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QCD radiation in transPlanckian scattering

N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315].

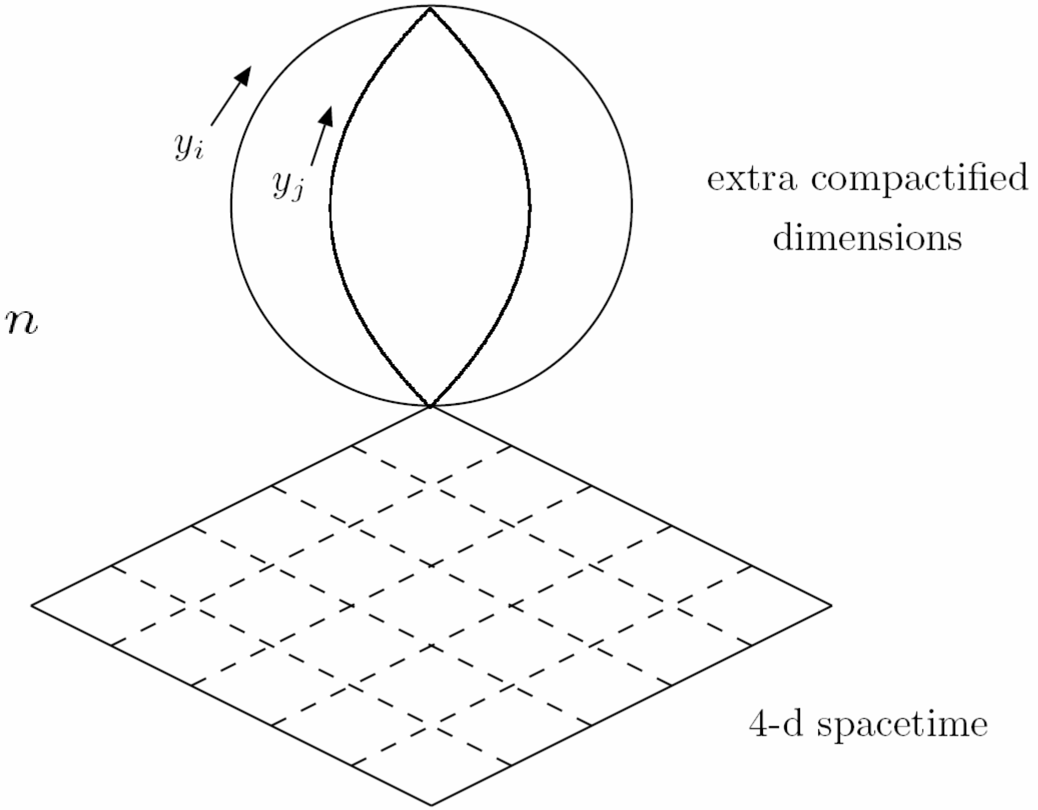
1) Large Extra Dimensions

$$S_{\text{bulk}} = -\frac{1}{2}\int d^{4+n}x\sqrt{-g^{(4+n)}}M_\star^{n+2}R^{(4+n)}$$

"integrating out" the extra dimensions:

$$M_{\text{Pl}}^2 = M_\star^{n+2}(2\pi r)^n$$

Quantum gravity scale could be much lower than M_{Pl} !



Fundamental ASSUMPTION:
only gravity propagates in the extra dimensions

- Expected deviations from Newtonian gravity:

$$V(r') = \begin{cases} -G_N^{(4+n)} \dfrac{m_1 m_2}{r'^{1+n}} & r' < r \\ -G_N \dfrac{m_1 m_2}{r'} & r' > r \end{cases}$$

(r = "size" of the (compactified) extra dimensions)

- Experimental bound: $r \leq 0.2 \text{ mm}$

For example:

$M_* \sim 10^3 \text{ GeV}$

$r \sim 2 \cdot 10^{-17} 10^{\frac{32}{n}} \text{ cm}$

$M_* \sim 10^3 \text{ GeV}$ and $n \geq 2$ is a serious possibility!

G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **630**, 293 (2002) [arXiv:hep-ph/0112161]

2) Transplanckian scattering

- Relevant scales: $D = 4 + n$

$$\lambda_P = \left(\frac{G_D \hbar}{c^3}\right)^{\frac{1}{n+2}} \qquad \lambda_B = \frac{4\pi\hbar c}{\sqrt{s}}$$

$$G_D = \frac{(2\pi)^{n-1}\hbar^{n+1}}{4c^{n-1}M_D^{n+2}} \qquad R_S = \frac{1}{\sqrt{\pi}}\left[\frac{8\Gamma\left(\frac{n+3}{2}\right)}{(n+2)}\right]^{\frac{1}{n+1}} \left(\frac{G_D\sqrt{s}}{c^4}\right)^{\frac{1}{n+1}}$$

- Peculiar feature: NEW SCALE

$$b_c \propto \left(\frac{G_D s}{\hbar c^5}\right)^{\frac{1}{n}}$$

- Cannot be defined if n=0
- Goes to infinity if $\hbar \to 0$ with G_D constant

⇒ "Size of classical region" (assume $b_c \gg R_S$)

Model independent features

(since QG not known)

- Impact parameter $b \gg R_S \rightarrow$ weak gravitational field (linearization)
- $\sqrt{s} \gg M_D \quad \Rightarrow \quad R_S \gg \lambda_P \gg \lambda_B$
  $\rightarrow$ quantum-gravity effects are small

(we can use just QM and linearized GR)

- Forward scattering at small angles
  $\rightarrow$ eikonal regime, predictive computation is possible

TransPlanckian eikonal regime:

$$M_D/\sqrt{s} \ll 1, \qquad -t/s \ll 1$$

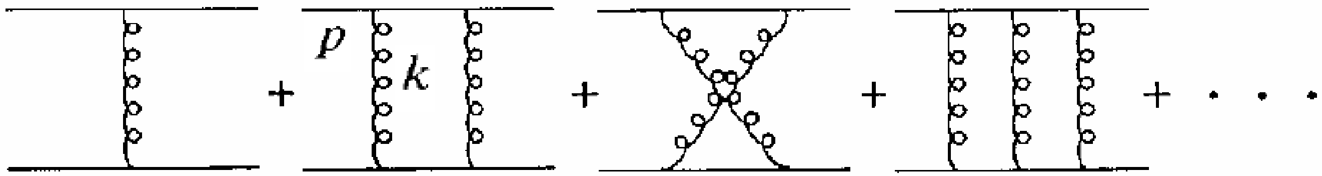
<u>REMARKABLE FACTS</u>:

- **Predictivity**
- **Model independence**

D. Kabat and M. Ortiz, Nucl. Phys. B **388**, 570 (1992)

FIRST APPROACH: Eikonalization

evaluate leading behaviour at high energy and small angles by summing an infinite number of diagrams



approximation:

- Take on shell vertices
- Use: $\frac{1}{(p+k)^2+m^2-i\epsilon} \approx \frac{1}{2p\cdot k - i\epsilon}$

(resummation) ⇒

$$\mathcal{A}_{\rm eik} = -2is\int d^2b_\perp e^{iq_\perp b_\perp}\left(e^{i\chi}-1\right)$$

$t = q^2 \simeq -q_\perp^2$

$$\chi(b_\perp) = \frac{1}{2s}\int\frac{d^2q_\perp}{(2\pi)^2}e^{-iq_\perp b_\perp}\mathcal{A}_{\rm Born}(q_\perp^2) = \left(\frac{b_c}{b}\right)^n$$

- No UV sensitivity
- Spin independent

G. 't Hooft, Phys. Lett. B198 (1987) 61
P.C. Aichelburg and R.U. Sexl, Gen. Rel. Grav. 2 (1971) 303

SECOND APPROACH: Shock wave (generalized)

- Replace one particle by its AS shock wave:

$$ds^2 = -dx^+ dx^- + \Phi(x_\perp)\delta(x^-)(dx^-)^2 + dx_\perp^2$$

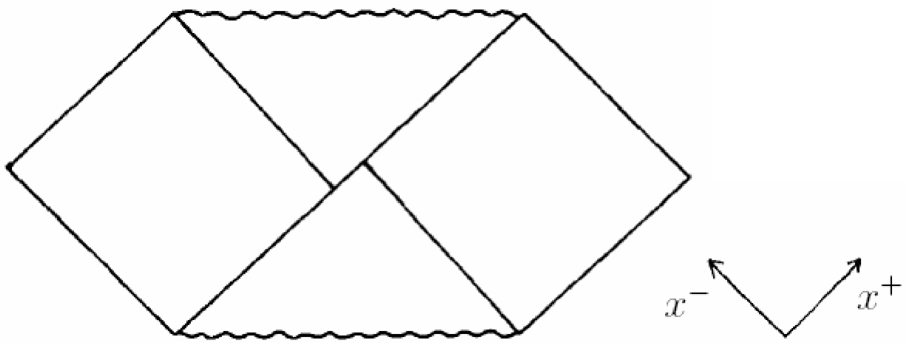
- Solve Einstein equations:

$$-\frac{1}{2}\partial_\perp^2 \Phi = 8\pi G_D E \delta^{(D-2)}(x_\perp)$$

(the particle moves in the positive z direction
$x_\perp$ denotes $D-2$ transverse directions)

$$\frac{\Phi}{8\pi G_D} = -\frac{E}{\pi}\log|x_\perp| \qquad (D=4)$$

$$= \frac{2E}{\Omega_{D-3}(D-4)|x_\perp|^{D-4}} \qquad (D>4)$$



AS shock wave: $$ds^2 = -dx^+dx^- + \Phi(x_\perp)\delta(x^-)(dx^-)^2 + dx_\perp^2$$

- Perform a (discontinuous) change of coordinates so that the metric becomes continuous across $x^- = 0$:

$$\begin{cases} x^+ = x'^+ + \theta(x^-)[\Phi(x_\perp) + \frac{(\partial_\perp \Phi(x_\perp))^2}{4} x^-] \\ x^- = x'^- \\ x^i = x'^i + \theta(x^-)\frac{\partial_i \Phi(x_\perp)}{2} x^- \qquad (i = 1, 2) \end{cases}$$

- Solve Klein-Gordon equation in these new coordinates (imposing continuity of the wavefunction):

$$\text{before collision:} \quad \psi(x) = \exp(+ip'x) = \exp(-iE'(z+t))$$

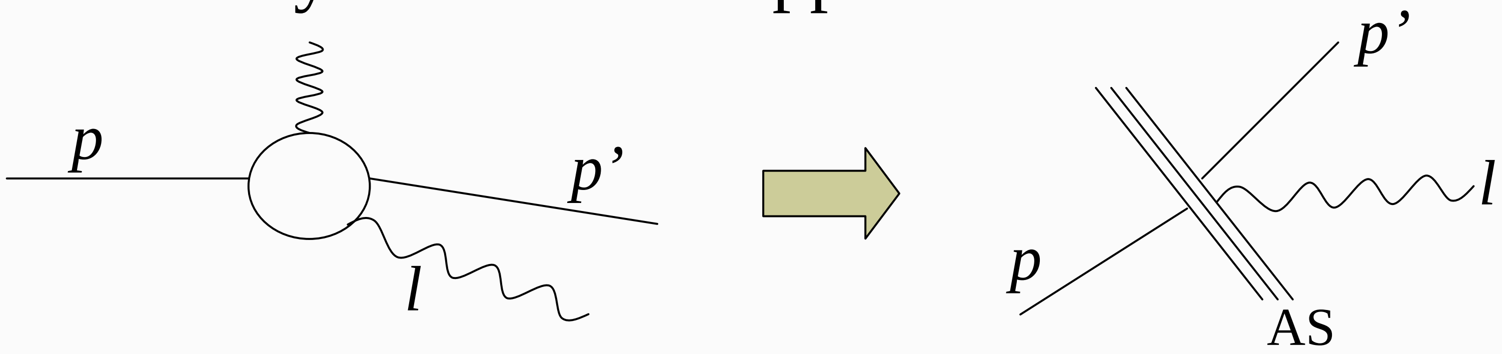
$$\text{after collision:} \quad \psi(x') = \exp(-iE'x^{+\prime}) = \exp(-iE'(x^+ - \Phi(x_\perp)))$$

⇒ eikonal amplitude: $\boxed{e^{i\chi} = \exp(iE'\Phi)}$

Same as before!

3) PROJECT: Including QCD radiation

- Best way: shock wave approach



- General formula: using Green's functions

$$M_{p\to p'+l} = \int d^4x \sqrt{g} g^{\mu\nu} G_{out}(p', x) \overleftrightarrow{\partial}_{\mu} G_{in}(x, p) G^{A}_{out}(l, \varepsilon, x)_{\nu}$$

- Main point: evaluation of this integral!
  (Green's functions are obtained via 't Hooft method)

STUDY OF *M*: leading log corrections

- Including all factors: total cross section = without gluon + large logarithms (IR singularity if quark massless)

→ AP equations
(the usual DIS scaling violation)

(redefine scale dependent parton distribution functions so that main corrections are reabsorbed)

- What we find:

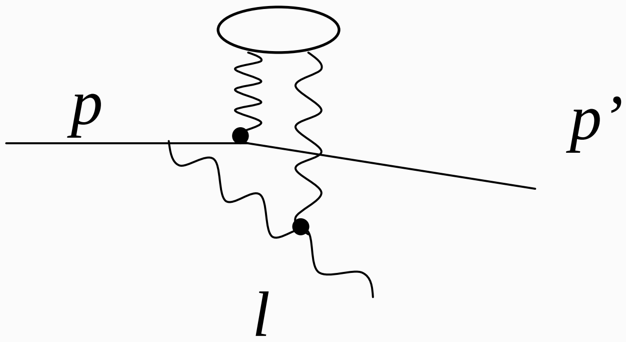
$q_\perp b_c \lesssim 1$ ⇒ scale = $q_\perp$ as usual (JJ)

$q_\perp b_c \gg 1$ ⇒ different scale! $\dfrac{(q_\perp b_c)^{\frac{1}{n+1}}}{b_c}$

[**Really a very peculiar fact**! Physical interpretation: saddle point]

4) Conclusions

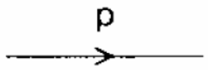
- Generalization of ‘t Hooft shock wave method in transPlanckian scattering in order to include radiation
- For large momentum transfer ($q_\perp b_c \gg 1$) the PDF must be normalized at a scale different than usual.
  Physical reason: RESCATTERING EFFECTS



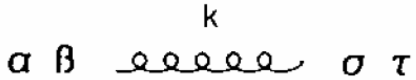
- Future developments: WORK IN PROGRESS

Extra Slides

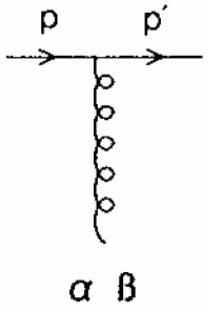
Feynman rules



$$i\Delta = -\frac{i}{p^2 + m^2 - i\epsilon}$$



$$iD^{\alpha\beta\sigma\tau} = -i\,\frac{16\pi G}{k^2 - i\epsilon}\left(\eta^{\alpha\sigma}\eta^{\beta\tau} + \eta^{\alpha\tau}\eta^{\beta\sigma} - \eta^{\alpha\beta}\eta^{\sigma\tau}\right)$$



$$\frac{i}{2}\left(p_\alpha p'_\beta + p_\beta p'_\alpha - \eta_{\alpha\beta}\left(p\cdot p' + m^2\right)\right)$$

Feynman rules in De Donder gauge.

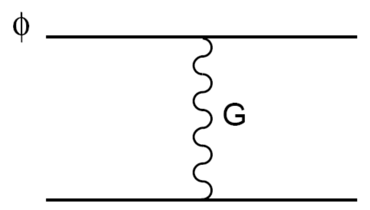
In evaluating the vertex factors, we ignore the recoil of the matter field

$$p_\mu p'_\nu + p_\nu p'_\mu - \eta_{\mu\nu}(p\cdot p' + m^2) \approx 2p_\mu p_\nu$$

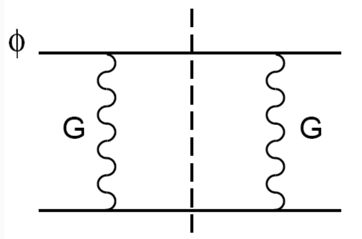
In the matter propagators, we ignore k^2 relative to $p\cdot k$

$$\frac{1}{(p+k)^2 + m^2 - i\epsilon} \approx \frac{1}{2p\cdot k - i\epsilon}$$

Eikonalization vs Shock wave: **eikonal resummation**



$$\mathcal{A}_{\rm Born}(-t) = \frac{s^2}{M_D^{n+2}} \int \frac{d^n q_T}{t - q_T^2} = \pi^{\frac{n}{2}} \Gamma(1 - n/2) \left(\frac{-t}{M_D^2}\right)^{\frac{n}{2}-1} \left(\frac{s}{M_D^2}\right)^2$$



$$\mathcal{A}_{1-{\rm loop}}(-q^2) = \frac{i}{4s} \int \frac{d^2 k_\perp}{(2\pi)^2} \mathcal{A}_{\rm Born}(k_\perp^2) \mathcal{A}_{\rm Born}[(q_\perp - k_\perp)^2]$$

$$\mathcal{A}_{\rm Born}(q_\perp^2) + \mathcal{A}_{1-{\rm loop}}(q_\perp^2) + \ldots = -2is \int d^2 b_\perp e^{i q_\perp b_\perp} \left(i\chi - \frac{1}{2}\chi^2 + \ldots\right)$$

$$\chi(b_\perp) = \frac{1}{2s} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i q_\perp b_\perp} \mathcal{A}_{\rm Born}(q_\perp^2).$$

$$\mathcal{A}_{\rm eik} = -2is \int d^2 b_\perp e^{i q_\perp b_\perp} \left(e^{i\chi} - 1\right)$$

$$\chi = \frac{\pi^{\frac{n}{2}-1} \Gamma(1 - n/2) s}{4 M_D^{n+2}} \int_0^\infty dq q^{n-1} J_0(qb) = \left(\frac{b_c}{b}\right)^n$$

$$b_c \equiv \left[\frac{(4\pi)^{\frac{n}{2}-1} s \Gamma(n/2)}{2 M_D^{n+2}}\right]^{1/n}.$$

Eikonalization vs shock wave:
shock wave computation:

Let us consider the plane wavefunction colliding with the classical AS shock wave. The wavefunction describes a particle of energy E' moving in the negative z direction. Thus $p' = (E', -E', 0)$ and before the collision the wavefunction is:

$$\psi(x) = \exp(+ip'x) = \exp(-iE'(z+t))$$

After the collision the wavefunction remains continuous in the x' coordinates, i.e.

$$\psi(x') = \exp(-iE'x^{+\prime}) = \exp(-iE'(x^+ - \Phi(x_\perp)) \quad , \quad x^- = \varepsilon > 0 \text{ small}$$

(i.e. immediately after collision). Thus we get eikonal amplitude:

$$\boxed{e^{i\chi} = \exp(iE'\Phi)}$$

$$\left(\Omega_k = \frac{2\pi^{(k+1)/2}}{\Gamma((k+1)/2)} \right)$$

Check:

$$\chi = 8\pi G_D \frac{2}{\Omega_{D-3}(D-4)b^n} EE' = \frac{(2\pi)^n}{M_D^{2+n}} \frac{\Gamma((k+1)/2)}{2\pi^{(k+1)/2}}\Big|_{k=n+1} \frac{2}{n}\frac{s}{4b^n}$$

$$= \frac{(2\pi)^n}{M_D^{2+n}} \frac{\Gamma(\frac{n}{2}+1)}{2\pi^{\frac{n}{2}+1}} \frac{2}{n}\frac{s}{4b^n} = \frac{2^{n-2}\pi^{\frac{n}{2}-1}}{M_D^{2+n}} \frac{\Gamma(\frac{n}{2})}{2}\frac{s}{b^n} = \frac{(4\pi)^{\frac{n}{2}-1}}{M_D^{2+n}} \frac{\Gamma(\frac{n}{2})}{2}\frac{s}{b^n} = \left(\frac{b_c}{b}\right)^n$$

G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **630**, 293 (2002) [arXiv:hep-ph/0112161]

Example of signal at the LHC

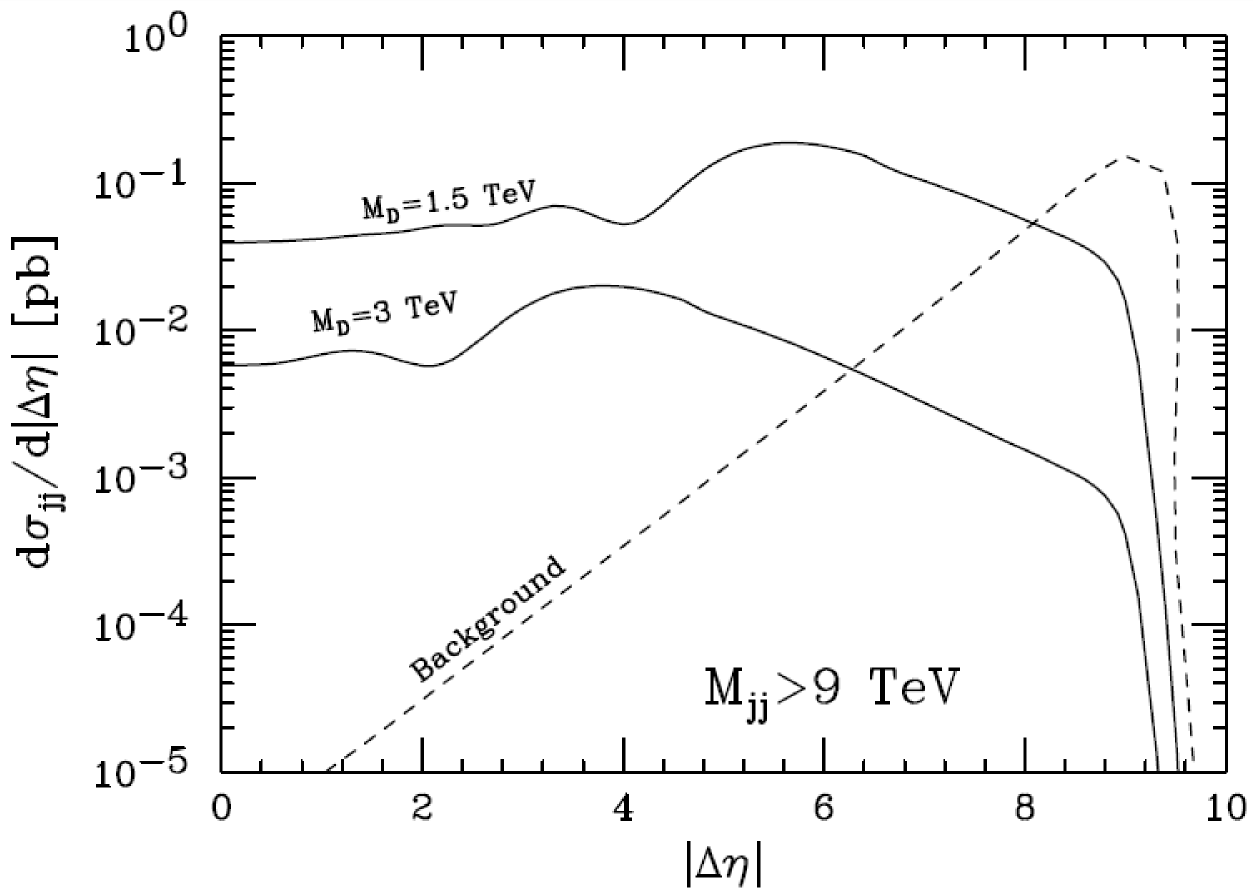


Figure 4: The di-jet differential cross section $d\sigma_{jj}/d|\Delta\eta|$ from eikonal gravity for $n = 6$, $M_{jj} > 9\,\text{TeV}$, when both jets have $|\eta| < 5$ and $p_T > 100\,\text{GeV}$, and for $M_D = 1.5\,\text{TeV}$ and $3\,\text{TeV}$. The dashed line is the expected rate from QCD.

Green's functions

- $G_{out}(p', x) = e^{-ip'x} \quad , \quad x^- > 0$
- $G^A_{out}(l, \varepsilon, x)_\mu = \varepsilon_\mu e^{-ilx} \quad , \quad x^- > 0 \,, \; \varepsilon l = 0$
- $G_{in}(p', x) = e^{+ipx} \quad , \quad x^- < 0$

$$e^{i\frac{p^-}{2}\Phi(x^i)} = \int \frac{d^2k}{(2\pi)^2} e^{ik^ix^i} A(k^i, p^-)$$

- $G_{in}(p, x) = e^{-i\left\{\frac{1}{2}p^-[x^+ - \Phi(x^i)] - p^ix^i\right\}}, x^- = 0^+$

$$= \int \frac{d^2k^i}{(2\pi)^2} e^{-i\left(\frac{(p^i+k^i)^2}{2p^-}x^- + \frac{p^-}{2}x^+ - (p^i+k^i)x^i\right)} A(k^i, p^-) \quad (x^- > 0)$$

- $G_{out}(p', x) = e^{i\left\{\frac{1}{2}p'^-[x^+ + \Phi(x^i)] - p'^ix^i\right\}}, x^- = 0^-$

$$= \int \frac{d^2k^i}{(2\pi)^2} e^{i\left(\frac{(p'^i-k^i)^2}{2p'^-}x^- + \frac{p'^-}{2}x^+ - (p'^i-k^i)x^i\right)} A(k^i, p'^-) \quad (x^- < 0)$$

- $$G^A_+ = \int \frac{d^2k^i}{(2\pi)^2} e^{-i\tilde{l}x} A(k^i, l^-)\varepsilon_+ \qquad G^A_- = \int \frac{d^2k^i}{(2\pi)^2} e^{-i\tilde{l}x} A(k^i, l^-)\left[\varepsilon_- + \frac{(k^i)^2}{(l^-)^2}\varepsilon_+ + \frac{\varepsilon_i k^i}{l^-}\right]$$

$$G^A_i = \int \frac{d^2k^i}{(2\pi)^2} e^{-i\tilde{l}x} A(k^i, l^-)\left[\varepsilon_i + \frac{2k^i}{l^-}\varepsilon_+\right] \qquad (\tilde{l}^-, \tilde{l}^+, \tilde{l}^i) = \left(l^-, \frac{(l^i - k^i)^2}{l^-}, l^i - k^i\right)$$

AP equations (roughly, many factors missing)

$$\frac{d\sigma}{d^2q_\perp dx} = \delta(x-1)\frac{1}{4\pi^2}\left|A_{reg}(q_\perp;p^-)\right|^2 + \frac{1}{(2\pi)^5}\delta(x-x(z))\frac{z\,dz}{1-z}\boxed{\int d^2l_\perp\left|A_i(q_\perp,l_\perp,z;p^-)\right|^2}$$

$$\propto \left\{\frac{1}{(n+1)}\log(b_c q)-\log(b_c\mu)\right\}|A_{reg}(q,zp^-)|^2 + \left\{\frac{1}{(n+1)}\log(b_c q)-\log(b_c\mu)\right\}|A_{reg}(q,(1-z)p^-)|^2 + \ldots$$

$$\frac{df_{PDF}}{d\log Q^2} = [\ldots] \quad\rightarrow\quad \boxed{\frac{df_{PDF}}{d\log Q^2} = \frac{1}{(n+1)}[\ldots]} \qquad Q^2 \gg \frac{1}{b_c^2}$$

($\leftrightarrow$ new scale)