New sum rules for nucleon and trinucleon total photoproduction cross-sections

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The subject of our contribution is to derive:

- sum rules relating the Dirac mean square radii and anomalous magnetic moments of the proton and neutron (or He^3 and H^3) to the integral over a difference of the total proton and neutron (or He^3 and H^3) photoproduction cross-sections

With this aim, we start with a consideration of the **very high energy peripheral electron-proton scattering** in one photon approximation

$$e^-(p_1) + p^+(p) \to e^-(p_1') + X \tag{1}$$

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Its **matrix element** is

$$M = i\frac{4\pi\alpha}{q^2}\bar{u}(p_1^{'})\gamma^{\mu}u(p_1) < X \mid J^{\nu} \mid p > g_{\mu\nu} \tag{2}$$

The produced hadronic state X is **moving closely to the direction of initial proton** and $M_X^2 = (p+q)^2$.

Further:

- Sudakov expansion

  V.V.Sudakov, Sov. Phys. JETP 3 (1956) 65

  of the **photon transferred 4-vector** q

$$q = \beta_q \tilde{p}_1 + \alpha_q \tilde{p} + q_{\perp} \tag{3}$$

  into the following **almost light-like vectors**

$$\tilde{p}_1 = p_1 - p_1^2 p/(2p_1 p), \quad \tilde{p} = p - p^2 p_1/(2p_1 p)$$

- and the **Gribov representation**

  V.N.Gribov: Lectures on the theory of complex angular momenta, Phys.-Tech. Inst., Kharkov (1970)

  of the metric tensor

$$g_{\mu\nu} = g^{\perp}_{\mu\nu} + \frac{2}{s}(\tilde{p}_\mu \tilde{p}_{1\nu} + \tilde{p}_\nu \tilde{p}_{1\mu}) \approx \frac{2}{s}\tilde{p}_\mu \tilde{p}_{1\nu},$$

$$s = (p_1 + p)^2 \approx 2p_1 p \gg Q^2 = -q^2$$

in the photon propagator

are applied.

Then transforming the **phase space volume of the final state** suitably, one obtains the **differential electroproduction cross-section**

$$\frac{d\sigma^{e^- p^+ \to e^- X}}{d\boldsymbol{q}^2} = \frac{\alpha \boldsymbol{q}^2}{4\pi^2} \int\limits_{2M_p m_\pi + m_\pi^2}^{\infty} \frac{ds_1}{s_1^2[\boldsymbol{q}^2 + (m_e s_1/s)^2]^2} \,\mathrm{Im}\tilde{A}(s_1, \boldsymbol{q}) \quad (4)$$

with

$$-t = Q^2 = \boldsymbol{q}^2,$$

$$s_1 = 2qp = M_X^2 + Q^2 - m_p^2$$

to be **expressed through the imaginary part** of the almost forward Compton scattering amplitude on the proton $\tilde{A}(s_1, \boldsymbol{q})$ with the intermediate state X.

For the case of **small photon momentum transfer squared** in (4) and by **using the optical theorem**

$$\mathrm{Im}\tilde{A}(s_1, \boldsymbol{q})\Big|_{\boldsymbol{q}^2 \to 0} = \mathrm{Im}A(s_1, \boldsymbol{q})\Big|_{\boldsymbol{q}^2 \to 0} \approx 4\pi s_1 \sigma_{tot}^{\gamma p \to X}(s_1) \quad (5)$$

one comes to the relation

$$\boldsymbol{q}^2 \frac{d\sigma^{e^- p^+ \to e^- X}}{d\boldsymbol{q}^2}\Bigg|_{\boldsymbol{q}^2 \to 0} = \frac{\alpha}{\pi} \int\limits_{2M_p m_\pi + m_\pi^2}^{\infty} \frac{ds_1}{s_1} \sigma_{tot}^{\gamma p \to X}(s_1), \qquad (6)$$

similar to the total cross-section of the electroproduction process on the proton in the **Weizsäcker–Williams approximation**

G.Weizsäcker, Z. Phys. 88 (1934) 612

E.Williams, Phys. Rev. 45 (1939) 729

$$\sigma_{tot}^{e^- p^+ \to e^- X}(s_1) = \frac{2\alpha}{\pi} \ln\left(\frac{s}{m_e m_\pi}\right) \int\limits_{2M_p m_\pi + m_\pi^2}^{\infty} \frac{ds_1}{s_1} \sigma_{tot}^{\gamma p \to X}(s_1). \qquad (7)$$

The just demonstrated procedure in elm. interactions is well known - **the method of equivalent photons**.

e.g. *A.I.Akhieser, V.B.Berestecky: Quantum electrodynamics, NAUKA, Moscow (1969)*

At the same time, for **very high energy the elastic electron-proton differential cross-section in one photon approximation** is

$$\frac{d\sigma^{e^- p^+ \to e^- p^+}}{d\boldsymbol{q}^2} = 4\frac{\alpha^2}{(\boldsymbol{q}^2)^2}\left[F_{1p}^2(\boldsymbol{q}^2) + \frac{\boldsymbol{q}^2}{4M_p^2} F_{2p}^2(\boldsymbol{q}^2)\right], \qquad (8)$$

where $F_{1p}(\boldsymbol{q}^2)$ and $F_{2p}(\boldsymbol{q}^2)$ are the proton **Dirac** and **Pauli** electromagnetic form factors.

Next step is an investigation of the **analytic properties** of the forward Compton scattering amplitude $A(s_1, \boldsymbol{q})$ in s_1-plane.

They consist in:

- **one-proton intermediate state pole** at $s_1 = Q^2$,
- the **right-hand cut** starting at the pion-nucleon threshold $s_1 = Q^2 + 2M_p m_\pi + m_\pi^2$
- and the u_1**-channel left-hand cut** starting from $s_1 = Q^2 - 8M_N^2$.

Now, one defines the **path integral** I

$$I = \int_C \frac{ds_1}{(\boldsymbol{q}^2)^2} \frac{p_1^\mu p_1^\nu \tilde{A}_{\mu\nu}}{s_1^2} \qquad (9)$$

from the gauge invariant light-cone projection $p_1^\mu p_1^\nu \tilde{A}_{\mu\nu}$ of the part $\tilde{A}_{\mu\nu}$ of the total Compton scattering tensor with photon first absorbed and then emitted along the fermion line, in s_1-plane, as presented in Fig. 1

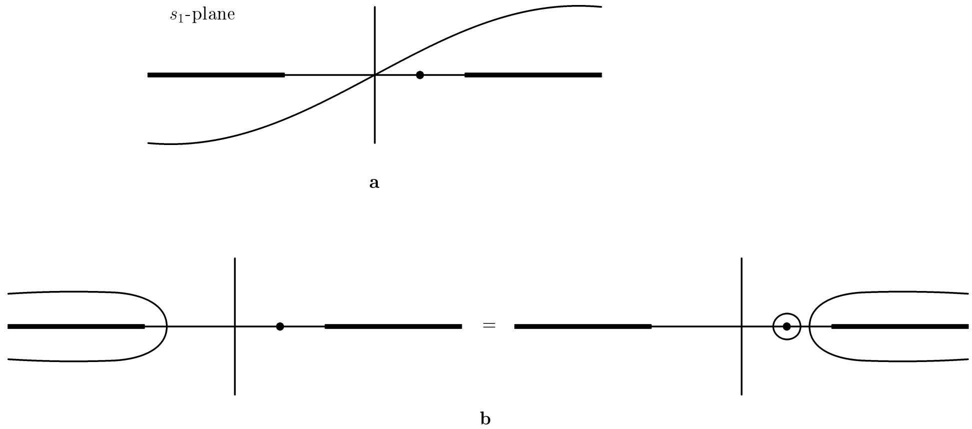

Fig. 1: Sum rule interpretation in s_1 plane.

Afterwards, once the contour C is closed to **upper half-plane**, another one to **lower half-plane**, the following sum rule

$$\pi \mathrm{Res} = \int\limits_{r.h.}^{\infty} \frac{ds_1}{s_1^2(\boldsymbol{q}^2)^2}\,\mathrm{Im}\tilde{A}(s_1,\boldsymbol{q}) - \int\limits_{l.h.}^{-\infty} \frac{ds_1}{s_1^2(\boldsymbol{q}^2)^2}\,\mathrm{Im}\tilde{A}(s_1,\boldsymbol{q}) \qquad (10)$$

appears with Res to be the one-proton intermediate state pole contribution.

Then, taking into account (4) and (8), one comes (for more details see Appendix D in *V.N.Baier, V.S.Fadin, V.A.Khoze, E.A.Kuraev, Physics Reports 78 (1981) 293*)

to the relation

$$1 - F_{1p}^2(-Q^2) - \frac{Q^2}{4M_p^2}F_{2p}^2(-Q^2) = \frac{(Q^2)^2}{4\pi\alpha^2}\frac{d\sigma^{e^-p^+\to e^-X}}{dQ^2} + LCC_p, \quad (11)$$

where the abbreviation LCC_p denotes the left-hand cut contribution in (10).

Repeating **the same procedure for the neutron**, one can write down a similar relation

$$-F_{1n}^2(-Q^2) - \frac{Q^2}{4M_n^2}F_{2n}^2(-Q^2) = \frac{(Q^2)^2}{4\pi\alpha^2}\frac{d\sigma^{e^-n\to e^-X}}{dQ^2} + LCC_n, \quad (12)$$

where the abbreviation LCC_n denotes the left-hand cut contribution in (10) for the neutron.

Finally, **neglecting a difference in the starting positions of the left-hand cuts contributions**, LCC_p and LCC_n, which is **caused by the small isotopic invariance violation** of the electromagnetic interactions, then **subtracting (12) from (11)**, one achieves a **mutual annulation of the left-hand cut proton and neutron contributions** as follows

$$1 - F_{1p}^2(-Q^2) + F_{1n}^2(-Q^2) - \frac{Q^2}{4M_p^2}F_{2p}^2(-Q^2) + \frac{Q^2}{4M_n^2}F_{2n}^2(-Q^2) =$$
$$\frac{(Q^2)^2}{4\pi\alpha^2}\left(\frac{d\sigma^{e^-p^+\to e^-X}}{dQ^2} - \frac{d\sigma^{e^-n\to e^-X}}{dQ^2}\right). \quad (13)$$

Moreover, **carrying a derivation** of both sides in (13) according to Q^2 and **employing the relation (6)** for both, proton and neutron, one comes for $Q^2 \to 0$ to the **new sum rule** relating Dirac mean squared radii and anomalous magnetic moments of the proton and the neutron to the integral over a difference of the total proton and neutron photoproduction cross-sections

$$\frac{1}{3}\left\langle r_{1p}^2\right\rangle - \frac{\mu_p^2}{4M_p^2} + \frac{\mu_n^2}{4M_n^2} = \frac{1}{2\pi^2\alpha}\int\limits_{\omega_N}^{\infty}\frac{d\omega}{\omega}\left[\sigma_{tot}^{\gamma p\to X}(\omega) - \sigma_{tot}^{\gamma n\to X}(\omega)\right] \quad (14)$$

with $\omega_N = m_\pi + m_\pi^2/2M_N$, in which just a **mutual cancellation of the rise of these total proton and neutron photoproduction cross-sections** for $\omega \to \infty$, created by the Pomeron exchanges, is achieved.

Here we would like to note, that the sum rule following from (11) by a derivation of both sides according to Q^2 and an application of the relation (6), is the known Gottfried sum rule [1]

K.Gottfried, Phys. Rev. Lett. 18 (1967) 1174

under the assumption that one neglects a contribution of LCC_p. The Gottfried sum rule, however, **suffers from a divergence of the corresponding integral** due to the well known rise of the total proton photoproduction cross-section at high energies created by the Pomeron exchanges at the competent amplitude.

If we consider instead of the proton and neutron, He^3 and H^3, which belong to the same isodoublet with spin 1/2, then all previous procedure can be repeated with the latter nuclei.

[1] We would like to thank S.Gerasimov for giving us this reference

As a result one obtains the sum rule of the following form

$$\frac{1}{3}\left[4\left\langle r^2_{1He^3}\right\rangle-\left\langle r^2_{1H^3}\right\rangle\right]-\frac{\mu^2_{He^3}}{4M^2_{He^3}}+\frac{\mu^2_{H^3}}{4M^2_{H^3}}=$$

$$=\frac{1}{2\pi^2\alpha}\int\limits_{\omega_{th}}^{\infty}\frac{d\omega}{\omega}\left[\sigma^{\gamma He^3\to X}_{tot}(\omega)-\sigma^{\gamma H^3\to X}_{tot}(\omega)\right] \tag{15}$$

Conclusions

Starting from:

- very high-energy elastic and inelastic electron nucleon (or electron-trinucleon)scattering with a production of a hadronic state X moving closely to the direction of initial hadron
- utilizing analytic properties of the forward Compton scattering amplitude on nucleons (or trinucleons)
- then for the case of small transferred momenta one can derive new sum rules relating Dirac radii and anomalous magnetic moments of the considered objects to the integral over a difference of the total proton and neutron (or He^3 and H^3) photoproduction cross-sections