MesoBioNano- Science

Radiation from a modulated positron beam in the Crystalline Undulator

Andriy Kostyuk
Andrei Korol
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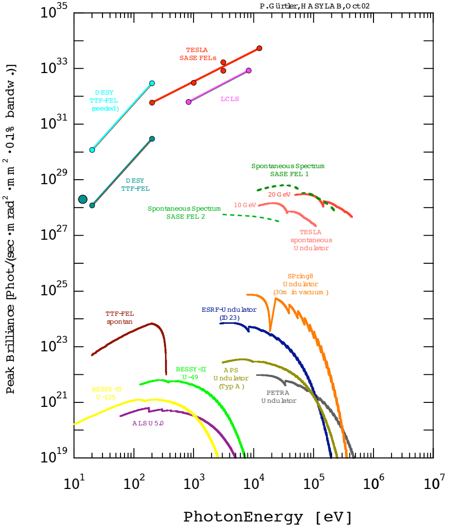
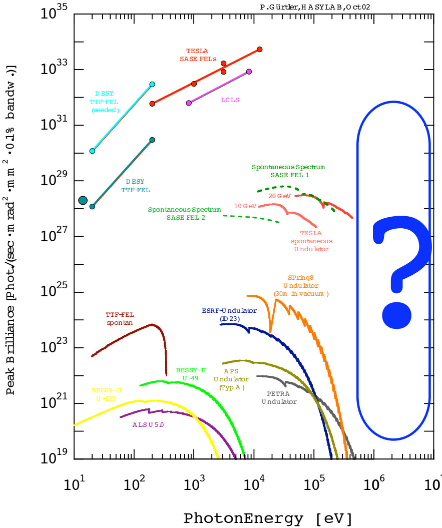
Motivation



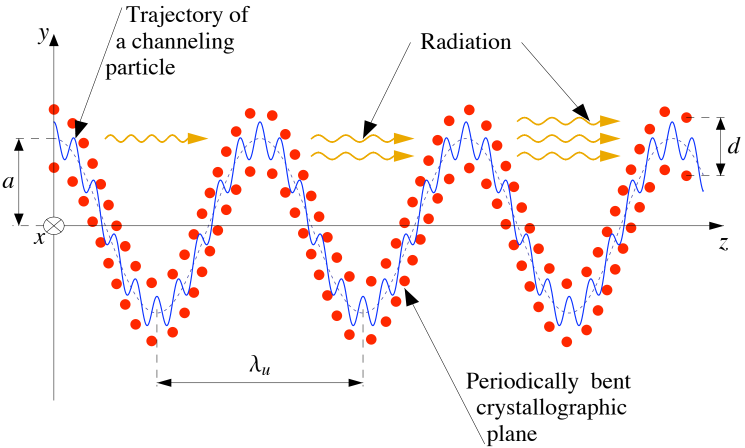
Based on
R.Brinkmann *et al.* (Ed.) 'Tesla FEL. Technical design report. Supplement.'

Motivation



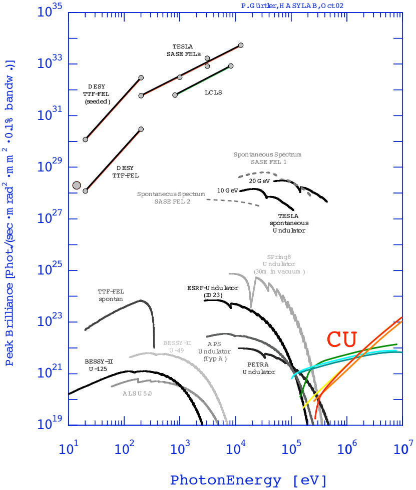
P.Gürtler, HASYLAB, Oct 02
Peak Brilliance [Phot./(sec·mrad²·mm²·0.1% bandw.)]
PhotonEnergy [eV]
TESLA SASE FELs
DESY TTF-FEL (seeded)
LCLS
DESY TTF-FEL
Spontaneous Spectrum SASE FEL 1
20 GeV
10 GeV
Spontaneous Spectrum SASE FEL 2
TESLA spontaneous Undulator
SPring8 Undulator (30m in vacuum)
TTF-FEL spontan
ESRF-Undulator (ID 23)
BESSY-II U-49
APS Undulator (Typ A)
PETRA Undulator
BESSY-II U-125
ALS U5.0
?

Crystalline Undulator



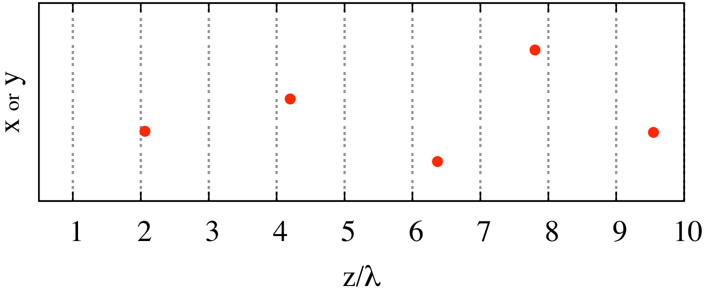
A.V. Korol, A.V. Solov'yov, W. Greiner,
J. Phys. G **24**, L45 (1998);
Int. J. Mod. Phys. E **8**, 49 (1999).

Crystalline Undulator



A.V. Korol, A.V. Solov'yov, W. Greiner Topics in Heavy Ion Phys. (2005) 73–86.

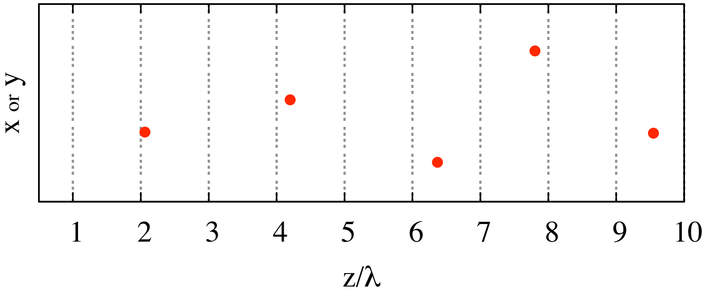
A.V. Korol, A.P. Kostyuk, A.V. Solov'yov, W. Greiner (2008), in preparation

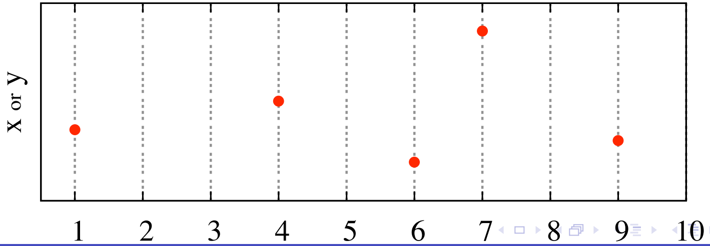


$$E_0^2 = E_{01}^2 + E_{02}^2 + E_{03}^2 + E_{04}^2 + \cdots \propto nE_{01}^2 \quad (1)$$

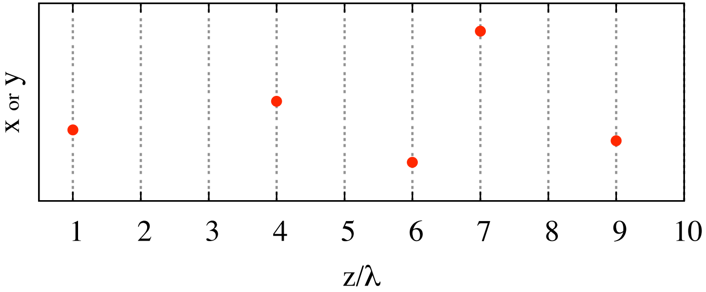
$$\text{Radiation Flux} = c\frac{E_0^2}{8\pi} \propto n \quad (2)$$

Undulator vs Laser





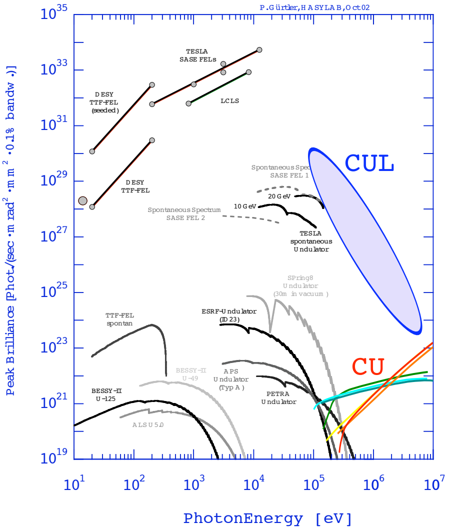
Particles in a Laser



$$E_0 = E_{01} + E_{02} + E_{03} + E_{04} + \cdots \propto nE_{01} \tag{3}$$

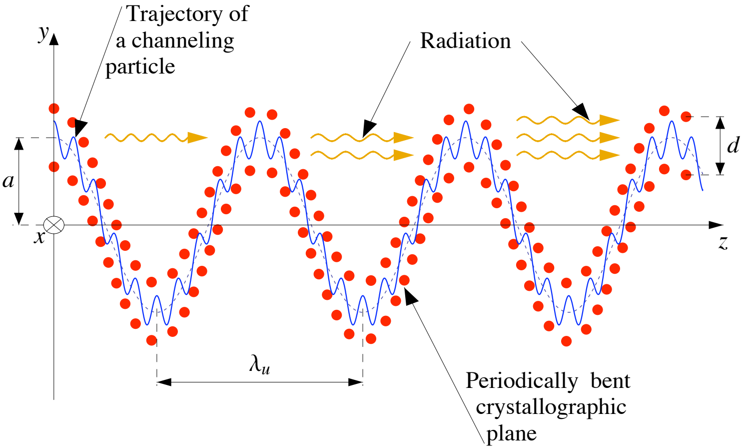
$$\text{Radiation Flux} = c\frac{E_0^2}{8\pi} \propto n^2 \tag{4}$$

Expected Brilliance

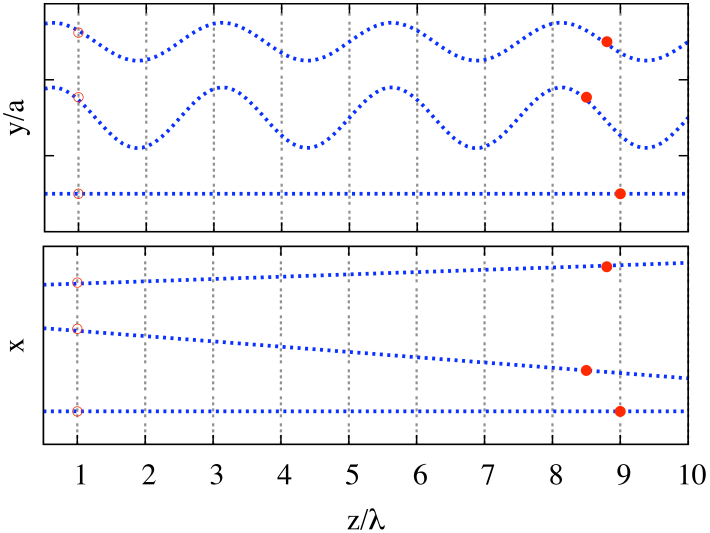


Will the beam preserve its modulation in a crystalline channel at sufficiently large penetration depth?

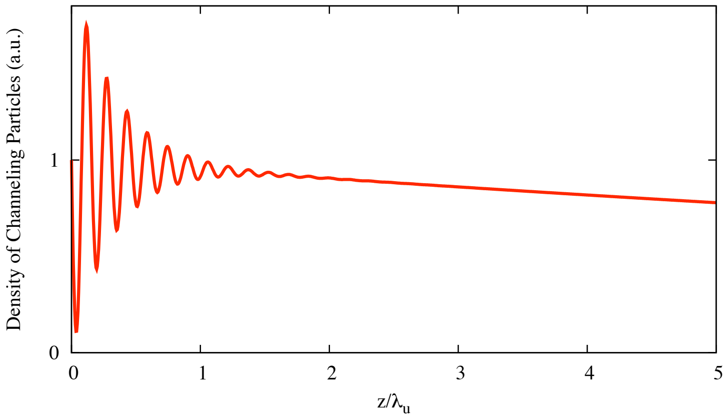
Channeling Oscillations



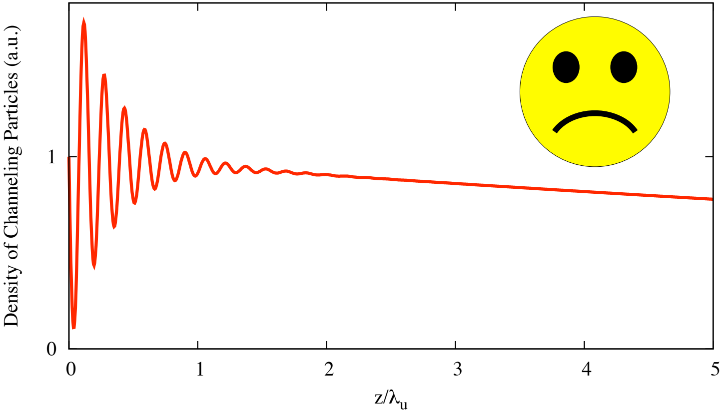
Beam Demodulation



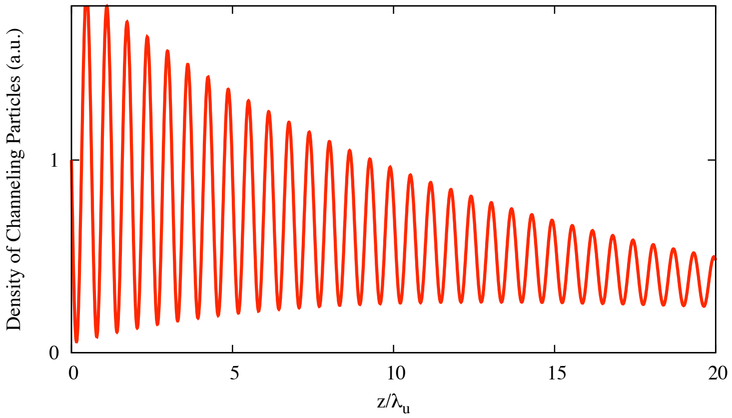
Demodulation within one Undulator Period



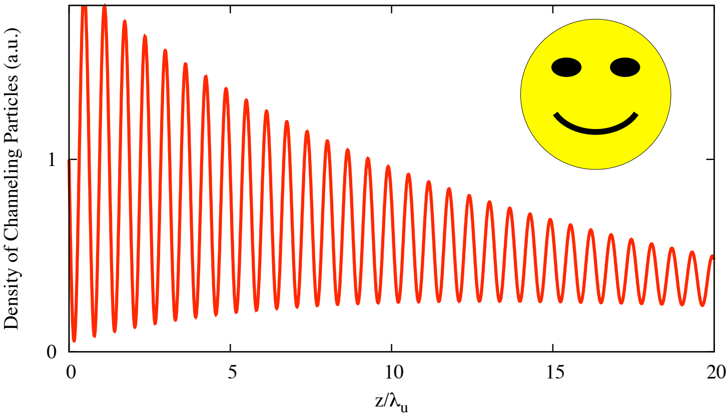
Demodulation within one Undulator Period



Demodulation within Dozens of Undulator Periods



Demodulation within Dozens of Undulator Periods



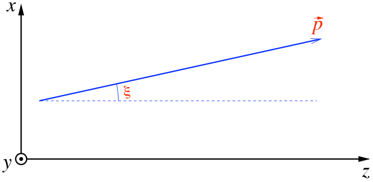
What is the characteristic length at which the beam gets demodulated?

Is it possible to build a crystalline undulator with $L_{\mathrm{dm}}/\lambda_u \gg 1$?

Particle Distribution

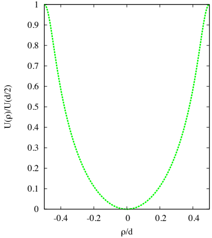
$f(t, z; \xi, E_y)$ is the distribution function of channeling particles with respect to the angle

$$\xi = \frac{p_x}{p}$$



and the transverse energy

$$E_y = \frac{p_y^2}{2E} + U(y)$$



The distribution depends on the penetration depth z and time t.

Kinetic Equation of Fokker-Planck Type

$$\frac{1}{c}\frac{\partial f}{\partial t} + \frac{\langle v_z \rangle}{c}\frac{\partial f}{\partial z} = D_0 \left[\frac{\partial}{\partial E_y} \left(E_y \frac{\partial f}{\partial E_y} \right) + \frac{1}{E}\frac{\partial^2 f}{\partial \xi^2} \right]$$

$$D_0 = \frac{mc^2}{8} \langle n_e \rangle \int d\Theta \frac{d\sigma}{d\Theta} \Theta^2$$

Longitudinal Velocity:

$$\begin{aligned} \langle v_z \rangle &= c\sqrt{1 - \frac{1}{\gamma^2}} \left(1 - \frac{\xi^2}{2}\right) \left(1 - \frac{E_y}{2E}\right) \\ &\approx c \left(1 - \frac{1}{2\gamma^2} - \frac{\xi^2}{2} - \frac{E_y}{2E}\right) \end{aligned}$$

Kinetic Equation of Fokker-Planck Type

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Excluding the Time Variable

If the initial beam is periodically modulated it can be represented as a Fourier series with respect to the time:

$$f(t, z; \xi, E_y) = \sum_{j=-\infty}^{\infty} g_j(z; \xi, E_y) \exp(ij\omega t).$$

where $g_j^*(z; \xi, E_y) = g_{-j}(z; \xi, E_y)$
It is sufficient to consider only one harmonic. We substitute

$$f(t, z; \xi, E_y) = g(z; \xi, E_y) \exp(i\omega t)$$

$$i\frac{\omega}{c} g(z; \xi, E_y) + \frac{\langle v_z \rangle}{c} \frac{\partial g}{\partial z} = D_0 \left[\frac{\partial}{\partial E_y} \left(E_y \frac{\partial g}{\partial E_y} \right) + \frac{1}{E} \frac{\partial^2 g}{\partial \xi^2} \right]$$

Separation of variables

$$\frac{D_0}{E}\frac{1}{\Xi(\xi)}\frac{d^2\Xi(\xi)}{d\xi^2} - i\omega\frac{\xi^2}{2} = \mathcal{C}_\xi$$

$$\frac{D_0}{\mathcal{E}(E_y)}\frac{d}{dE_y}\left(E_y\frac{d\mathcal{E}(E_y)}{dE_y}\right) - i\omega\frac{E_y}{2E} = \mathcal{C}_y$$

$$\frac{1}{\mathcal{Z}(z)}\frac{d\mathcal{Z}(z)}{dz} + \frac{i\omega}{2\gamma^2} = \mathcal{C}_z$$

$$\mathcal{C}_z = \mathcal{C}_y + \mathcal{C}_\xi$$

The Solution

$$g(z;\xi,E_y) = \exp\left(-i\frac{\omega}{c}z\right)\sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\mathfrak{c}_{n,k}\Xi_n(\xi)\mathcal{E}_k(E_y)\mathcal{Z}_{n,k}(z),$$

$$\Xi_n(\xi) = H_n\left(\mathrm{e}^{i\pi/8}\sqrt[4]{\frac{\omega E}{2cD_0}}\,\xi\right)\exp\left(-\frac{1+i}{4}\sqrt{\frac{\omega E}{cD_0}}\xi^2\right).$$

$H_n(\dots)$ are Hermite's polynomials.

$$\mathcal{E}_k(E_y)=\exp\left(-\frac{1+i}{2}\sqrt{\frac{\omega}{cD_0E}}E_y\right)L_{\nu_k}\left((1+i)\sqrt{\frac{\omega}{cD_0E}}E_y\right)$$

$L_{\nu_k}(\dots)$ is Laguerre's function.

The Solution

$$g(z;\xi,E_y) = \exp\left(-i\frac{\omega}{c}z\right)\sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\mathfrak{c}_{n,k}\Xi_n(\xi)\mathcal{E}_k(E_y)\mathcal{Z}_{n,k}(z),$$

$$\mathcal{Z}_{n,k}(z) = \exp\left\{-\frac{z}{L_\mathrm{d}}\left[\alpha_k(\kappa) + (2n+1)\frac{\sqrt{\kappa}}{j_{0,1}}\right] -i\omega z\left[\frac{1}{2\gamma^2} + \theta_\mathrm{L}^2\beta_k(\kappa) + \theta_\mathrm{L}^2\frac{(2n+1)}{2j_{0,1}\sqrt{\kappa}}\right]\right\}$$

Dechanneling length:

$$L_\mathrm{d} = 4U_\mathrm{max}/(j_{0,1}^2 D_0) \qquad J_0(j_{0,1}) = 0$$

Lidhard's angle:

$$\theta_\mathrm{L} = \sqrt{2U_\mathrm{max}/E}$$

$$\kappa = \pi\frac{L_\mathrm{d}}{\lambda}\theta_\mathrm{L}^2$$

The Transcendental Equation for $\alpha_k(\kappa)$ and $\beta_k(\kappa)$

$$\kappa = \pi \frac{L_\mathrm{d}}{\lambda} \theta_\mathrm{L}^2$$

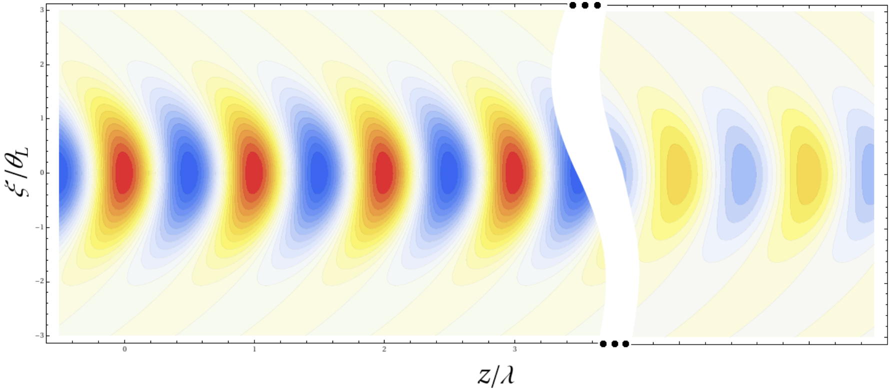
$$L_{\nu_k}\left(\frac{(1+i)}{2} j_{0,1}\sqrt{\kappa}\right) = 0.$$

$$\begin{aligned}
\alpha_k(\kappa) &= \frac{\sqrt{\kappa}}{j_{0,1}}\left(2\Re\left[(1+i)\nu_k\right]+1\right) \\
\beta_k(\kappa) &= \frac{1}{2 j_{0,1}\sqrt{\kappa}}\left(2\Im\left[(1+i)\nu_k\right]+1\right)
\end{aligned}$$

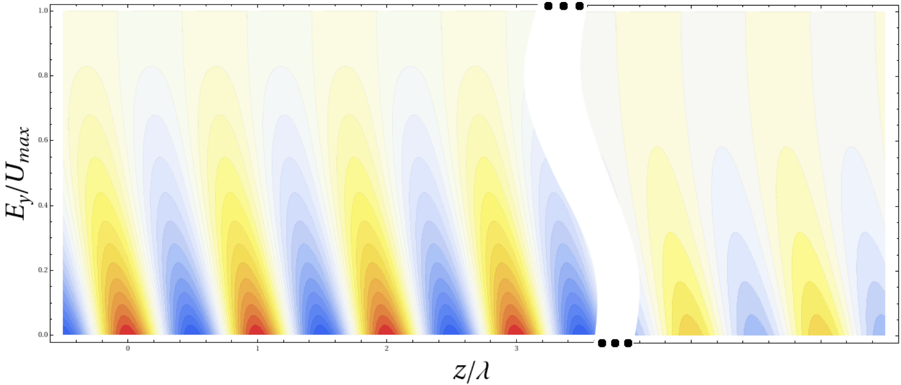
$$g(z;\xi,E_y) = \exp\left(-i\frac{\omega}{c}z\right)\sum_{n=0}^{\infty}\sum_{k=1}^{\infty}\mathfrak{c}_{n,k}\Xi_n(\xi)\mathcal{E}_k(E_y)\mathcal{Z}_{n,k}(z),$$

$$\begin{aligned}\mathcal{Z}_{n,k}(z) \quad = \quad \exp\Bigg\{ & -\frac{z}{L_\mathrm{d}}\left[\alpha_k(\kappa) + (2n+1)\frac{\sqrt{\kappa}}{j_{0,1}}\right] \\ & -i\omega z\left[\frac{1}{2\gamma^2} + \theta_\mathrm{L}^2\beta_k(\kappa) + \theta_\mathrm{L}^2\frac{(2n+1)}{2j_{0,1}\sqrt{\kappa}}\right]\Bigg\}\end{aligned}$$

$$g(z;\xi,E_y) \asymp \exp\left(-i\frac{\omega}{c}z\right)\mathfrak{c}_{0,1}\Xi_0(\xi)\mathcal{E}_1(E_y)\mathcal{Z}_{0,1}(z),$$


ξ/θ_L
z/λ
3
2
1
0
−1
−2
−3
0
1
2
3

Asymptotic Particle Distribution vs. E_y and z for $\kappa = 10$



Asymptotic Behaviour

$$g(z;\xi,E_y) \propto \exp\left(-z/L_{\mathrm{dm}} - i\omega/u_z\, z\right)$$

Demodulation Length:

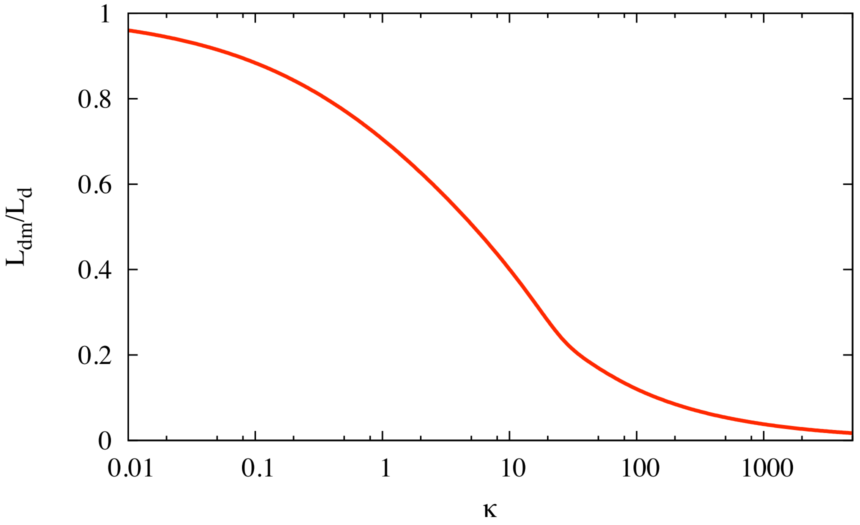
$$L_{\mathrm{dm}} = \frac{L_{\mathrm{d}}}{\alpha_1(\kappa) + \sqrt{\kappa}/j_{0,1}}$$

Phase velocity:

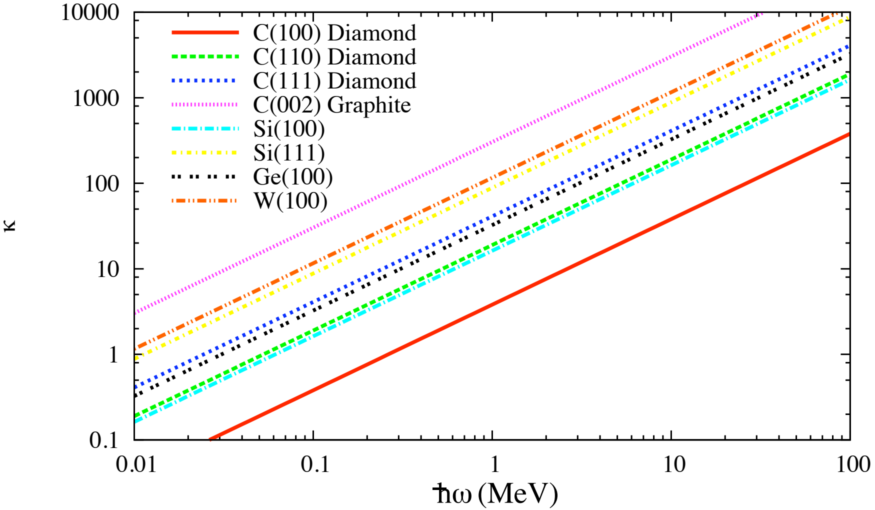
$$u_z = \left[1 + \frac{1}{2\gamma^2} + \theta_{\mathrm{L}}^2\left(\beta_k(\kappa) + \frac{1}{2j_{0,1}\sqrt{\kappa}}\right)\right]^{-1}$$

$$\kappa = \pi\frac{L_{\mathrm{d}}}{\lambda}\theta_{\mathrm{L}}^2$$

Demodulation Length



Parameter κ



Summary

- Beam demodulation is an important factor determining the physical limits on the applicability domain of the crystalline undulator based hard X ray and gamma ray laser.
- The beam evolution can be described by a partial differential equation of Fokker-Planck type which can be solved analytically for a parabolic interplanar potential.
- **A channeling beam can preserve its modulation at sufficiently large penetration depths so that coherent radiation in the crystalline undulator is feasible.**
- The bent crystal laser is most suitable for the application in the photon energy range of hundreds keV where the demodulation length not much smaller than the dechanneling length.

I thank the organizers for this great conference in this beautiful place.

Thank you for your attention.

This work has been supported by the European Commission (the PECU project, Contract No. 4916 (NEST)).

Supplement

Gamma Klystron