A photon-in-cell code for laser wakefield accelerators

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Alpha-X Theory Group

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Background

Nonlinear wave driven processes in plasmas are normally described by

- **A monochromatic pump wave coupling to other mono-chromatic waves**
- **Random phase waves coupling to other random phase waves**

Alternative:

- **Random or broadband wave couples to monochromatic or coherent structures.**

Introduction

- **Wave kinetics: a novel approach to the interaction of waves with plasmas [1-5].**

- **A monochromatic wave is described as a compact distribution of quasi-particles.**

- **Broadband turbulence is described as a gas of quasi-particles (photons, plasmons, driftons, etc.)**

[1] R. Trines *et al.*, in preparation (2004).

[2] J. T. Mendonça, R. Bingham, P. K. Shukla, Phys. Rev. E 68, 0164406 (2003).

[3] L.O. Silva *et al.*, *IEEE Trans. Plas. Sci.* **28**, 1202 (2000).

[4] R. Bingham *et al.*, Phys. Rev. Lett. **78**, 247 (1997).

[5] R. Bingham *et al.*, Physics Letters A **220**, 107 (1996).

For complex phenomena

- Photon acceleration/deceleration;
- Photon wave breaking
- Photon turbulence;
- Modulational instabilities
- Steepening/compression of short pulses

Applications

Applications in laser-plasma interaction…

- Laser-plasma accelerators
- Broadband laser-plasma interactions – Induced Spatial Incoherence
- Strong plasma turbulence – Langmuir wave collapse

… and many other fields

- Zonal flows in a "drifton gas" – anomalous transport driven by density/temperature gradients
- Turbulence in planetary, stellar atmospheres – Rossby waves
- Plasma waves driven by neutrino bursts
- The list goes on

Motivation

- Develop a PIC-type code for quasi-particle model of short-wavelength modes (photons, drift modes, …) in a fluid plasma
- Study of photon-driven wakefields for accelerators.

Basic features

- Less computationally demanding than a full-PIC description;
- A much simpler description than the "slowly evolving envelope approximation;"
- Gives better physical insight into the EM field dynamics;
- Allows for an easy description of broadband, incoherent light pulses

Liouville theory

We define the quasiparticle density as the wave energy density divided by the quasiparticle energy:

$$N(\vec{k},\vec{r},t) = W(\vec{k},\vec{r},t)/\omega(k)$$

The number of quasiparticles is conserved:

$$\frac{d}{dt}\int N(\vec{k},\vec{r},t)\,d\vec{r}\,d\vec{k} = 0$$

Then we can apply Liouville's theorem:

$$\frac{d}{dt}N(\vec{k}(t),\vec{r}(t),t) = \left(\frac{\partial}{\partial t} + \vec{v}\frac{\partial}{\partial \vec{r}} + \vec{F}\frac{\partial}{\partial \vec{k}}\right)N(\vec{k}(t),\vec{r}(t),t) = 0$$

where $\vec{v} = d\vec{x}/dt = \partial\omega/\partial\vec{k}$ **and** $\vec{F} = d\vec{k}/dt = -\partial\omega/d\vec{x}$ **are obtained from the q.p. dispersion relation**

Condensed theory

- **Fluid model for the plasma (density n, momentum p):**

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}\left(\frac{np}{\gamma}\right) = 0 \qquad \frac{\partial p}{\partial t} + \frac{\partial \gamma}{\partial x} = -E \qquad \frac{\partial E}{\partial x} = 1 - n \qquad \gamma = \sqrt{1 + p^2 + A^2}$$

- **For a given photon density N(t,x,k), we have:**

$$A^2 = \int \frac{N(t,x,k)}{\omega_k} dk$$

- **Particle model for the photons:**
  - **Photon number conservation;**
  - **Hamiltonian:** $\omega_i = \sqrt{\omega_p^2(t,x_i) + k_i^2}$
  - **Equations of motion:**

$$\frac{dx_i}{dt} = \frac{k_i}{\omega_i} \qquad \frac{dk_i}{dt} = \frac{1}{2\omega_i}\frac{\partial}{\partial x}\left(\frac{n}{\gamma}\right)(x_i,t)$$

Example: solitons (solitary pulses)

- **Soliton: a pulse exciting a relativistic plasma wave of only one bucket [6];**
- **All the pulse's photons are confined to that bucket;**
- **Simultaneous acceleration and deceleration of photons keeps the pulse together;**
- **Solitons are particularly well suited to be studied through the photon kinetic approach.**

[6] P.K. Kaw, A. Sen, and T. Katsouleas, *Phys. Rev. Lett.* **68**, 3172, 1992.

1-D Hybrid Code

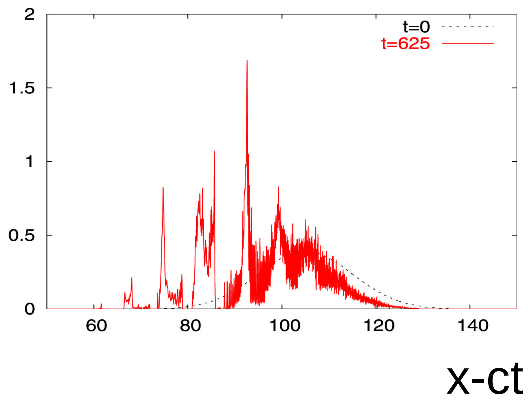
- Monospaced grid, 3000 cells, up to 100 000 particles;
- Second order Runge-Kutta methods to solve both particle and fluid equations;
- Quadratic splines for photon and field projection;
- A four-pass binomial filter on the vector potential;
- Moving simulation window: important for wakefield studies, Raman instabilities, etc.;
- Arbitrary shapes for the initial plasma and photon density profiles;
- The non-linear approach allows a peak plasma density of $n/n_0 = 10$ before breakdown occurs!

Results 1: long pulse

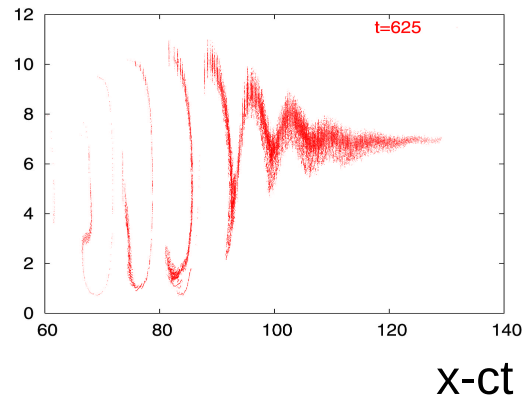
Modulational instability

- A long, low-intensity laser pulse has been injected into an underdense plasma ($k_0/k_p = 7$);
- Modulational instability clearly visible in all plots;
- In addition, “wave breaking” of a photon wave, photon bunching, and photon turbulence, can be seen.

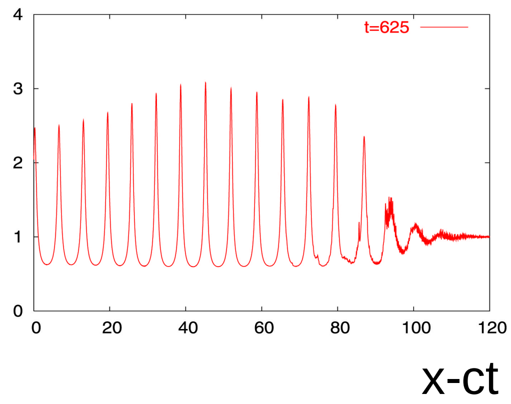
(a) Pulse envelope



(b) Photon phase space



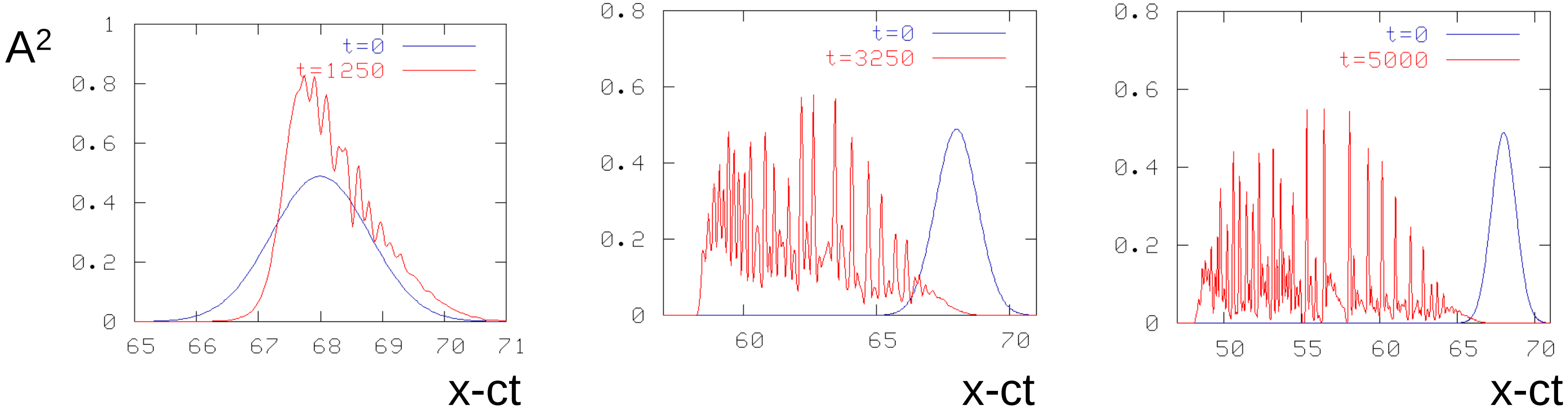
(c) Plasma electron density



Results 2: single short pulse

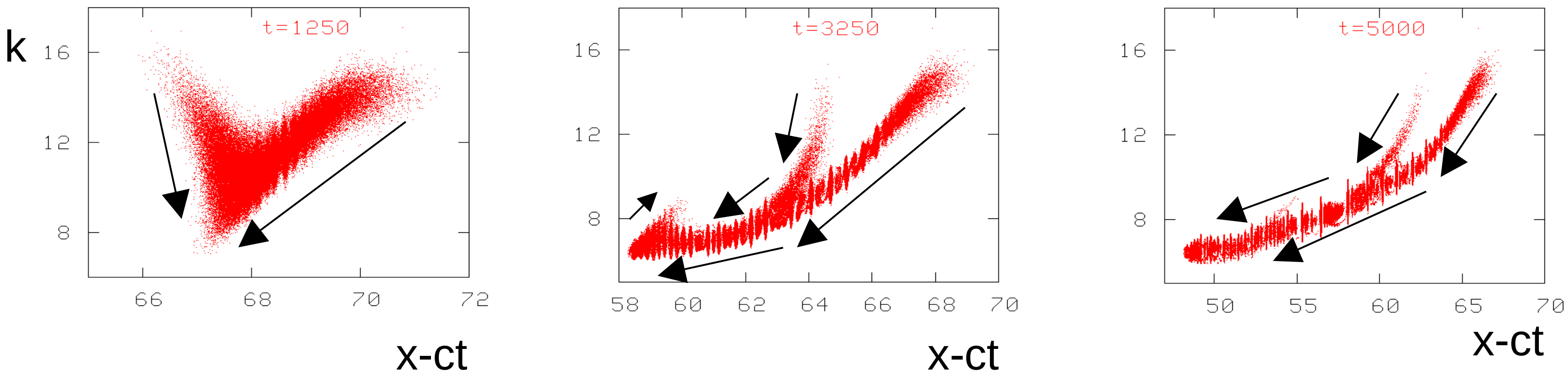
Wakefield excitation

- **A short, low-intensity laser pulse has been injected into an underdense plasma (k_0/k_p = 14);**
- **Plots of the pulse envelope show initial steepening…**
- **…followed by stretching and flattening of the pulse.**



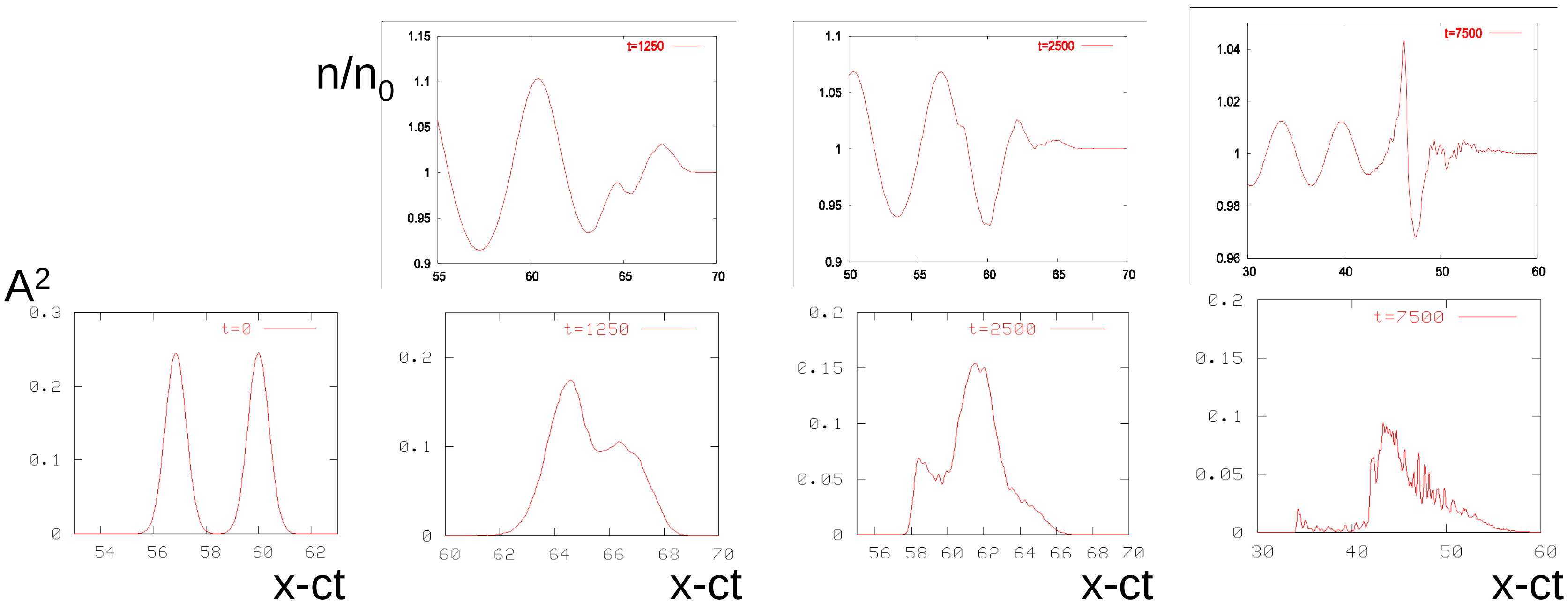
Single short pulse (cont'd)

- In photon phase space, there is initial photon deceleration, explaining the pulse steepening…
- …while later on, the photon bunch is stretched and chirped, explaining the pulse stretching;
- Clearly we are far removed from a soliton solution.



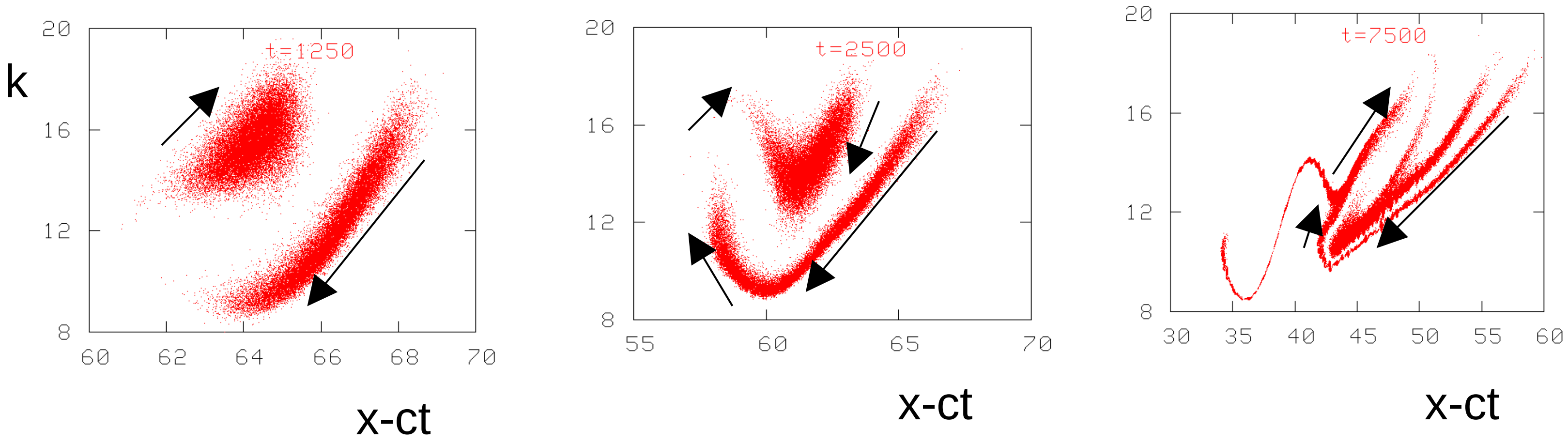
Results 3: double short pulse

- Non-linear soliton solutions often display a train of peaks in the pulse envelope...
- ...which motivated us to use a pulse with two very short peaks, crammed into a single plasma period;
- Pulse envelope plots already show that the pulse energy is confined better.



Double short pulse (cont'd)

- In photon phase space, we see that when one peak is decelerated, the other is accelerated...
- ...leading to a much more balanced energy exchange between photons and plasma...
- and we obtain a mostly stable pulse:
- Towards the soliton!



Conclusions

Photon kinetics…

- Is well suited for the study of interaction of EM waves with underdense plasmas,
- For 'old' phenomena: pulse steepening / compression and modulational instabilities,
- And 'new' phenomena: photon acceleration, photon wave breaking and photon turbulence, soliton formation,…
- Leads to improved physical insight through extended diagnostics,

Future work:

- 2-D / 3-D simulations
- other types of fast waves (drift waves, Langmuir waves)

Photon kinetics: a valuable new perspective on wave-plasma interaction!