Purely Flavored Leptogenesis at the TeV Scale

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Outline

- ★ The SM with the SEE-SAW
- ★ The Sakharov conditions
- \star Flavor
- ★ TeV Scale Leptogesis only from Flavor Effects
- ★ The Bolztmann Equations (BE)
- ★ Rescaling
- ★ Conclusions

Minimal extension of the SM: add 3 right handed neutrinos, $\alpha = 1, 2, 3$.

$$\mathcal{L}_Y = -\frac{1}{2} M_{N_\alpha} \bar{N}^c_\alpha N^c_\alpha - (\lambda_{\alpha i} \,\overline{N}_\alpha \,\ell_i \,\widetilde{\Phi}^\dagger + h_i \,\overline{e}_i \,\ell_i \,\Phi^\dagger + \text{h.c.})$$

Basis: $M_N = \text{diag}(M_1, M_2, M_3)$; diagonal charged lepton Yukawas h_i . Explain the suppression of ν masses via the seesaw: $\mathcal{M}_{\nu} = -(v \lambda)^T \frac{1}{M_N} (v \lambda)$ In terms of the diagonal light ν mass-matrix $m_{\nu} \equiv \text{diag}(m_1, m_2, m_3)$:

$$\lambda_{\alpha j} = \frac{1}{v} \left[\sqrt{M_N} \cdot R \cdot \sqrt{m_\nu} \cdot U^{\dagger} \right]_{\alpha j}$$

(with $R^T R = 1$, $UU^{\dagger} = 1$) [Casas Ibarra NPB618 (2001) hep-ph/0103065]

The seesaw model has 18 independent parameters (3 M_{α} , 3 m_{ν_i} , 3+3 complex angles in R and U). 3+6 parameters can be measured (in principle) at low energy, 3+6 are confined to high energy.

The Sakharov conditions, $\Rightarrow Y_B \equiv \frac{n_B - \bar{n}_B}{s} \approx (8.7 \pm 0.4) \times 10^{-11}$

In the SM with the seesaw the conditions are naturally satisfied:

- 2. *CP*: Complex Yukawa couplings $\lambda_{\alpha i}$ induce CP violation in the interference between tree level and loop decay amplitudes. E.g. for $N \rightarrow \ell \Phi$ decays:



- **3.** For a mass scale $M_N \sim 10^{11\pm3} \text{ GeV}$ deviations from thermal equilibrium in the primeval expanding Universe can occur at the time the N_{α} decay: $\Gamma_N(N \to \ell \Phi) < H(T \sim M_N)$
- **4. EW-Sphalerons** are SM processes that, in the EW symmetric phase, violate *B* and *L* (but conserve $\Delta_i = B/3 L_i$) and convert part of the *L*-asymmetry into a *B*-asymmetry.

Whether 'SM+SeeSaw' leptogenesis is able to explain the Baryon Asymmetry of the Universe is just a quantitative question.

Flavor

We have two types of CP violating effects in N_1 decays

 $\Gamma_1 \equiv \Gamma(N_1 \to \ell_i \Phi), \quad \overline{\Gamma}_1 \equiv \Gamma(N_1 \to \overline{\ell}'_i \overline{\Phi}), \ (i = 1, 2, 3)$

- (1) Leptons ℓ and antileptons $\overline{\ell'}$ are produced at different rates: $\Gamma_1 \neq \overline{\Gamma}_1$
- (2) The states ℓ and $\overline{\ell'}$ produced are not CP conjugate: $CP(\overline{\ell'}) \equiv \ell' \neq \ell$

The second effect is important when the reaction rates of charged lepton Yukawas $(h_{\tau,\mu,e})$ are faster than the rates of N_1 Yukawas (λ_{1i}) ($T \ll 10^{13}$ GeV) and of the Universe expansion:

$$\Gamma(\Phi \leftrightarrow \ell_{\tau} \bar{\tau}_{R}) > \Gamma_{1}(N_{1} \leftrightarrow \ell_{i} \Phi)$$

$$\&$$

$$\Gamma(\Phi \leftrightarrow \ell_{\tau} \bar{\tau}_{R}) > H.$$

Then a *density matrix* for $\ell_{e,\mu,\tau}$ builds up.

For medium-low mass scales ($M_N < 10^{12}$ GeV) flavor effects determine the final value of the Baryon asymmetry through (1) and - in a more subtle and interesting way - inducing new and possibly large effects related to (2).

It is quite remarkable that the different effects of the two types of CP violation can be disentangled rather clearly

TeV Scale Leptogesis only from Flavor Effects.

A particular realization of these possibilities occurs when an additional mass scale related to the breaking of a new symmetry, is presented below the mass M_{N_1} of the lightest Majorana neutrino.

[Aristizabal, Nardi, Losada PLB659 (2008) hep-ph/0103065]

Assume that at a scale close to the leptogenesis scale a *horizontal* $U(1)_X$ symmetry forbids direct couplings between ℓ and $N \Rightarrow \overline{\ell} P_R N \Phi$.

$$-\mathcal{L} = \dots + \bar{F}_a M_a F_a + h_{ia} \bar{\ell}_i P_R F_a \Phi + \lambda_{\alpha a} \bar{N}_{\alpha} F_a S + h.c.$$

e.g. as enforced by the assignment: $X(\ell_i, F_{R_a}, F_{L_a}) = +1$, X(S) = -1 and $X(N_{\alpha}, \Phi) = 0$.

 $F_a \equiv (F_{R_a}, F_{L_a})^T$, are heavy vectorlike fields, singlets with respect to $SU(2)_L \times U(1)_Y$ and charged under $U(1)_X$. S, is responsible for the breaking of $U(1)_X$ with $\langle S \rangle \equiv \sigma$.

All the interaction terms also preserve a U(1) (accidental) global symmetry L: $L(\ell_L, F_{R_a}, F_{L_a}, N_R) = +1$ and $L(S, \Phi) = 0$. After $U(1)_X$ and electroweak symmetry breaking

$$-\mathcal{M}_{ij} = \left[h^* \frac{\sigma}{M_F} \lambda^T \frac{v^2}{M_N} \lambda \frac{\sigma}{M_F} h^\dagger\right]_{ij}, \qquad -\mathcal{M}_{ij} = \left[\tilde{\lambda}^T \frac{v^2}{M_N} \tilde{\lambda}\right]_{ij}, \qquad \tilde{\lambda} = \left(\lambda \frac{\sigma}{M_F} h^\dagger\right)_{\alpha i}$$

The light neutrino masses have an aditional suppression factor $(\frac{\sigma}{M_F})^2$ and are of fourth order in the fundamental couplings λ and h. The matrices λ and h both contain physical phases that are important to leptogenesis. Besides the electroweak breaking scale v, we have the following new scales: the mass scale of vectorlike fields M_F , the lepton number breaking scale M_{N_1} , the horizontal symmetry breaking scale σ .

The different possibilities, $M_{N_1} < M_{N_2} < M_{N_3}$

* Standard leptogenesis: $M_F, \sigma >> M_{N_1}$ After integrating out the *F* fields one obtains the standard seesaw Lagrangian containing the effective operator: $\tilde{\lambda}_{\alpha i} \bar{N}_{\alpha} \ell_i \Phi$. Then *CP* asymmetry comes from the inter-

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$$\epsilon_{N_1} = \frac{3}{16\pi[\tilde{\lambda}\tilde{\lambda}^{\dagger}]_{11}} \sum_{\beta=1}^{3} \frac{M_{N_1}}{M_{N_{\beta}}} \Im m[\tilde{\lambda}\tilde{\lambda}^{\dagger}]_{\beta}$$

the amount of B asymmetry that can be generated from N_1 dynamics can be written as:

$$\frac{n_B}{s} \sim -\epsilon_{N_1} \eta$$

 η is the efficiency factor that accounts for amount of *L* asymmetry that survives the washout process.

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$$\epsilon_j = \frac{3}{128\pi[\tilde{\lambda}\tilde{\lambda}^{\dagger}]_{11}} \sum_{i=1}^3 \Im m[[hr^2h^{\dagger}]_{ij}\tilde{\lambda}_{1i}\tilde{\lambda}_{1j}^*]$$

$$\Im m[\tilde{\lambda}hr^2h^{\dagger}\tilde{\lambda}^{\dagger}]_{11} = 0 \Rightarrow \epsilon_{N_1} = \sum_j \epsilon_j = 0.$$

The loop is *L*-conserving (Dirac type diagram) $\Rightarrow \epsilon_{N_1} = 0$.

Leptogenesis can occur only through the effects of lepton flavor dynamics combined with the violation of lepton number that is provided by the washout processes.

Others cases see: [Aristizabal, Nardi, Losada PLB659 (2008) hep-ph/0103065]

Boltzmann's Equations ($\sigma < M_{N_1}$ and $M_{F_a} > M_{N_1}$)

The set of Boltzmann's Equations (BE) for Y_{N_1} and $Y_{\Delta L_i} = Y_{\ell_i} - Y_{\bar{\ell}_i}$ is

$$\dot{Y}_{N_{1}} = -(y_{N_{1}}-1) \sum_{i=e,\mu,\tau} \left(\gamma_{S\Phi\ell_{i}}^{N_{1}} + \gamma_{\Phi\ell_{i}}^{N_{1}\bar{S}} + \gamma_{S\ell_{i}}^{N_{1}\bar{\Phi}} + \gamma_{\bar{S}\bar{\Phi}}^{N_{1}\ell_{i}} \right) \dot{Y}_{\Delta L_{i}} = \epsilon_{i} \left(y_{N_{1}}+1 \right) \sum_{i=e,\mu,\tau} \left(\gamma_{S\Phi\ell_{i}}^{N_{1}} + \gamma_{\Phi\ell_{i}}^{N_{1}\bar{S}} + \gamma_{S\ell_{i}}^{N_{1}\bar{\Phi}} + \gamma_{\bar{S}\bar{\Phi}}^{N_{1}\ell_{i}} \right) - \Delta y_{\ell_{i}} \left(\gamma_{S\Phi\ell_{i}}^{N_{1}} + \gamma_{\Phi\ell_{i}}^{N_{1}\bar{S}} + \gamma_{S\ell_{i}}^{N_{1}\bar{\Phi}} + y_{N_{1}}\gamma_{\bar{S}\bar{\Phi}}^{N_{1}\ell_{i}} \right)$$

where $\dot{Y} = zHs\frac{dY}{dz}$, $z = \frac{M_1}{T}$, $Y = \frac{n}{s}$ and $y = \frac{Y}{Y^{eq}}$. The relevant processes (decay and scatterings) are:



They are of order $(\lambda h)^2$. The important point is that decay and scatterings are the same order (differently from the standard case $\gamma_D \sim \lambda^2$, $\gamma_S \sim \lambda^2 g^2$).

The factor $(y_N + 1)$ in the source term, when N_1 is equilibrium we can get $Y_{\Delta L_i} \neq 0$ which violate the Sakarov's criteria. We need to include the processes the order higher, which have a intermediate Mayorana neutrino N_1 on-shell, in this model the processes with N_1 on-shell are $[3 \leftrightarrow 3]$ and $[2 \leftrightarrow 4]$, they have terms that in the source term are the order $\mathcal{O}((\lambda h^{\dagger})^2)$.

The on-shell processes have been written as (remember $\gamma = \gamma^{on} + \gamma^{off}$):

$\gamma^{on} {}^{\Phi \ell S}_{\bar{\Phi} \bar{\ell} \bar{S}}$	=	$P^{N}_{\Phi\ell S}\gamma^{N}_{\bar{\Phi}\bar{\ell}\bar{S}} + P^{\bar{S}N}_{\Phi\ell}\gamma^{SN}_{\bar{\Phi}\bar{\ell}} + P^{\bar{\Phi}N}_{S\ell}\gamma^{\Phi N}_{\bar{S}\bar{\ell}}$
	+	$P^{ar{\ell}N}_{S\Phi}\gamma^{\ell N}_{ar{S}ar{\Phi}}$
$\gamma^{on}{}^{S\ell S}_{\bar{\Phi}\bar{\ell}\bar{\Phi}}$	=	$P^{ar{\Phi}N}_{S\ell}\gamma^{SN}_{ar{\ell}ar{\Phi}}$
$\gamma^{on} \Phi^{SS}_{\bar\ell\bar\ell} \bar\Phi$	=	$P^{ar{\ell}N}_{\Phi S}\gamma^{SN}_{ar{\ell}ar{\Phi}}$
$\gamma^{on} \Phi S \Phi_{\bar{\ell}\bar{\ell}\bar{S}}$	=	$P^{ar{\ell}N}_{\Phi S}\gamma^{\Phi N}_{ar{\ell}ar{S}}$

here $P_{b_2}^{a_2N} = \frac{\gamma_{b_2}^{a_2N}}{\gamma_T}$ where γ_T is the sum the γ_B^A in the same order: $\gamma_T = \sum \gamma_{S\Phi\ell}^N + \gamma_{\bar{S}\bar{\Phi}\bar{\ell}}^N + \gamma_{\ell\Phi}^{\bar{S}N} + \gamma_{\bar{\ell}\bar{\Phi}}^{\bar{N}N} + \gamma_{\ell\bar{S}}^{\Phi N} + \gamma_{\bar{\ell}\bar{S}}^{\Phi N} + \gamma_{\bar{L}\bar{S}}^{\Phi N} + \gamma_{\bar{L}\bar{S}}^{\Phi$ diagrams for $|\Delta L_i| = 2$:







$[S\ell_i S \leftrightarrow \bar{\Phi} \bar{\ell}_i \bar{\Phi}], [\Phi SS \leftrightarrow \bar{\ell}_i \bar{\ell}_i \bar{\Phi}], [\Phi S\Phi \leftrightarrow \bar{\ell}_i \bar{\ell}_i \bar{S}].$





 $\left[\ell_i S \leftrightarrow \bar{\Phi} \bar{S} \bar{\ell}_i \bar{\Phi}\right], \left[\ell_i \Phi \leftrightarrow \bar{S} \bar{S} \bar{\ell}_i \bar{\Phi}\right], \left[\Phi S \leftrightarrow \bar{\ell}_i \bar{S} \bar{\ell}_i \bar{\Phi}\right]$

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where $\dot{Y} = zHs\frac{dY}{dz}$, $z = \frac{M_1}{T}$, $Y = \frac{n}{s}$ and $y = \frac{Y}{Y^{eq}}$. The relevant processes (decay and scatterings) are:



The $[3 \leftrightarrow 3]$ and $[2 \leftrightarrow 4]$ processes give the contribution to $Y_{\Delta L_i} = \cdots - 2\sum_i \left(\gamma_{S\Phi\ell_i}^{N_1} + \gamma_{\Phi\ell_i}^{N_1\bar{S}} + \gamma_{S\ell_i}^{N_1\bar{\Phi}} + \gamma_{\bar{S}\bar{\Phi}}^{N_1\ell_i}\right)$ so we can get the $(y_{N_1} - 1)$ factor in the B.E.

but:

We have written for the decay $\frac{\gamma}{zHs}$ $\gamma_{S\ell_i\Phi}^{N_1} = N_{N_1}^{eq}(z) \frac{K_1(z)}{K_2(z)} \Gamma_{S\ell_i\Phi}^{N_1}$ 10⁴ $\frac{\gamma}{zHs} = 1$ $\Gamma^{N_1}_{S\ell_i\Phi} = \frac{M_{N_1}}{192\pi^3} \sum_{a,b} r_a r_b h^*_{ib} h_{ia} \lambda_{1a} \lambda^*_{1b}$ 10^{-5} and scatterings $\gamma = \frac{M_1^4}{512\pi^5 z} \int_1^\infty dx \sqrt{x} K_1\left(z\sqrt{x}\right) \hat{\sigma}\left(x\right)$ 10^{-14} $\gamma^{N_1}_{S\ell_e\Phi}$ $\hat{\sigma} = \sum_{a} r_{a}^{2} h_{ia}^{*} h_{ai} \lambda_{1a}^{*} \lambda_{a1} F^{a} (x)$ $\gamma_{\ell_e\Phi}^{\bar{S}N_1}$ $\cdots \qquad \gamma_{\Phi S}^{\bar{\ell}_e N_1}$ + $2\sum_{a < b} r_a r_b \Re e[h_{ia}^* h_{bi} \lambda_{1a}^* \lambda_{b1} G^{a,b}(x)].$ 10⁻²³ 10^{-1} 10^{0} 10¹ With the transformations, $h \rightarrow \kappa h$ and $\lambda \rightarrow$ $\kappa^{-1}\lambda$ so that \mathcal{Z}

 \bigstar the rates of *e* are $\xi_{S_i} > 1$ and $\xi_{D_i} < 1$.

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The Numerical solution of the BE, simple example

With a set of couplings $h/(4\pi) < 1$ and $\lambda/(4\pi) < 1$, the solution the BE is:



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The final values of the asymmetry densities (at z >> 1) are:

$$Y_{\Delta L_e} = -7.1 \times 10^{-10}$$

 $Y_{\Delta L_{\mu}} = -2.0 \times 10^{-10}$
 $Y_{\Delta L_{\tau}} = +0.3 \times 10^{-10}$

Rescaling

Rescaling of λ and h that leave \mathcal{M}_{ν} invariant play an important role:

$$egin{array}{ccc} \lambda & \longrightarrow & rac{1}{\kappa}\lambda \ h & \longrightarrow & \kappa h \end{array}$$

the low energy parameters remains invariant:

 $\tilde{\lambda}_{ij} \longrightarrow \tilde{\lambda}_{ij} \Rightarrow \mathcal{M}_{ij} \longrightarrow \mathcal{M}_{ij}$

but the *CP* asymmetries rescale as:

 $\epsilon_i \longrightarrow \kappa^2 \epsilon_i \quad [\epsilon_i \sim (hr^2 h^{\dagger})].$

This rescaling defines large parameter space regions (λ, h) compatible with low energy ν data and allow to enhance: $Y_{\Delta_{\ell_i}} \longrightarrow \kappa^2 Y_{\Delta_{\ell_i}}$ $(\kappa > 1)$.

The strong condition $\sum_i \epsilon_i = 0$, $\Longrightarrow Y_B$!

Rescaling

Rescaling of λ and h that leave \mathcal{M}_{ν} invariant play an important role:

 \downarrow



the relation $Y'_{\ell_i}(\kappa) = \kappa^2 Y_{\ell_i}(\kappa = 1)$.

global rescaling $\kappa_i \rightarrow \kappa$.

Conclusions

The following points are relevant:

- **4.** $M_1 = 2$ TeV implies a low scale for leptogenesis in contrast with the standard case ($M_{N_1} > 10^8$ Gev).
- **.** The flavor *CP* asymmetries is $\epsilon_i \neq 0$, but $\sum_i \epsilon_i = 0$. In this case leptogenesis is due to flavor dynamics.
- •. Succesful TeV scale leptogenesis.
- \heartsuit . Rescaling of λ and h that leave \mathcal{M}_{ν} invariant play an important role.
- **★.** Test at the LHC seems difficult.

Backup: Real Intermediate Substrate (RIS)

In the intervale Δz and with the notation: $\gamma_j^i \equiv \gamma \ (i \rightarrow j)$

the **BE** can be written

$$\Delta Y_{\ell} = \left(Y_{\bar{\ell}} \tilde{\gamma}_{\ell}^{\bar{\ell}} + Y_N \gamma_{\ell}^N - Y_{\ell} \tilde{\gamma}_{\bar{\ell}}^{\ell} - Y_{\ell} \gamma_N^{\ell} \right) \Delta z$$

where : $\gamma^\ell_{\bar{\ell}} \equiv \hat{\gamma}^\ell_{\bar{\ell}} + \tilde{\gamma}^\ell_{\bar{\ell}}$

and in same way

$$\Delta Y_{\bar{\ell}} = \left(Y_{\ell} \tilde{\gamma}_{\bar{\ell}}^{\ell} + Y_N \gamma_{\bar{\ell}}^N - Y_{\bar{\ell}} \tilde{\gamma}_{\ell}^{\bar{\ell}} - Y_{\bar{\ell}} \gamma_N^{\bar{\ell}} \right) \Delta z.$$

For $\Delta Y_{\ell} - \Delta Y_{\bar{\ell}} \equiv \Delta Y_{\mathcal{L}}$

$$\Delta Y_{\mathcal{L}} = \left[\gamma_D \left(Y_{N_1} - 1\right)\epsilon - 2Y_{\mathcal{L}}\gamma_{N_s}\right] \Delta z$$

we have written: $\hat{\gamma}^\ell_{\bar{\ell}} = \gamma^\ell_N B^N_{\bar{\ell}}$ y $\hat{\gamma}^{\bar{\ell}}_\ell = \gamma^{\bar{\ell}}_N B^N_\ell$, with

$$B^N_\ell = \frac{\gamma^N_\ell}{\gamma^N_\ell + \gamma^N_{\bar{\ell}}}, \qquad B^N_{\bar{\ell}} = \frac{\gamma^N_{\bar{\ell}}}{\gamma^N_\ell + \gamma^N_{\bar{\ell}}}$$



