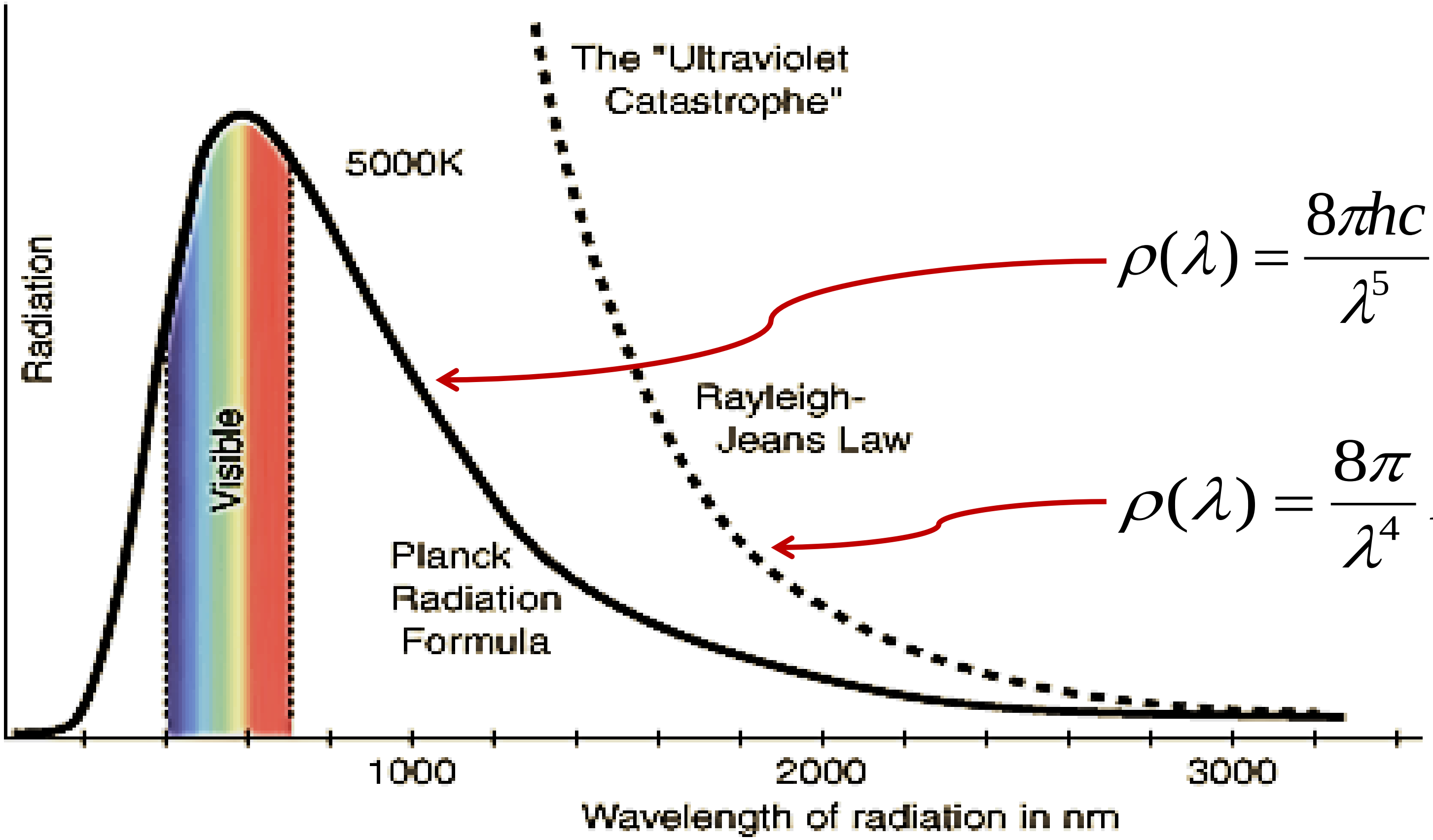


Light bulb

Sunday, October 7, 1900 teatime.

- Max Planck solves the puzzle of the spectrum of black body radiation.
- In its calculations, Planck simply replaced an **integral** with a **sum**, in order to get things right.
- The implications of this "mathematical trick" revealed a bizarre behaviour of nature.

Sum vs. integral:



$$\rho(\lambda)=\frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda KT}}-1}$$

$$\rho(\lambda)=\frac{8\pi}{\lambda^4}KT$$

Planck's law explains the black-body spectrum even at short wavelengths (small λ). The Rayleigh-Jeans law goes to infinity when λ goes to zero.

What was Planck's idea?

Planck assumed that the inner walls of the black body behaved like many tiny oscillators.
They absorbed and emitted electromagnetic waves at **all wavelengths and all energies**.

From this hypothesis, he calculated the **mean energy** <*E*> emitted by each oscillator, "multiplied by" (actually integrated over) the number of oscillators.

Max Planck's calculation (1/2)

Planck disagreed with atomic theories, and did not like statistical thermodynamics…

Nevertheless, Planck took his start from Boltzmann's statistical approach:

Distribution

Probability of finding a particle with energy between *E* and *E*+d*E*

$$P(E)dE = \frac{e^{-\frac{E}{kT}}}{\int_0^\infty e^{-\frac{E}{kT}}\,dE}\,dE$$

Normalization

In the same fashion as in thermodynamics, this equation provides the probability to find a particle of energy *E* within a gas at temperature *T*.

How much do you understand this formula?

Max Planck's calculation (2/2)

The mean value of a distribution is given by its **integral** "weighted" with the value of the energy. This mean value is $\langle E \rangle = kT$.

Using this results, one obtains the Rayleigh-Jeans' black-body spectrum.

$$\langle E \rangle = \frac{\int_0^\infty E e^{-\frac{E}{kT}} dE}{\int_0^\infty e^{-\frac{E}{kT}} dE} dE = \cdots = kT \quad \Rightarrow \quad \rho(\lambda) = \frac{8\pi}{\lambda^4} KT$$

Planck used a sum instead. To do that, he had to replace $\int_0^\infty E e^{-\frac{E}{kT}} dE$ with $\sum_0^\infty nh\nu\, e^{-\frac{nh\nu}{kT}}$. So the energy is no longer a continuous variable E, but rather n times the value $h\nu$.

$$\langle E \rangle_{Planck} = \frac{\sum_0^\infty nh\nu\, e^{-\frac{nh\nu}{kT}}}{\sum_0^\infty e^{-\frac{nh\nu}{kT}}} = \cdots = \frac{hc}{\lambda} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \quad \Rightarrow \quad \rho(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda KT}} - 1}$$

“Quantum” of energy

It would seem that not any amount of energy can be emitted and absorbed. Only certain discrete values are permitted by nature: the multiples of the so-called **“Quanta”** (one quantum, two quanta)

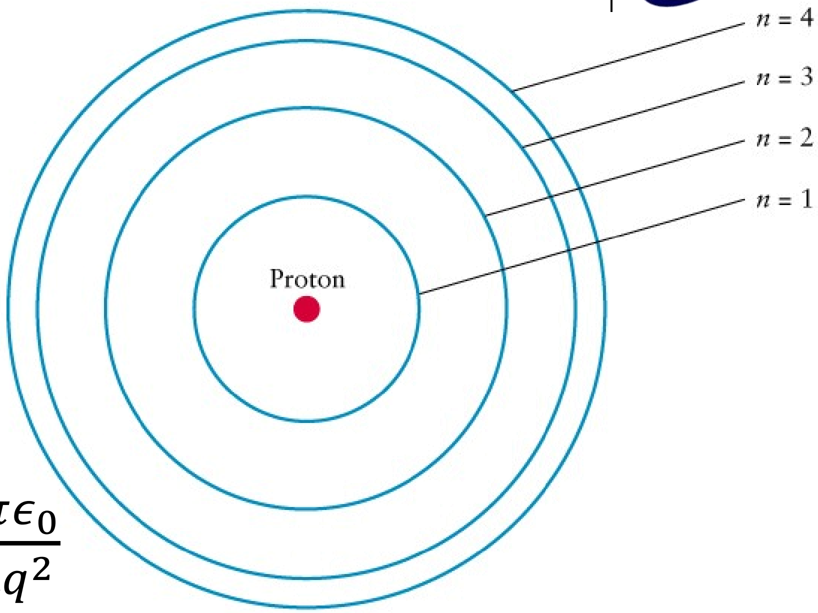
$$E = h\nu$$

$$h = 6.62618\ 10^{-34}\ \mathrm{Js}$$

Nature behaves in a bizarre way, is there anything else that can be explained with this *quantum mechanics*?

Bohr's atom (1913)

A charge acceleratin within an electric field emits radiation and loses some enrgy.
So why electrons do not fall over the nucleus?



$$F_c = F_E \rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{r^2}$$

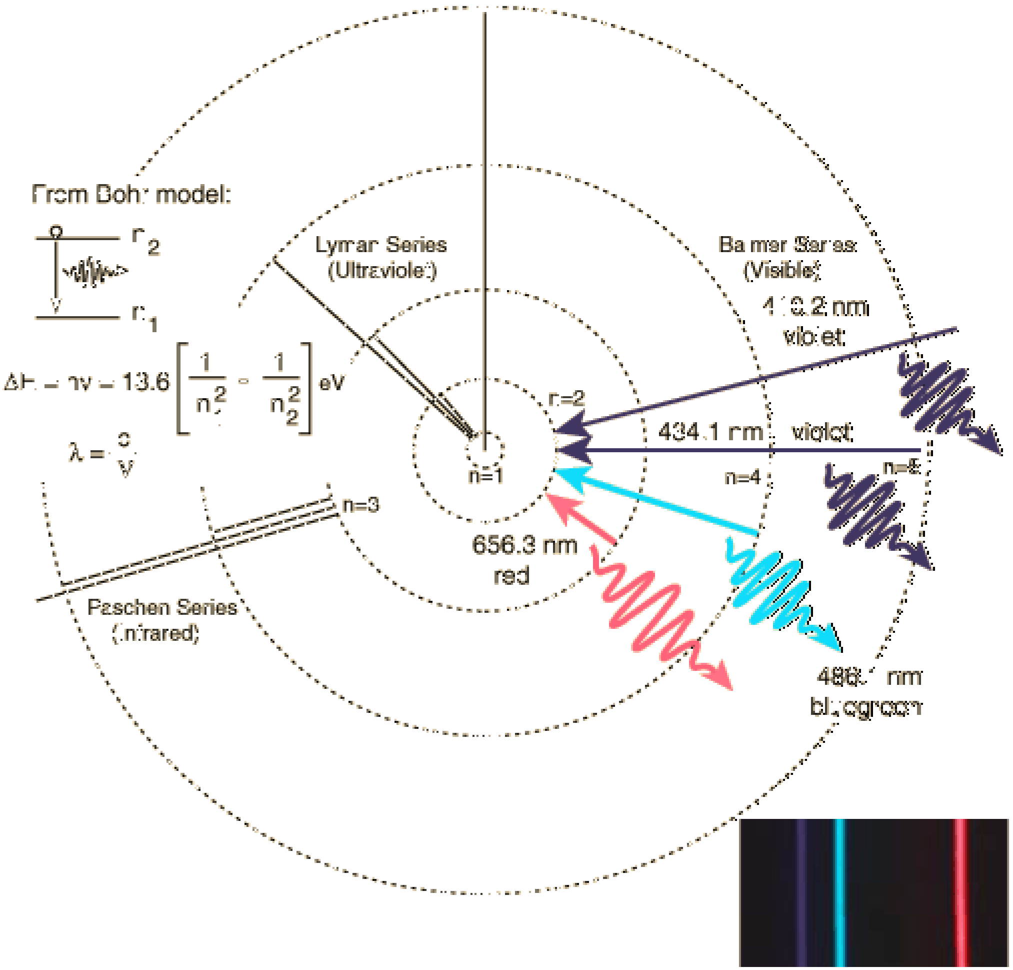
$$\frac{m^2v^2r^2}{r\,mr^2} = \frac{L^2}{mr^3} = \frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{r^2} \rightarrow r = L^2\cdot\frac{4\pi\epsilon_0}{mq^2}$$

What would we obtain if L were quantized ($L = n\frac{h}{2\pi} = n\hbar$)?

Clearly the radius would be quantized as well:

$$r = n^2\hbar^2\cdot 4\frac{\pi\epsilon_0}{mq^2}$$

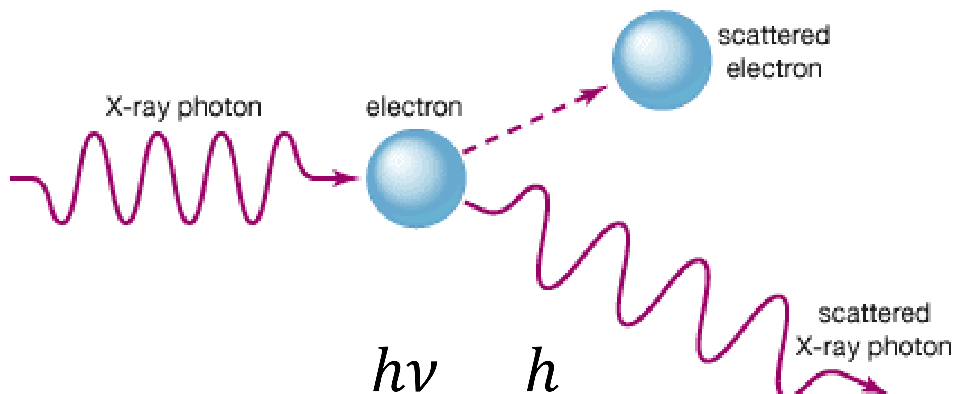
Discrete atom spectra



The quantization of angular momentum explains also the well known and yet inexplicable lines in the emission spectra of atoms.

The Compton effect

The Compton effect is the result of a high-energy radiation hitting an electron. The scattered radiation experiences a **wavelength shift** that cannot be explained in terms of classical wave theory, thus lending support to **Einstein's photon theory**. The effect was first demonstrated in 1923 by **Arthur Holly Compton** (Nobel Prize in 1927).



$$E = h\nu \qquad p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

De Broglie's hypothesis

- For a photon, the following formulas hold:

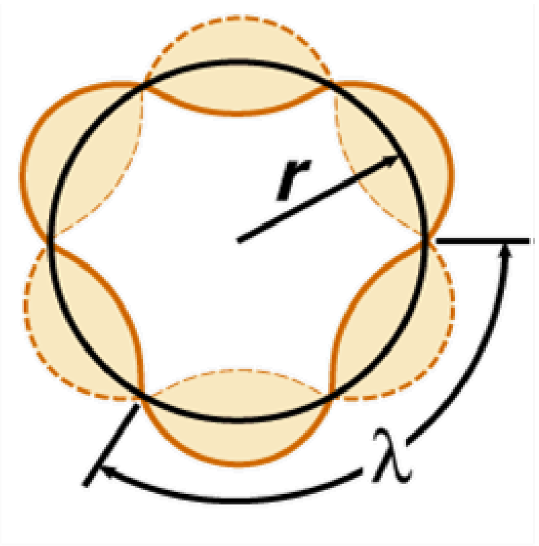
$$E = h\nu \quad \rightarrow \quad p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

- Louis De Broglie (Nobel in 1929) stated that a material particle should behave in the same way. It's only because of the small value of *h* and the small dimension of particles that **we don't experience quantum mechanics** in everyday life
- In the end if an **electron is a wave**, what is its wavelength?

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- This means that an electron can only fit in an orbit if it has a circumference being a multiple of the **"electron wavelength"** $n\lambda = 2\pi r$:

$$L = rmv = rp = \frac{n\lambda}{2\pi} \cdot \frac{h}{\lambda} = \frac{nh}{2\pi} = n\hbar$$



Conclusion

Quantum mechanics describes a lot of mysterious behaviours of nature.

There is no classical way to describe phenomena like: black-body spectrum, atom discrete spectra, Compton effect, photoelectric effect, double-slit experiment, and much more…

But how can it be like that?

Nobody knows how it can be like that.

Thanks for your attention