Divergence Estimation Procedure and Calculation

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Abstract

Link budgets for many of the ILRS sites are estimated using divergence values that are derived from the site logs. Actual data for calculating the station divergence is often incomplete or very optimistic and based on diffraction theory from the full size of the primary mirror for monostatic systems or the full size of the Coude path and beam expander for bistatic systems.

Accurate divergence measurements and a standard method of measuring the divergence is needed by the ILRS for several reasons, including GNSS array requirements and performance prediction and reliable prediction of the energy density delivered on target for the entire ILRS network to deal with requests for information for potential new satellites.

A procedure was developed and presented at the last ILRS meeting for scanning over azimuth and elevation on satellites and using a graphical procedure to estimate the divergence. In this presentation, an equation has been derived from the laser radar equation for the number of photoelectrons detected which allows calculation of the $1/e^2$ divergence from the scan data directly without estimating from graphs. This method will reduce the subjectivity in the estimation and will also allow the measurement to be automated. Data from several stations which responded last year with divergence scan data has been used to test this method and results will be presented.

Divergence equation discussion and derivation

Assume that scans are done on two satellites at different ranges but *under very similar atmospheric conditions*. The scan is done in AZ and EL up to the points where NPE ~ 0 on both sides of the scan.

At the peak of the return power, NPE for both satellites is proportional to σ/R^4 , but NPE_1 does not equal NPE_2. NPE refers to the number of photoelectrons in the laser radar equation, σ is the satellite cross section, and R is the slant range. At the half angle points of the scan, we still have NPE proportional to σ/R^4 but also with $NPE_1 = NPE_2 \sim 0$. Therefore by equating the expressions for NPE and assuming that most of the terms in the equation for NPE are approximately the same during the measurements, we can derive an expression for the divergence. This is why it is important to take the measurements fairly quickly under the same sky conditions and, if possible, in the same region of the sky. This will allow atmospheric propagation parameters in the laser radar equation to be treated as approximately constant for the two different satellite scans.

In the laser radar equation, the expression for transmitter gain is $G_t = (8/\theta_t^2)_*\exp[-2(\theta/\theta_t)^2]$ where θ_t is the far field divergence half angle between the beam center and the $1/e^2$ intensity point and θ is the beam pointing error, or in this case, the half angle of the measured scan. If we assume that the scan measurements are taken on two satellites in the same region of the sky quickly enough, then all factors in the laser radar expression for NPE are approximately constant with the exception of σ, R, and G_t which changes due to the pointing error change (i.e. due to the scan). The expression for NPE is:

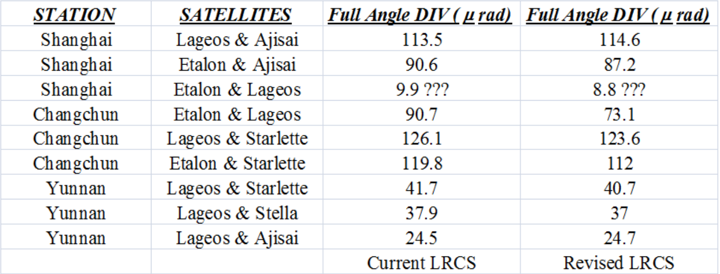
$NPE = \eta_{e*}(E_{r*}\lambda/hc)_*\eta_{t*}G_{t*}\sigma_*(1/4\pi R^2)^2{}_*A_{r*}\eta_{r*}T_a^2{}_*T_c^2$. Under the above assumption, this expression can be reduced to $NPE = K_*(\sigma/R^4)_*\exp[-2(\theta/\theta_t)^2]$ where K is an approximate constant for the two scans.

If we equate the NPE expressions for the endpoints of the scans on the two satellites, $NPE_1 = NPE_2 = 0$, then we can solve for θ_t. The result is $\theta_t^2 = 2(\theta_1^2 - \theta_2^2) / \ln[(\sigma_1/\sigma_2)_*(R_2^4/R_1^4)]$. This is an estimate of the divergence in terms of known quantities: the satellite cross-sections, satellite ranges, and the measured scan angles. The expression derivation and the scanning procedure are discussed in more detail in the attached PowerPoint slides.

Divergence estimates from contributed scanning data

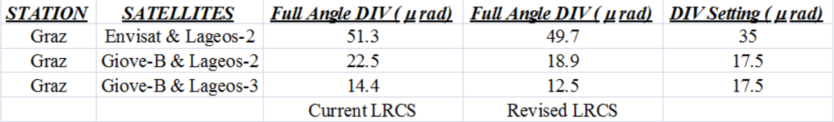
Several stations responded to last year's request for scanning data and this data has been used to test the derived expression for estimating the full angle divergence of the transmitted beam. In some cases, some of the necessary data such as slant range and / or elevation angle was not sent and these had to be estimated. This is shown in more detail in the attached PowerPoint slides. The majority of the contributed data yielded reasonable results that were fairly consistent with the exception of one scan submitted from the Shanghai station. The cause of this inconsistency is currently unknown. The results from the submitted data are shown below.

Divergence estimates from data reported by Chinese Stations:

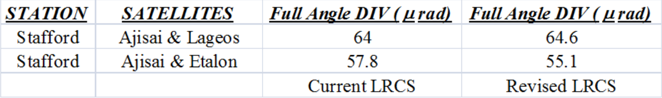


STATION	SATELLITES	Full Angle DIV (µ rad)	Full Angle DIV (µ rad)
Shanghai	Lageos & Ajisai	113.5	114.6
Shanghai	Etalon & Ajisai	90.6	87.2
Shanghai	Etalon & Lageos	9.9 ???	8.8 ???
Changchun	Etalon & Lageos	90.7	73.1
Changchun	Lageos & Starlette	126.1	123.6
Changchun	Etalon & Starlette	119.8	112
Yunnan	Lageos & Starlette	41.7	40.7
Yunnan	Lageos & Stella	37.9	37
Yunnan	Lageos & Ajisai	24.5	24.7
		Current LRCS	Revised LRCS

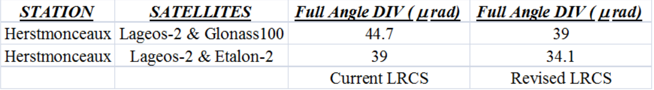
Divergence estimates from data reported by Graz, Stafford, & Herstmonceux:



STATION	SATELLITES	Full Angle DIV (µ rad)	Full Angle DIV (µ rad)	DIV Setting (µ rad)
Graz	Envisat & Lageos-2	51.3	49.7	35
Graz	Giove-B & Lageos-2	22.5	18.9	17.5
Graz	Giove-B & Lageos-3	14.4	12.5	17.5
		Current LRCS	Revised LRCS	



STATION	SATELLITES	Full Angle DIV (µ rad)	Full Angle DIV (µ rad)
Stafford	Ajisai & Lageos	64	64.6
Stafford	Ajisai & Etalon	57.8	55.1
		Current LRCS	Revised LRCS



STATION	SATELLITES	Full Angle DIV (µ rad)	Full Angle DIV (µ rad)
Herstmonceaux	Lageos-2 & Glonass100	44.7	39
Herstmonceaux	Lageos-2 & Etalon-2	39	34.1
		Current LRCS	Revised LRCS

Concluding Remarks

A simple calculation for estimation of divergence has been derived from the standard laser radar equation for number of photoelectrons. Assumptions made in the derivation require care in taking the data for the estimation to be valid. The results will differ depending on atmospheric transmission and other conditions at the SLR station. This method should be used as an estimate to obtain values for average divergence, maximum and minimum, and to determine the general health of the station optical train.

References

1. “Millimeter Accuracy Satellite Laser Ranging: A Review”, John J. Degnan, Contributions of Space Geodesy to Geodynamics: Technology Geodynamics 25, American Geophysical Union, 1993.
2. “Cross section of ILRS satellites”, David A. Arnold