Young Researchers Workshop
"Physics Challenges in the LHC Era"

QCD AMPLITUDES WITH THE GLUON EXCHANGE AT HIGH ENERGIES (AND GLUON REGGEIZATION PROOF)

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1. TERMINOLOGY: REGGEIZATION

- **Reggeization of any particle** assumes the *signaturized* amplitude to acquire the asymptotic behavior like $s^{j(t)}$, when exchanging this particle in the t-channel in Regge limit ($t \ll s$). Here $j(t) \equiv 1 + \omega(t)$ is the Regge trajectory with $\omega(0) = 0$.
- **Signature** in the channel t_l for multi-particle production means the (anti-) symmetrization with respect to the substitution $s_{i,j} \leftrightarrow -s_{i,j}$, for $i < l \leq j$.

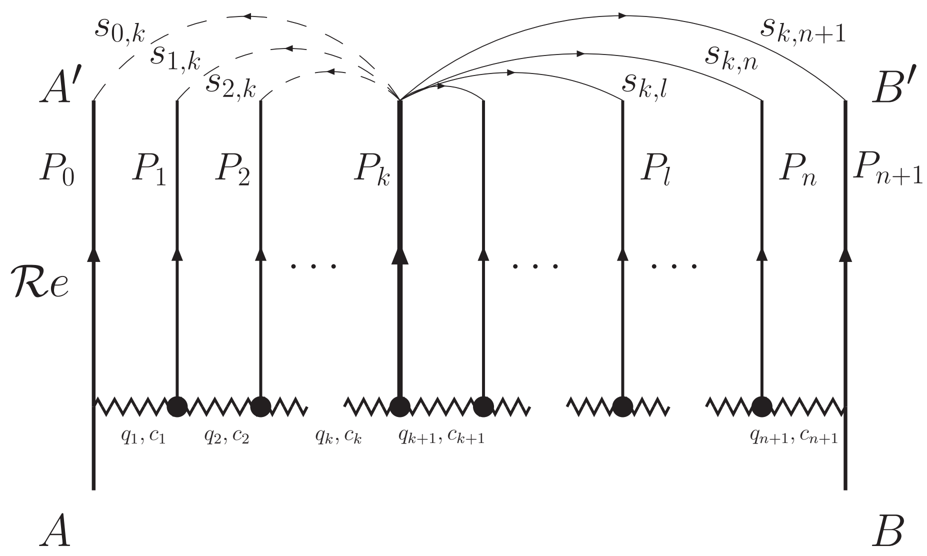


Fig.1 The amplitude for the process $2 \to n + 2$.

- **Hypothesis of the gluon reggeization** claims that in Regge limit the real part of the NLA-amplitude $2 \to n + 2$ with negative signature and with octet in all t_i-channels has universal form:

$$\mathcal{R}e\mathcal{A}_{AB}^{A'B'+n} = \bar{\Gamma}_{A'A}^{R_1} \left(\prod_{i=1}^{n} \frac{e^{\omega(q_i^2)(y_{i-1}-y_i)}}{q_{i\perp}^2} \gamma_{R_i R_{i+1}}^{J_i} \right) \frac{e^{\omega(q_{n+1}^2)(y_n - y_{n+1})}}{q_{(n+1)\perp}^2} \Gamma_{B'B}^{R_{n+1}}, \tag{1}$$

where $y_i = \frac{1}{2}\ln(\frac{k_i^+}{k_i^-})$ — particle (P_i) rapidities, and $\gamma_{R_i R_{i+1}}^{J_i}$, $\Gamma_{P'P}^{R}$ — known effective vertices.

2. THE PROOF IDEA: BOOTSTRAP RELATIONS AND CONDITIONS

- **Bootstrap relations:** There is infinite number of necessary and sufficient conditions for compatibility of the Regge amplitude form (1) with **unitarity**:

$$\mathcal{R}e\Big[\frac{1}{-2\pi i}\Big(\sum_{l=k+1}^{n+1}\text{disc}_{s_{k,l}}-\sum_{l=0}^{k-1}\text{disc}_{s_{l,k}}\Big)\mathcal{A}_{AB}^{A'B'+n}\Big]=\frac{1}{2}(\omega(t_{k+1})-\omega(t_k))\mathcal{R}e\mathcal{A}_{AB}^{A'B'+n} \qquad (2)$$

- **Bootstrap conditions:** There is finite number of identities making **all** bootstrap relations fulfilled. These conditions restrict effective vertices and the gluon trajectory.

The main goal is to prove them hold true.

- **Elastic** conditions constrain the kernel ($\hat{\mathcal{K}}$) and the effective vertex describing the transition of the initial particle (B) to the final one (B'):

$$|\bar{B}'B\rangle = g\Gamma_{B'B}^{R_{n+1}}|R_\omega(q_{B\perp})\rangle,$$

$$\hat{\mathcal{K}}|R_\omega(q_\perp)\rangle=\omega(q_\perp^2)|R_\omega(q_\perp)\rangle,$$

- **Inelastic** conditions appear in elementary ($2\longrightarrow 3$) inelastic amplitudes.

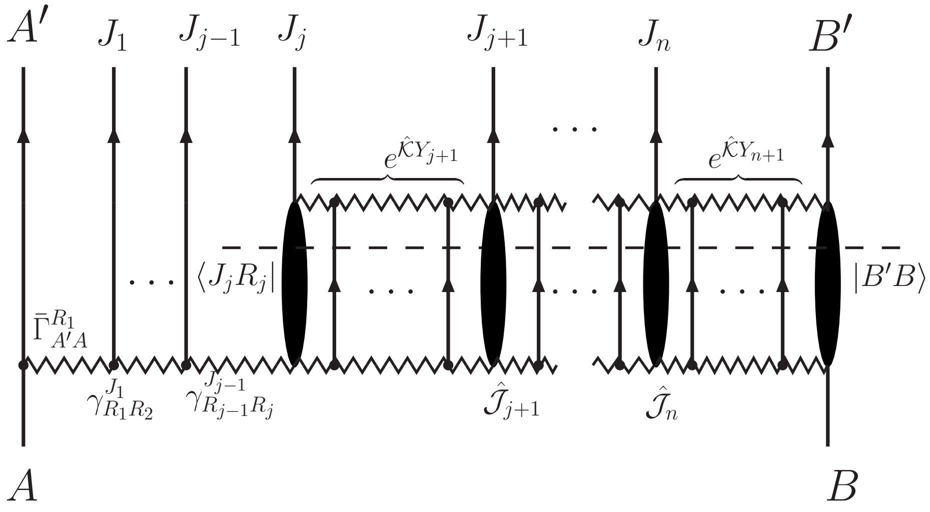
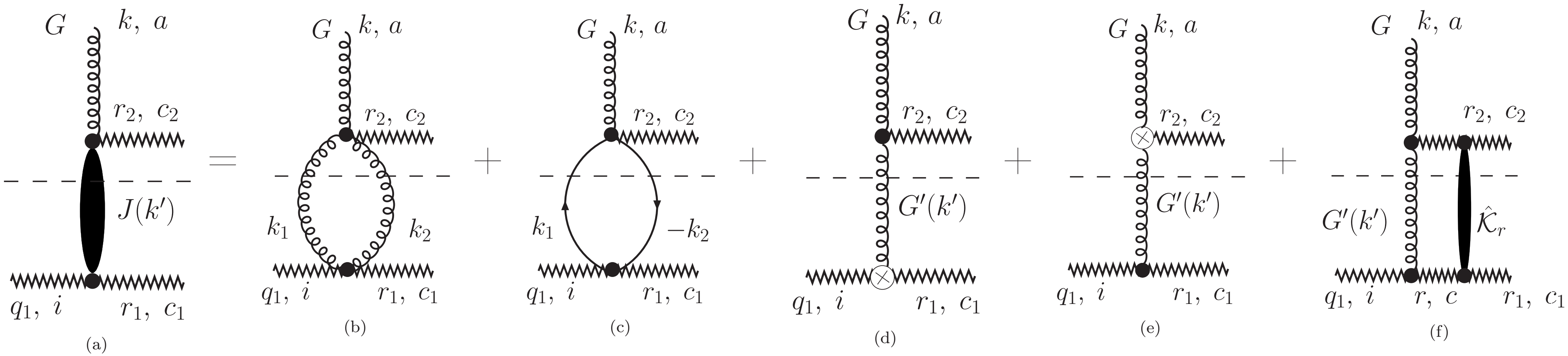


Fig.2 $s_{j,n+1}$-channel discontinuity calculation via unitarity relation

$$|\bar{J}_iR_{i+1}\rangle+\hat{\mathcal{J}}_i\,|R_\omega(q_{(i+1)\perp})\rangle\;g\;q_{(i+1)\perp}^2=|R_\omega(q_{i\perp})\rangle\;g\;\gamma_{R_iR_{i+1}}^{J_i}, \qquad (3)$$

3. INELASTIC CONDITION: $|\bar{J}_i R_{i+1}\rangle$ — NLO IMPACT-FACTOR

- **Impact factor of the jet production** is the first component of the inelastic bootstrap condition, intrinsically it appears as logarithmically non-enhanced term in the discontinuity (in $s_{0,1}$ or $s_{1,2}$ channel) for the process $2 \longrightarrow 3$.



Further we consider the most nontrivial NLO impact-factor: $J_j = G(k)$

$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle = \langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{v.c.} + \langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{loop}. \quad (4)$$

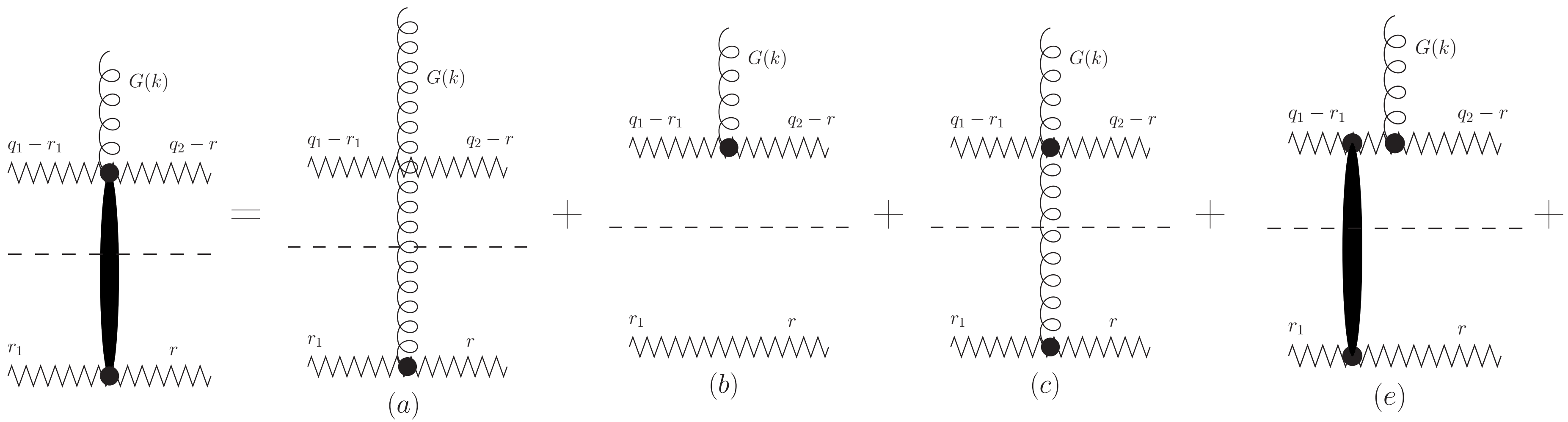
$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{v.c.} = \frac{\delta(q_{1\perp} - k_\perp - r_{1\perp} - r_{2\perp})}{2k^+} \sum_{G'} \Big[\gamma^{G'(C)}_{R_1\mathcal{G}_1}\Gamma^{\mathcal{G}_2(B)}_{GG'} + \gamma^{G'(B)}_{R_1\mathcal{G}_1}\Gamma^{\mathcal{G}_2(C)}_{GG'} - \gamma^{G'(B)}_{R_1\mathcal{G}_2}\Gamma^{\mathcal{G}_1(C)}_{GG'} - \gamma^{G'(C)}_{R_1\mathcal{G}_2}\Gamma^{\mathcal{G}_1(B)}_{GG'} + \gamma^{G'(B)}_{R_1\mathcal{G}_1}\Gamma^{\mathcal{G}_2(B)}_{GG'} \times$$

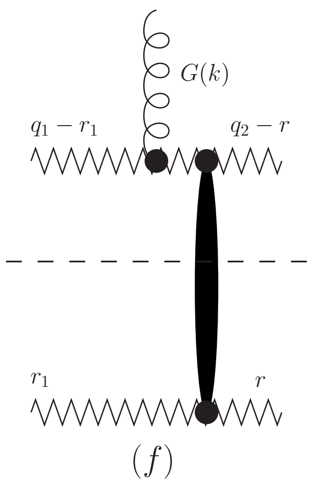
$$\times \left\{ \frac{1}{2}(\omega^{(1)}(q_1) - \omega^{(1)}(r_1)) \ln\left[\frac{k_\perp^2}{(q_1 - r_1)_\perp^2}\right] - \frac{\omega^{(1)}(r_2)}{2} \ln\left[\frac{k_\perp^2}{r_{2\perp}^2}\right] \right\} - \gamma^{G'(B)}_{R_1\mathcal{G}_2}\Gamma^{\mathcal{G}_1(B)}_{GG'} \Big\{ r_1 \leftrightarrow r_2 \Big\} \Big].$$

$$\langle GR_1|\mathcal{G}_1\mathcal{G}_2\rangle^{loop} = \frac{\delta(q_{1\perp} - k_\perp - r_{1\perp} - r_{2\perp})}{2k^+} \sum_{f=2g,q\bar{q}} \int \left(\gamma^{\{f\}}_{R_1\mathcal{G}_1}\Gamma^{\mathcal{G}_2}_{G\{f\}} - \gamma^{\{f\}}_{R_1\mathcal{G}_2}\Gamma^{\mathcal{G}_1}_{G\{f\}} \right) d\phi^{\Delta}_{\{f\}} - \Delta\langle GR_1|\hat{\mathcal{K}}^B_r|\mathcal{G}_1\mathcal{G}_2\rangle^B.$$

4. INELASTIC CONDITION: ONE GLUON PRODUCTION OPERATOR

- **One gluon production operator** is the second component of the inelastic bootstrap condition, describing the transition reggeon-reggeon state to gluon-reggeon-reggeon.





$$\langle \mathcal{G}_1'\mathcal{G}_2'|\hat{\mathcal{J}}_i^{\Delta}|\mathcal{G}_1\mathcal{G}_2\rangle = \delta(r_{1\perp}+r_{2\perp}-k_{i\perp}-r'_{1\perp}-r'_{2\perp})[\gamma^{J_i}_{\mathcal{G}_1\mathcal{G}_1'}\delta(r_{2\perp}-r'_{2\perp})r_{2\perp}^{2}\delta_{\mathcal{G}_2\mathcal{G}_2'}+$$

$$+\gamma^{J_i}_{\mathcal{G}_2\mathcal{G}_2'}\delta(r_{1\perp}-r'_{1\perp})r_{1\perp}^{2}\delta_{\mathcal{G}_1\mathcal{G}_1'}] + \sum_{G}\int_{y_i-\Delta}^{y_i+\Delta}\frac{dz_G}{2(2\pi)^{D-1}}(\gamma^{\{J_iG\}}_{\mathcal{G}_1\mathcal{G}_1'}\gamma_G^{\mathcal{G}_2\mathcal{G}_2'}+\gamma^{G}_{\mathcal{G}_1\mathcal{G}_1'}\gamma^{\mathcal{G}_2\mathcal{G}_2'}_{J_iG}).$$

5. PREVIOUSLY OBTAINED RESULTS:

- By the direct calculation we (V.S., M.G., A.V.) demonstrated that the inelastic bootstrap condition was fulfilled being projected onto the colour octet in the t-channel:

$$f^{ac'_1c'_2}\Big[\langle \mathcal{G}'_1\mathcal{G}'_2|\hat{\mathcal{J}}_i\,|R_\omega(q_{(i+1)\perp})\rangle\, g\, q^2_{(i+1)\perp} + \langle \mathcal{G}'_1\mathcal{G}'_2|\bar{J}_i R_{i+1}\rangle\Big] = f^{ac'_1c'_2}\langle \mathcal{G}'_1\mathcal{G}'_2|R_\omega(q_{i\perp})\rangle\, g\, \gamma^{J_i}_{R_iR_{i+1}}.$$

- We introduced the operator formulation of the bootstrap reggeon formalism: for bootstrap relations and conditions. Further we conjectured that the following condition is valid for arbitrary color representation:

$$\hat{\mathcal{J}}_i\,|R_\omega(q_{(i+1)\perp})\rangle\, g\, q^2_{(i+1)\perp} + |\bar{J}_i R_{i+1}\rangle = |R_\omega(q_{i\perp})\rangle\, g\, \gamma^{J_i}_{R_iR_{i+1}}. \quad (5)$$

- In V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko, Phys. Lett.**B** 639 (2006) using the direct discontinuity calculation through the unitarity in terms of our components

$$\begin{aligned} &-4i(2\pi)^{D-2}\delta(q_{(j+1)\perp} - q_{i\perp} - \sum_{l=i}^{l=j} k_{l\perp})\ \mathrm{disc}_{s_{i,j}} A^{\mathcal{S}}_{2\to n+2} = \bar{\Gamma}^{R_1}_{A'A}\frac{e^{\omega(q_1)(y_0-y_1)}}{q^2_{1\perp}}\left(\prod_{l=2}^{i}\gamma^{J_{l-1}}_{R_{l-1}R_l}\frac{e^{\omega(q_l)(y_{l-1}-y_l)}}{q^2_{l\perp}}\right)\times \\ &\times\langle J_iR_i|\left(\prod_{l=i+1}^{j-1} e^{\hat{\mathcal{K}}(y_{l-1}-y_l)}\hat{\mathcal{J}}_l\right)e^{\hat{\mathcal{K}}(y_{j-1}-y_j)}|\bar{J}_jR_{j+1}\rangle\left(\prod_{l=j+1}^{n}\frac{e^{\omega(q_l)(y_{l-1}-y_l)}}{q^2_{l\perp}}\gamma^{J_l}_{R_lR_{l+1}}\right)\frac{e^{\omega(q_{n+1})(y_n-y_{n+1})}}{q^2_{(n+1)\perp}}\Gamma^{R_{n+1}}_{B'B}, \end{aligned} \quad (6)$$

  and applying granted elastic and inelastic NLO bootstrap conditions we proved all bootstrap relations to be fulfilled. So the last millstone on the way of reggeization proof was the validity of (5).

6. DIFFERENT COLOUR REPRESENTATIONS IN T-CHANNEL:

This last unproved bootstrap condition (J_i is one gluon) can be present in projected form:

$$\langle \mathcal{G}_1' \mathcal{G}_2' | \hat{\mathcal{J}}_i \, | R_\omega(q_{(i+1)\perp}) \rangle \; g \, q_{(i+1)\perp}^2 + \langle \mathcal{G}_1' \mathcal{G}_2' | \bar{J}_i R_{i+1} \rangle = \langle \mathcal{G}_1' \mathcal{G}_2' | R_\omega(q_{i\perp}) \rangle \; g \, \gamma_{R_i R_{i+1}}^{J_i},$$

First of all, it was proved for the octet (the most important) in the t-channel. There is no r.h.s. for any other t-channel representations $R \neq 8$, since from the explicit form of $|R_\omega(q_{i\perp})\rangle$:

$$\mathcal{P}_{c_1',c_2'}^{(R)c_1,c_2} \langle \mathcal{G}_1' \mathcal{G}_2' | R_\omega(q_{i\perp}) \rangle \; g \, \gamma_{R_i R_{i+1}}^{J_i} = 0. \tag{7}$$

From the explicit form of the effective vertices it is easy to see that there are only THREE nontrivial colour structures into the operator of the gluon production $\langle \mathcal{G}_1' \mathcal{G}_2' | \hat{\mathcal{J}}_i \, | R_\omega(q_{(i+1)\perp}) \rangle$ and into the impact-factor $\langle \mathcal{G}_1' \mathcal{G}_2' | \bar{J}_i R_{i+1} \rangle$. The optimal choice is the "trace-based":

$$\mathrm{Tr}[T^{c_2} T^a T^{c_1} T^i], \;\; \mathrm{Tr}[T^a T^{c_2} T^{c_1} T^i], \;\; \mathrm{Tr}[T^a T^{c_1} T^{c_2} T^i] \tag{8}$$

- The first colour structure is symmetric with respect to c_1, c_2 and can be reduced: $\mathrm{Tr}[T^{c_2} T^a T^{c_1} T^i] = \frac{N_c^2}{2} \mathcal{P}^{(0)} + \frac{N_c+2}{2} \mathcal{P}^{(27)} + \frac{2-N_c}{2} \mathcal{P}^{(N_c>3)}$ Corresponding coefficient had not been calculated before. **It is the last problem on the reggeization proof way.**
- The coefficient at second colour structure $\mathrm{Tr}[T^a T^{c_2} T^{c_1} T^i]$ is very similar to the octet case, and whereby is considered to be calculated.
- The last coefficient (at $\mathrm{Tr}[T^a T^{c_1} T^{c_2} T^i]$) can be easily obtained from the previous one.

7. RESULTS AND PLANS:

- We formulated the gluon reggeization proof in operating form through the bootstrap approach based on unitarity.
- We proved all bootstrap conditions (necessary for reggeization proof) to be correct when projecting on octet colour representation. But it is not sufficient for the final proof!
- We proved all bootstrap conditions to be correct for second and third colour structure and for all colour structures in fermionic sector of the inelastic bootstrap condition.
- We calculated in the dimensional regularization all components of the last coefficient at the symmetric colour structure for the inelastic bootstrap condition.
- For this structure we demonstrated the cancellation of all singular terms (collinear regularization, $\frac{1}{\epsilon^2}$, and $\frac{1}{\epsilon}$), rational, and logarithmic terms.
- **We are planing** to demonstrate the cancellation for dilogarithmic and double logarithmic terms in the inelastic bootstrap condition in the nearest future.

THANKS FOR YOUR ATTENTION!